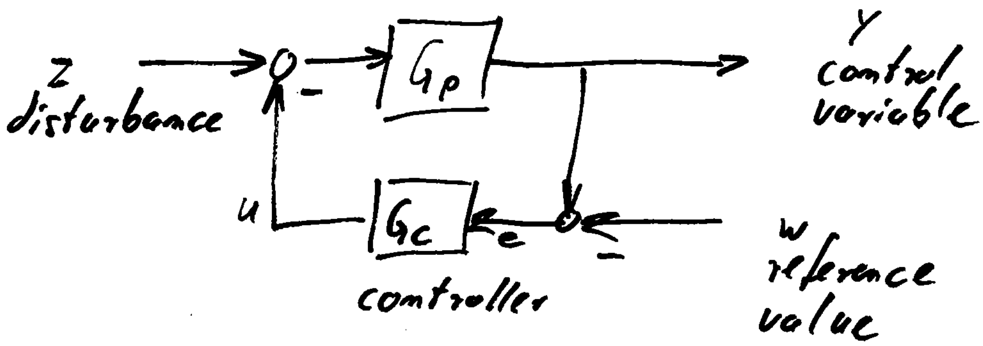
Problem 1

a) 

b) Closed-loop control uses feedback, open-loop control does not.

c) • SISO: <u>One</u> input and <u>one</u> output.

• Differential equations
• Transfer function

d) ODE $\circ\!\!-\!\!\bullet$ $a_3[s^3 y(s) - s^2 a - sb - c] + a_2[s^2 y(s) - sa - b] +$

$$+ a_1[s\,y(s) - a] + a_0\, y(s) = b_0\, u(s)$$

e) The parameters T_1 and T_t describe two different kinds of delay: a) A delay caused by inertia $\rightarrow T_1$
b) A time delay $\rightarrow T_t$

Problem 2

a) The fund behaves like a P-element.

b) Prop.: $y_s(t) \sim \bar{u} = \text{const.}$

Int.: $y_s(t) \sim \int_0^t \bar{u}\, d\tilde{t} = \bar{u} t$

Diff.: $y_s(t) \sim \frac{du(t)}{dt}$

The index s denotes the steady state condition.

c) A controller with I-part, so that $\lim_{t \to \infty} e(t) = 0$

to achieve disturbance rejection and asymptotic tracking.

d) $PIDT_2$: $T_2 \ddot{x}_a(t) + T_1 \dot{x}_a(t) + x_a(t) = k\left[x_e(t) + \frac{1}{T_I}\int x_e(t)dt + T_D \frac{dx_e(t)}{dt}\right]$

$$G(s) = \frac{X_a(s)}{X_e(s)} = k \frac{1 + \frac{1}{T_I}\frac{1}{s} + T_D s}{1 + sT_1 + s^2 T_2}$$

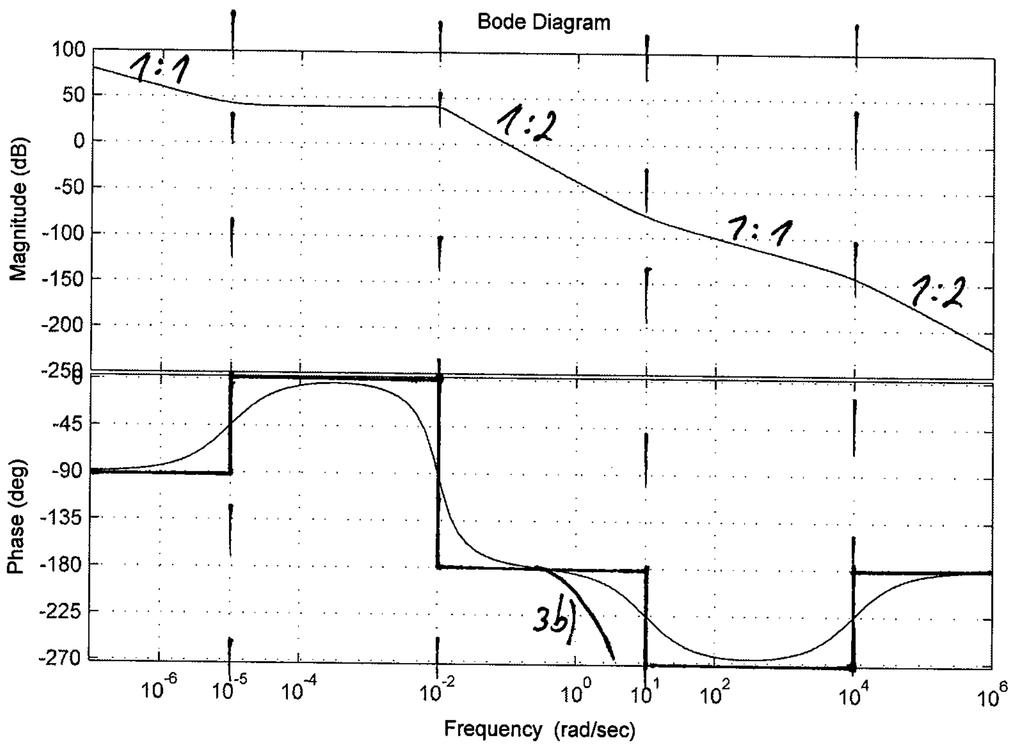
x_e: Input

x_a: Output

e) $Re\{T_{P1}, T_{P2}, T_{P3}\} \leq 0$

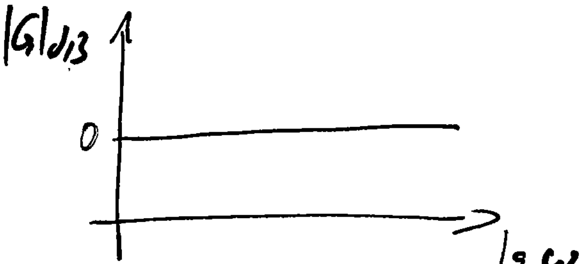
Problem 3

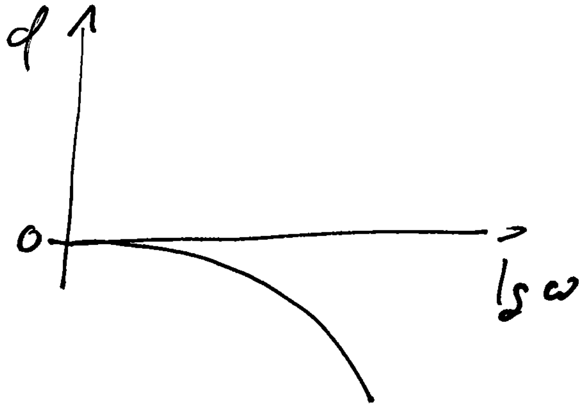
a) / b)



3b): No changes in the magnitude plot

b) Time delay system:





Problem 3

c) $G_o(s) = G_{plant}(s) \cdot G_{controller}(s)$

whereby $|G_o|_{dB} = |G_p|_{dB} + |G_c|_{dB}$

$\arg(G_o) = \arg(G_p) + \arg(G_c)$

The obtained Nyquist plot can be analyzed with the Nyquist criterion.

d) • ODE of the plant $\rightarrow G_p(s)$

PIDT$_2$ controller $\rightarrow G_c(s)$

• Characteristic equation of the closed-loop: $1+G_o(s) = 1 + G_p(s) \cdot G_c(s) = 0$

• The closed-loop is stable iff all poles of the closed-loop have negative real parts: $Re\{s_i\} < 0;\ i = 1,2,\dots,n$

→ This can be examined with the Hurwitz criterion.

→ If necessary the controller parameters have to be changed

e) $G_o(s) = G_p(s) \cdot G_c(s)$ → Calculation of the poles and zeros

→ Draw the root locus diagram

→ The position of the root locus provides information about the stability of the closed-loop system

→ The damping of an eigenvalue increases with decreasing distance to the real axis.

Problem 4

a) $$G(s) = \frac{Y(s)}{U(s)} = \frac{G_1 \cdot G_2 \cdot G_4}{1 + G_2 \cdot G_3 \cdot G_4}$$

$$= \frac{(1+T_I s)\, s\, K_p K_2 K_1}{(1+T_1 s)\left(K_I K_2 K_p + s + s^2(T_2 + T_I) + s^3(T_3^2 + T_I T_2) + s^4 T_I T_3^2\right)}$$

b) $$G(s) = \frac{4s}{4 + s + s^2 + 1{,}25 s^3 + 0{,}5 s^4}$$

$\boxed{DT_4\text{-System}}$

Hurwitz: $s^4 + 2{,}5 s^3 + 2s^2 + 2s + 8 = 0$

$$H_1 = 2{,}5 > 0$$

$$H_2 = \begin{vmatrix} 2{,}5 & 2 \\ 1 & 2 \end{vmatrix} = 3 > 0$$

$$H_3 = \begin{vmatrix} 2{,}5 & 2 & 0 \\ 1 & 2 & 8 \\ 0 & 2{,}5 & 2 \end{vmatrix} = -44 < 0$$

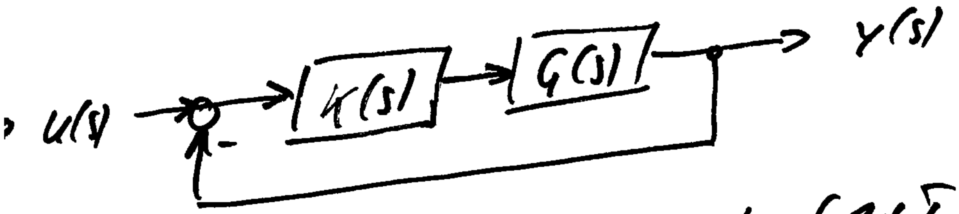
$\Rightarrow$ system is not stable!

Problem 4

c) • Simplified system $G = \frac{G_1}{1+G_1 \cdot G_2} = \frac{-4s}{s-12}$

• Zeros : $s_{01} = 0$

• Poles : $s_1 = 12 \Rightarrow$ System is unstable



• $G_{loop}(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{-4K_R(1+T_R s)}{-4K_R - 12T_R + s(-4K_R T_R + T_R)}$

$= \frac{-4K_R s - 4K_R}{-4K_R - 12 + s(1-4K_R)}$

Pole: $s_1 = \frac{-4(K_R+3)}{4K_R - 1}$

• Range for K_R in which the closed loop system is stable

$\frac{-4(K_R+3)}{4K_R-1} \overset{!}{<} 0$

i) $4K_R - 1 > 0 \Leftrightarrow \boxed{K_R > \frac{1}{4}}$

$-4(K_R+3) < 0 \Leftrightarrow K_R > -3$

ii) $4K_R - 1 < 0 \Leftrightarrow K_R < \frac{1}{4}$

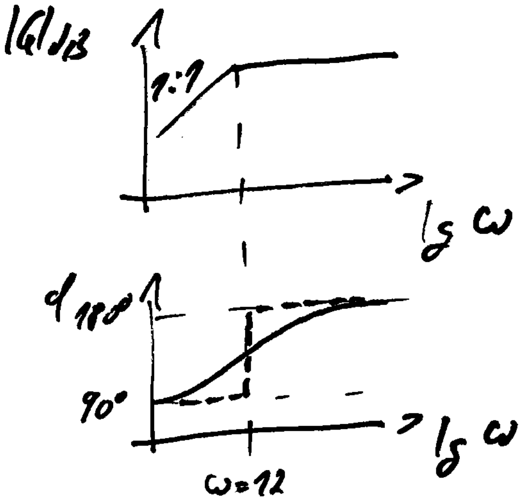
$-4(K_R+3) > 0 \Leftrightarrow \boxed{K_R < -3}$

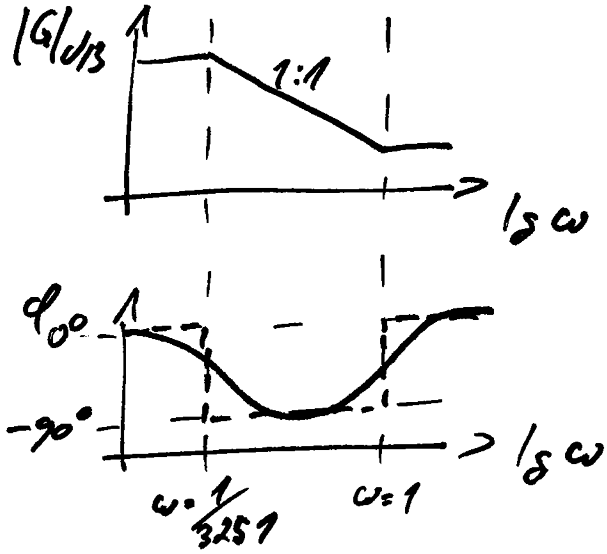
$\Rightarrow K_R > \frac{1}{4} \vee K_R < -3$

Problem 4

d) $G(s) = \frac{-45}{s-12}$

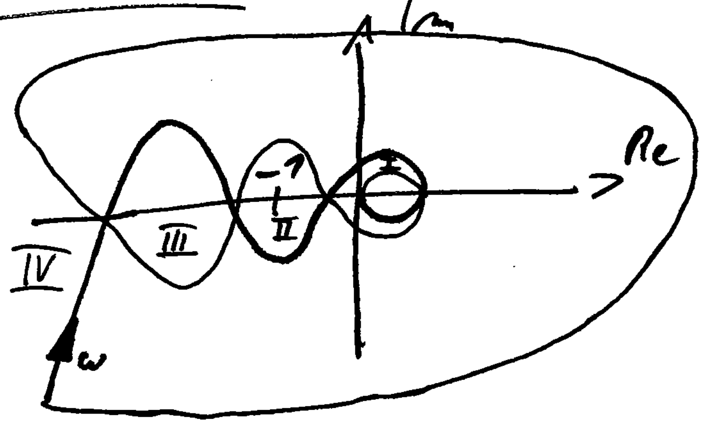
$G_{loop}(s) = \frac{12{,}004(1+s)}{0{,}004+13{,}004s} = \frac{3001(1+s)}{1+3251s}$





Nyquist criterion

e)



The systems has no pole in the right half of the s-plane.

I : $0-(-2)=2$

II : $0-(1-1)=0$ => stable if the loop is closed

III : $0-(-1-1)=2$

IV : $0-0=0$ => stable

„Number of closed-loop poles in the right half of the s-plane = number of open-loop poles in the right half of the s-plane − counterclockwise revolutions around the critical point."

Problem 5

a) Characteristic equation of the closed-loop system:

$$s^3 + \frac{1}{T_1} s^2 + \frac{K_P K_I}{T_1} s + \frac{K_P K_I}{T_1 T_m} = 0$$

Hurwitz:

$$H_1 = \frac{1}{T_1} > 0 \qquad (T_1 > 0)$$

$$H_2 = \begin{vmatrix} \frac{1}{T_1} & \frac{K_I K_P}{T_m T_1} \\ 1 & \frac{K_I K_P}{T_1} \end{vmatrix} = \frac{K_I K_P}{T_1^2} - \frac{K_I K_P}{T_m T_1}$$

i) $\left.\begin{matrix} K_I, K_P > 0 \\ \vee \\ K_I, K_P < 0 \end{matrix}\right\} \Rightarrow \frac{K_I K_P}{T_1^2} - \frac{K_I K_P}{T_m T_1} > 0$
$\Leftrightarrow T_1 < T_m$

ii) $\left.\begin{matrix} K_I > 0 \wedge K_P < 0 \\ \vee \\ K_I < 0 \wedge K_P > 0 \end{matrix}\right\} \Rightarrow T_m < T_1$

$$H_3 = \begin{vmatrix} \frac{1}{T_1} & \frac{K_I K_P}{T_m T_1} & 0 \\ 1 & \frac{K_I K_P}{T_1} & 0 \\ 0 & \frac{1}{T_1} & \frac{K_I K_P}{T_m T_1} \end{vmatrix} = \frac{K_I K_P}{T_m T_1} \cdot H_2$$

$$\frac{K_I K_P}{T_m T_1} \cdot H_2 > 0 \qquad (H_2 > 0)$$

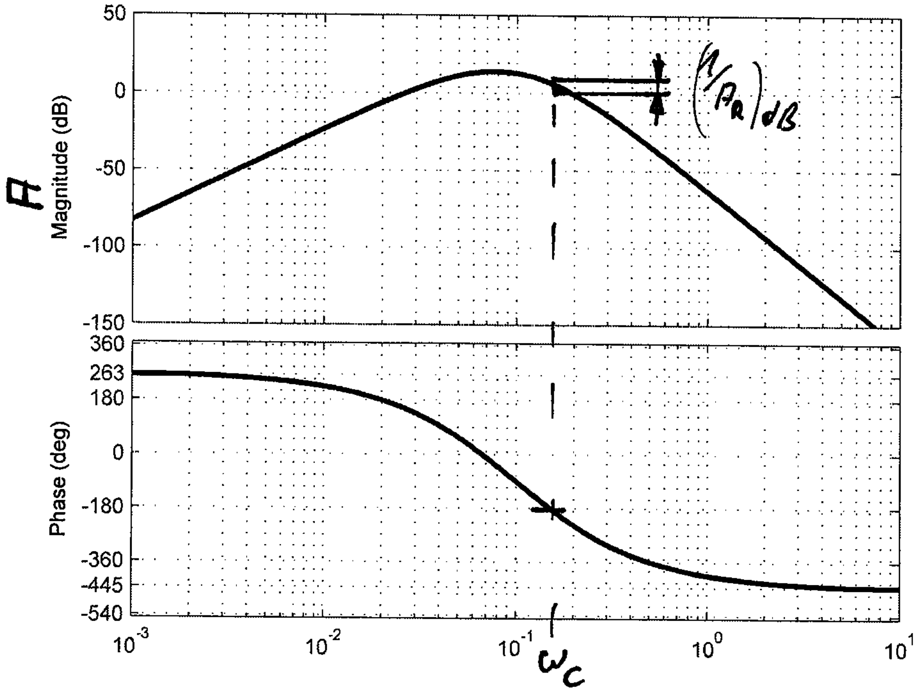
$$\Leftrightarrow \frac{K_I K_P}{T_m T_1} > 0$$

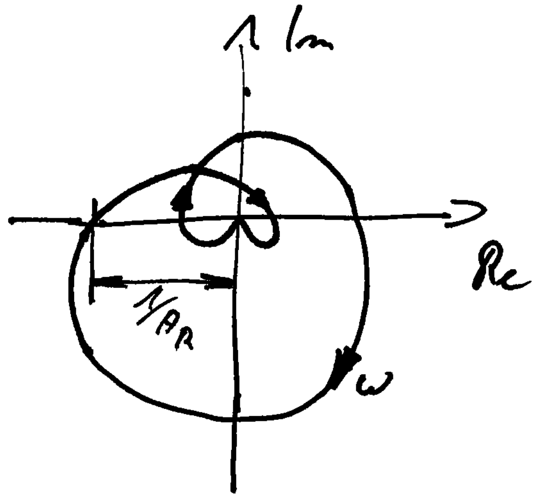
$$\Leftrightarrow K_I K_P > 0 \qquad (T_m, T_1 > 0)$$

Result: $\underline{\underline{T_1 > 0 \wedge K_I K_P > 0 \wedge T_1 < T_m}}$

Problem 5

b)





$$(A(\omega_c))_{dB} > 0 \Rightarrow A > 1 \Rightarrow A_R = \frac{1}{A} < 1$$

$\Rightarrow$ closed loop is unstable

Problem 5

c) $G_0(s) = G_S(s) \cdot G_R(s)$

$$= \frac{2K_p(3s+1)(2s+1)}{(2+(2-2i))(s+(2+2i))(s+4)(4s+1)s}$$

zeros: $s_{01} = -\frac{1}{3}$

$s_{02} = -\frac{1}{2}$

poles: $s_1 = -2+2i$

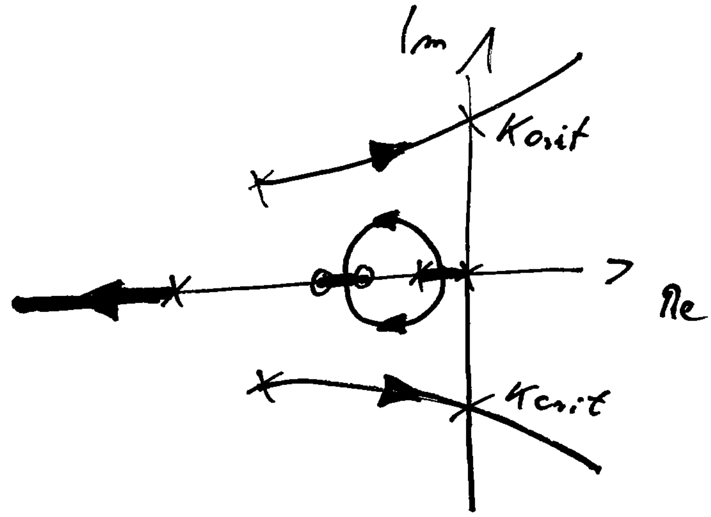
$s_2 = -2-2i$

$s_3 = -4$

$s_4 = -\frac{1}{4}$

$s_5 = 0$ $\Rightarrow$ The system is boundary stable

d)



e) i) Derive $G_0 + 1 = G_C \cdot G_P + 1$ (Characteristic equation)

ii) Use the Hurwitz criterion in dependence on K_P

iii) Find the K_P for that two roots become zero.

Problem 6

a) $y(s) = F_S(s) \cdot \mathcal{L}\{\delta(t)\}$

$$\lim_{t\to+0} y(t) = \lim_{s\to\infty} s\cdot y(s) = \lim_{s\to\infty} K\frac{s+\frac{1}{T_I}}{1+T_1 s} = \frac{K}{T_1}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\cdot y(s) = \lim_{s\to 0} K\frac{s+\frac{1}{T_I}}{1+T_1 s} = \frac{K}{T_I}$$

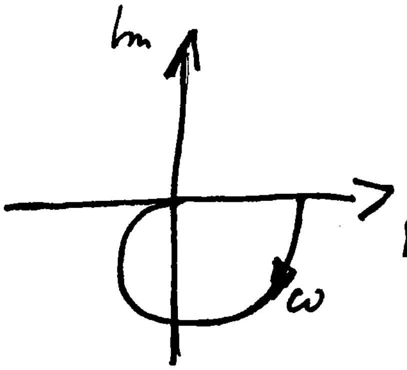
$$\frac{K}{T_1} = 2 \; ; \; \frac{K}{T_I} = \frac{2}{3} \; ; \; T_1 = \frac{1}{2} \Rightarrow \underline{\underline{K = 1 ; \; T_I = \frac{3}{2}}}$$

b) $$F_S(s) = \frac{1}{1+T_1 s} + \underline{\underline{\frac{1}{T_I s(1+T_1 s)}}} = \frac{F_3(s)}{1+F_3(s)} + \underline{\underline{F_4(s)}}$$

$$\Rightarrow \underline{\underline{F_3(s) = \frac{1}{T_1 s}}}$$

c) $$F_S(s) = \frac{F_1(s)}{1+F_1(s)F_2(s)} \qquad F_S(s) = \frac{1}{s+2+\frac{1}{s}} \cdot \frac{s+1}{s(1+T_1 s)} = \frac{s+1}{(s+1)^2(1+T_1 s)}$$

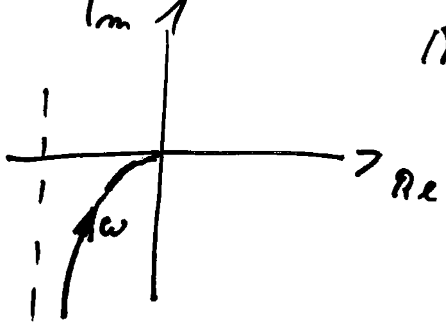
$$= \underline{\underline{\frac{1}{(1+s)(1+T_1 s)}}}$$



d) $$F_0(s) = F_S(s)\cdot F_R(s) = \frac{1}{(1+s)(1+T_1 s)} \cdot \frac{K_R(1+s)}{s} = \frac{K_R}{s(1+T_1 s)}$$

$$F_0(j\omega) = \frac{K_R}{-T_1\omega^2 + j\omega}$$

IT_1-System



System is for negative feedback always stable, because $(-1, j0)$ is always to the left of $F_0(j\omega)$.

Problem 6

e) Without $F_K(s)$:

$$F_z(s) = \frac{F_S(s)}{1 + \frac{F_1(s)}{1+F_1(s)F_2(s)} \cdot F_S(s) \cdot F_R(s)} = \frac{1+s}{T_1 s^2 + s + K_R}$$

for $w(t) \equiv 0$, $z(t) = 1(t)$ and $0 < K_R < \infty$:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s) = \lim_{s\to 0} F_z(s) = \frac{1}{K_R}$$

=> Steady state deviation from the desired value
=> $F_K(s)$ is necessary

with $F_K(s)$:

$$F_z(s) = \frac{F_S(s) - F_K(s) \frac{F_1(s)}{1+F_1(s)F_2(s)} \cdot F_S(s)}{\dots} \overset{!}{=} 0$$

$$\Rightarrow \quad 1 - F_K(s) \frac{F_1(s)}{1+F_1(s)F_2(s)} = 0$$

$$\Rightarrow \quad F_K(s) = \frac{1+F_1(s)F_2(s)}{F_1(s)}$$

Problem 6

b) Sketch the qualitative step response of F_5.