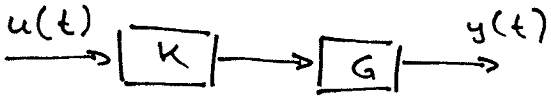
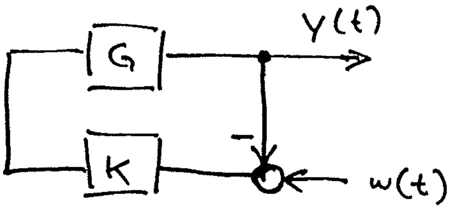
1 a) Feed-forward regards open loop control

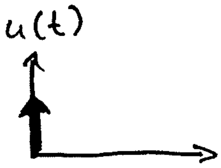


Feed-back regards closed loop control

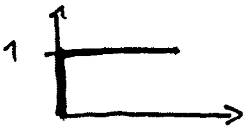


b) An experimental model of a system is used for analyzing the system behavior.

c)



Dirac (Impulse) function $F(s) = 1$



step function $F(s) = \frac{1}{s}$

Ramp function $F(s) = \frac{1}{s^2}$

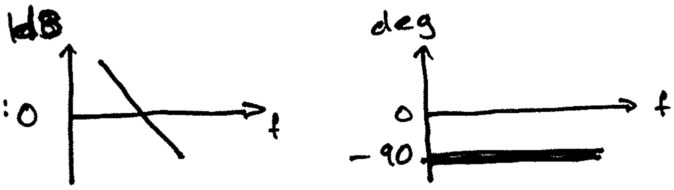
d) The step response describes how the system reacts to a step funktion as input.
A graphical plot of the output in the time-domain is used for classification.

e) 1) Eigenvalues 2) Root locus 3) Nyquist

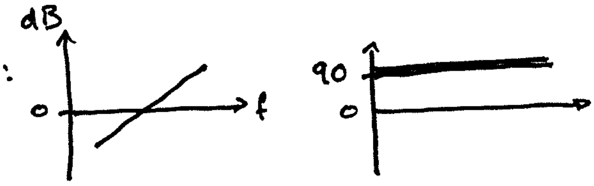
The use of stability criteria is to check whether a system is stable or not and to tune controller parameters to improve the dynamics of the system.

2a)

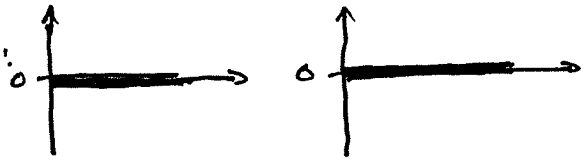
Integral:



Derivative:



Proportional:



b) Stability of a transfer system is a conclusion of the charcter of the system output in the time-domain. By giving an input to the system and analyzing the output, it is possible to determine the stability of the system.

c) Diff. eq. of a PDT_2-system:

$$T_2 \ddot{x}_a + T_1 \dot{x}_a + x_a = K\left[x_e + T_D \dot{x}_e\right]$$

Transfer function:

$$G(s) = K \cdot \frac{1 + T_D s}{T_2 s^2 + T_1 s + 1}$$

Step response: (final- and initial-value theorem)

$$\lim_{y \to 0} f(t) = \lim_{s \to \infty} s \cdot F(s)$$; (for a step $\left(\frac{1}{s}\right)$, multiply)

$$\Rightarrow \lim_{s \to \infty} \frac{1}{s} \cdot s \cdot K \cdot \frac{1 + T_D s}{T_2 s^2 + T_1 s + 1} = 0,$$ and

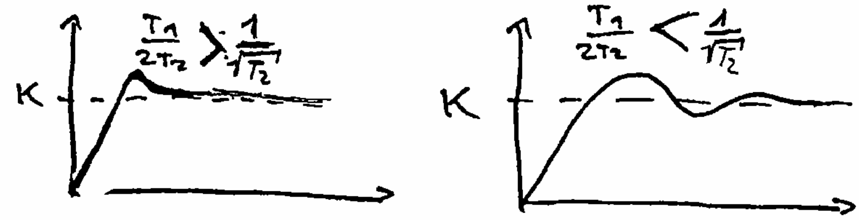
$$\lim_{y \to \infty} \Rightarrow \lim_{s \to 0} \frac{1}{s} \cdot s \cdot K \cdot \frac{1 + T_D s}{T_2 s^2 + T_1 s + 1} = K$$

$\Rightarrow$ Initial value $= 0$, Final value K

Complex roots in the charcteristic equation gives oscillations. (No complex roots $\Rightarrow$ no oscillations)

$$\Rightarrow \quad T_2 s^2 + T_1 s + 1 = 0 \quad \Rightarrow \quad s = -\frac{T_1}{2T_2} \pm \sqrt{\left(\frac{T_1}{2T_2}\right)^2 - \frac{1}{T_2}}$$

if $\frac{T_1}{2T_2} < \frac{1}{\sqrt{T_2}} \Rightarrow$ complex roots (oscillations); $T_2 > 0$

$\Rightarrow$ 

d) Transfer function of a PIT_1-system is:

$$G = K \cdot \frac{1 + \frac{1}{T_I s}}{1 + T_1 s} = (\text{with insertions}) = \frac{1}{s}$$

Transfer function of controller 1:

$G_1 = K_P \quad \Rightarrow$ open loop system: $G_{01} = G_1 \cdot G = \frac{K_P}{s}$

Trasfer function of controller 2:

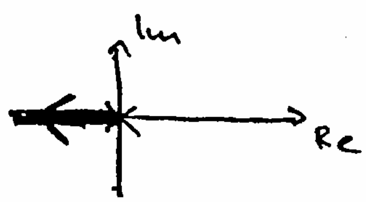
$G_2 = K_{PR} + \frac{K_{PR}}{T_{IR}\, s} \Rightarrow$ open loop system: $G_{02} = K_{PR} \frac{s+1}{s^2}$

Root locus alway checked with open loop system!

G_1: poles $= 0$; no zeros

number of asymptotes $= n - m = 1$

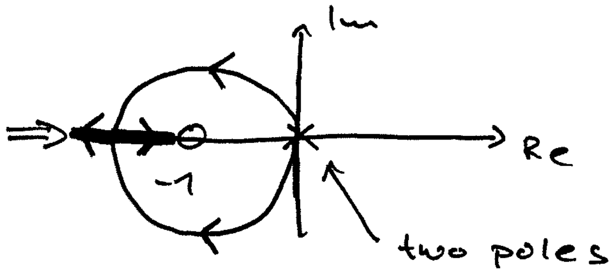
Directon of asymptote $= \frac{2r-1}{n-m} \cdot \pi \; (r = 1, 2, \ldots m-n) = \pi$

$\Rightarrow$ 

G_2: poles: $s_1 = 0$ $s_2 = 0$; zeros $z_1 = -1$

number of asymptotes $= n - m = 1$

Direction of asymptotes $= \frac{2r-1}{n-m} = \pi$



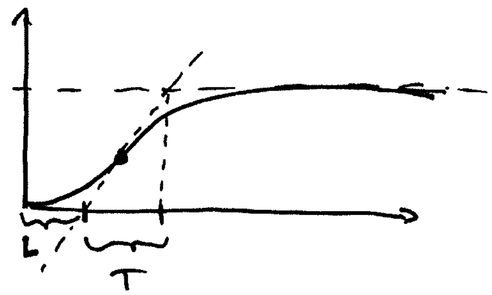
G_1 is asymptotically stable for $K > 0$ and no oscillations.

G_2 is asymptotically stable for $K > 0$ but has oscillations for some $0 < K < \infty$.

e) Ziegler-Nichols method (2 methods)

1) This method is used for systems with "S-shaped" step response (PT_2 with no oscillations).

Two constants are derived from the step response L = delay time and T = time constant.



Then the tuning follows:

$K_p = 1{,}2 \cdot \frac{T}{L}$, $T_I = 2 \cdot L$ and $T_D = 0.5\, L$

2) Disconnect the I-part and the D-part ($T_I = \infty$ $T_D = 0$)

Increase K_p until oscillations with constant amplitude.

Then, the period time is denoted T_0 and $K_p = K_0$

Then, the tuning follows:

$K_p = 0{,}6 \cdot K_0$, $T_I = \frac{T_0}{2}$, and $T_D = \frac{T_0}{8}$

3a)

Choosing the states $x_1 = x$ and $x_2 = \dot{x}$ gives

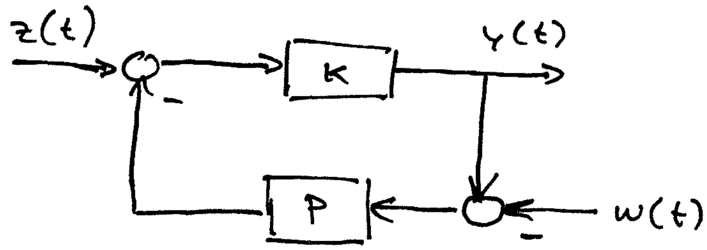
$\dot{x}_1 = x_2$; $\dot{x}_2 = \ddot{x}$ $\Rightarrow$

$$\Rightarrow \quad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t)$$

Measurnent of the state $x = x_1$ gives

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) Block diagram:



Transfer function from $w(t)$ to $y(t)$ (no disturbances $z(t)=0$)

$$G_{wy} = \frac{KP}{1 + KP} = \left(\text{with insertions } P = K_P \text{ ; } K = \frac{1}{T_I s}\right) \Rightarrow$$

$$G_{wy} = \frac{K_P}{T_I s + K_P} \quad \Rightarrow \quad PT_1\text{-behavior}$$

Transfer function from $z(t)$ to $y(t)$ $\Rightarrow$

$$\Rightarrow G_{zy} = \frac{1}{T_I s + K} \quad \Rightarrow \quad PT_1\text{-behavior}$$

c) Laplace transform gives:

$$4Xs^2 - 5Xs + 6X = F \Rightarrow$$

$$G = \frac{1}{s^2 - 5s + 6}$$

Stodola:

Coefficients $< 0 \Rightarrow$ unstable system.

d) Final value theorem on the control error:

$$\lim_{t \to \infty} \{y(t) - w(t)\} = \lim_{s \to 0} s \cdot [Y(s) - W(s)] \; ; \text{ where}$$

$$W(s) = \frac{1}{s} \quad \text{(step function)}$$

Transfer function from $w(t)$ to $y(t)$ $\Rightarrow$

$$\Rightarrow G_{wy}(s) = -\frac{T_D K_p s}{1 + T_1 s - T_D K_p s} \; ; \text{ since } Y(s) = G(s) \cdot W(s)$$

insertion gives: $\lim_{s \to 0} s[Y(s) - W(s)] =$

$$= \lim_{s \to 0} s[G(s) \cdot W(s) - W(s)] = \lim_{s \to 0} s \cdot W(s)[G(s) - 1] =$$

$$= \lim_{s \to 0} \frac{s}{s} \left[-\frac{T_D K_p s}{1 + T_1 s - T_D K_p s} - 1 \right] = -1$$

Final value theorem on the output (affected by disturbance step)

$$\lim_{t \to \infty} \{y(t)\} = \lim_{s \to 0} s\left[Y(s)\right];$$

$$Y(s) = G_{zy}(s)\, Z(s)$$

Transfer function from $z(t)$ to $y(t)$:

$$G_{zy}(s) = \frac{T_D s}{1 + T_1 s + T_D K_P s}$$ and insertion (with $Z(s) = \frac{1}{s}$ (step))

$$\lim_{s \to 0} s\left[Y(s)\right] = \lim_{s \to 0} s\left[G_{zy}(s) \cdot Z(s)\right] = \lim_{s \to 0} \frac{s}{s}\left[G(s)\right] =$$

$$= \lim_{s \to 0} \left[\frac{T_D s}{1 + T_1 s + T_D K_P s}\right] = 0$$

e) Caution: Positive feedback!

Transfer function from $w(t)$ to $y(t)$:

$$y(t) = G_1 G_2 \left(G_3 y(t) - w(t)\right) \Rightarrow$$

$$\Rightarrow G_{wy} = \frac{y(t)}{w(t)} = -\frac{G_1 G_2}{1 - G_1 G_2 G_3}$$

Transfer function from $z(t)$ to $y(t)$:

$$y(t) = G_1 \left(z(t) - G_2 G_3 y(t)\right) \Rightarrow$$

$$\Rightarrow G_{zy} = \frac{y(t)}{z(t)} = \frac{G_1}{1 - G_1 G_2 G_3}$$

Problem 41

a) $G_0(s) = G_S(s) \cdot G_R(s) = \dfrac{k}{s(1+T_2 s)(1+T_1 s)}$

Zeros : –

Poles : $0;\ \frac{-1}{T_2};\ \frac{-1}{T_1}$

b) $1 + G_0(s) = 0$

$\Leftrightarrow s(1+T_2 s)(1+T_1 s) + k = 0$

$\Leftrightarrow T_1 T_2 s^3 + (T_1+T_2)s^2 + s + k = 0$

HURWITZ

$$H = \begin{vmatrix} T_1+T_2 & k & 0 \\ T_1T_2 & 1 & 0 \\ 0 & T_1+T_2 & k \end{vmatrix}$$

$H_1 = T_1 + T_2 > 0$ (1)

$H_2 = T_1 + T_2 - k T_1 T_2 > 0$ (2)

$H_3 = k \cdot H_2 > 0$ (3)

(1): $T_1 + T_2 > 0$

(3): $k > 0$

(2): $\dfrac{T_1+T_2}{T_1T_2} > k \quad (T_1T_2 > 0)$

$\dfrac{T_1+T_2}{T_1T_2} < k \quad (T_1T_2 < 0)$

$$\Rightarrow \begin{cases} \dfrac{T_1+T_2}{T_1T_2} > k > 0 \ \wedge\ T_1+T_2 > 0 \ \wedge\ T_1T_2 > 0 \\ \vee \\ k > 0 \ \wedge\ T_1+T_2 > 0 \ \wedge\ T_1T_2 < 0 \end{cases} \Rightarrow 2 > k > 0$$

c) $|G_0(j\omega)| = \dfrac{|K|}{\omega \cdot \sqrt{1+T_1^2\omega^2}\sqrt{1+T_2^2\omega^2}}$

$\phi_0(j\omega) = -\frac{\pi}{2} - \arctan(T_1\omega) - \arctan(T_2\omega)$

d) $|G_0(j\omega_s)| = \dfrac{5/8}{\omega_s(1+\omega_s^2)} \overset{!}{=} 1$

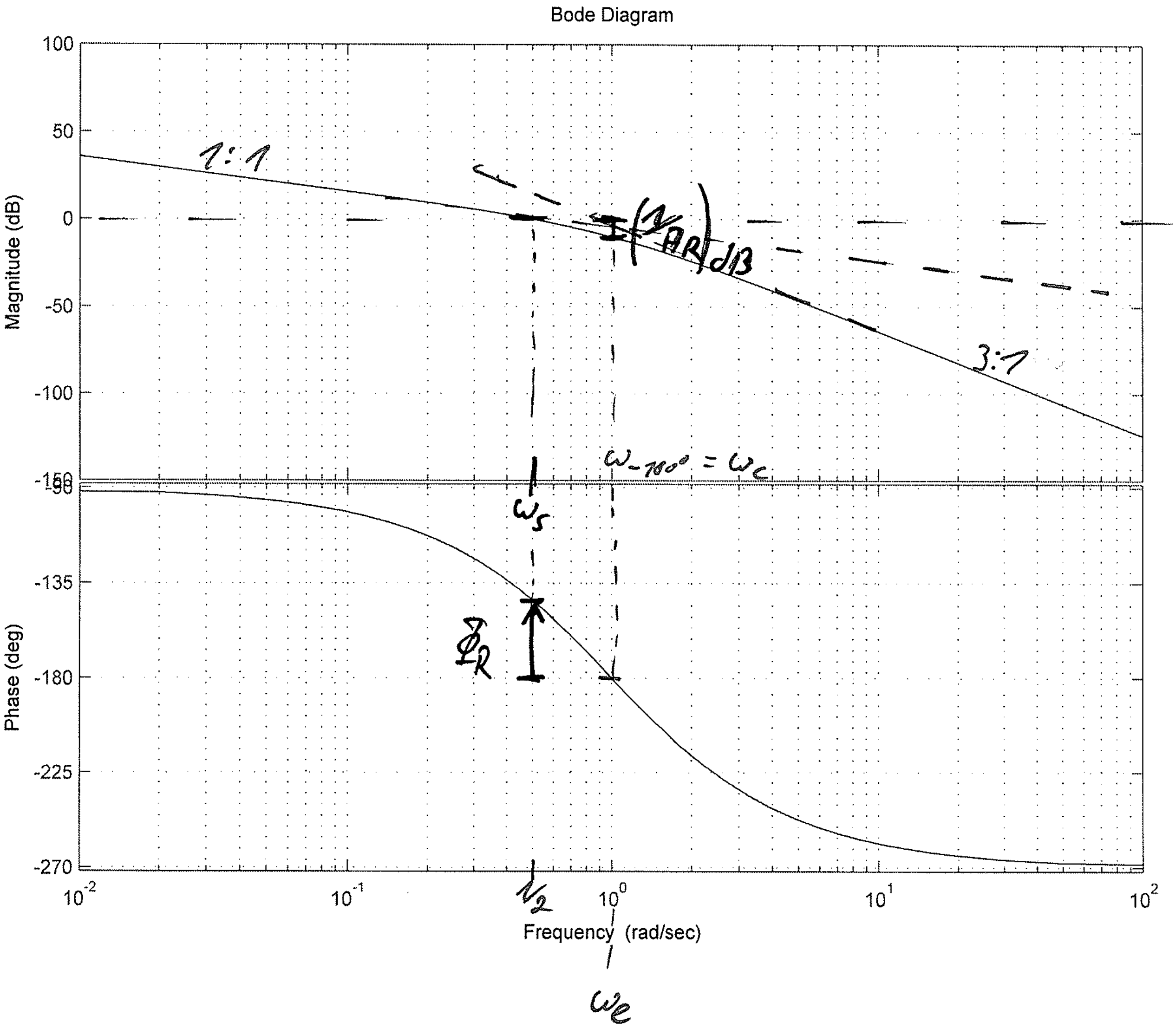
$\Leftrightarrow \omega_s^3 + \omega_s - \frac{5}{8} = 0$

$\Leftrightarrow \omega_{s_1} = \frac{1}{2} \wedge \omega_{s_2} = -\frac{1}{4} + \frac{1}{4}i\sqrt{19} \wedge \omega_{s_3} = -\frac{1}{4} - \frac{1}{4}i\sqrt{19}$

$\Rightarrow \underline{\underline{\omega_s = \frac{1}{2}}}$ (must be a real value)

$\phi_R = 180° - |\phi(\omega_s)| \approx 180° - 90° - 2 \cdot 26{,}56° = \underline{\underline{36{,}87°}}$

e)


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
1:1
3:1
$(1/A_R)$ dB
$\omega_{-180°} = \omega_c$
ω_s
Φ_R
ω_e

Problem 5

a) • Linearization: $\delta B = \left.\frac{dB}{dA}\right|_{A_0} \delta A$

$$= \left(\frac{2}{3}A_0 - 1\right)\delta A$$

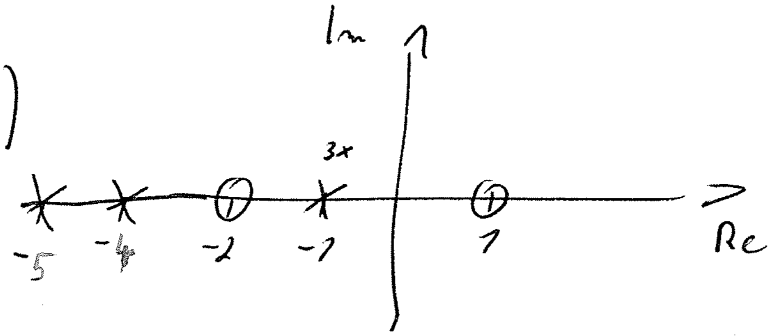
$$\longrightarrow B = \left(\frac{2}{3}A_0 - 1\right)A \longrightarrow \frac{B}{A} = \frac{2}{3}A_0 - 1$$

• $$G_0(s) = G_R \cdot G_S \cdot \left(\frac{2}{3}A_0 - 1\right)$$

$$= \underbrace{4\left(\frac{2}{3}A_0 - 1\right)}_{K} \frac{(s+2)(s-1)}{(s+1)^3(s+4)(s+5)}$$

$$= \underbrace{\frac{-2}{5}\left(\frac{2}{3}A_0 - 1\right)}_{K_{stat}(A_0)} \frac{\left(\frac{s}{2}+1\right)(-s+1)}{(s+1)^3\left(\frac{s}{5}+1\right)\left(\frac{s}{4}+1\right)}$$

b)



c) $A_0 < \frac{3}{2}$ -> Mitkopplung ($K < 0$) (positive feedback)

$A_0 > \frac{3}{2}$ -> Gegenkopplung ($K > 0$) (neg. feedback)

d) z.B. Regelungsnormalform: (control canonical form)

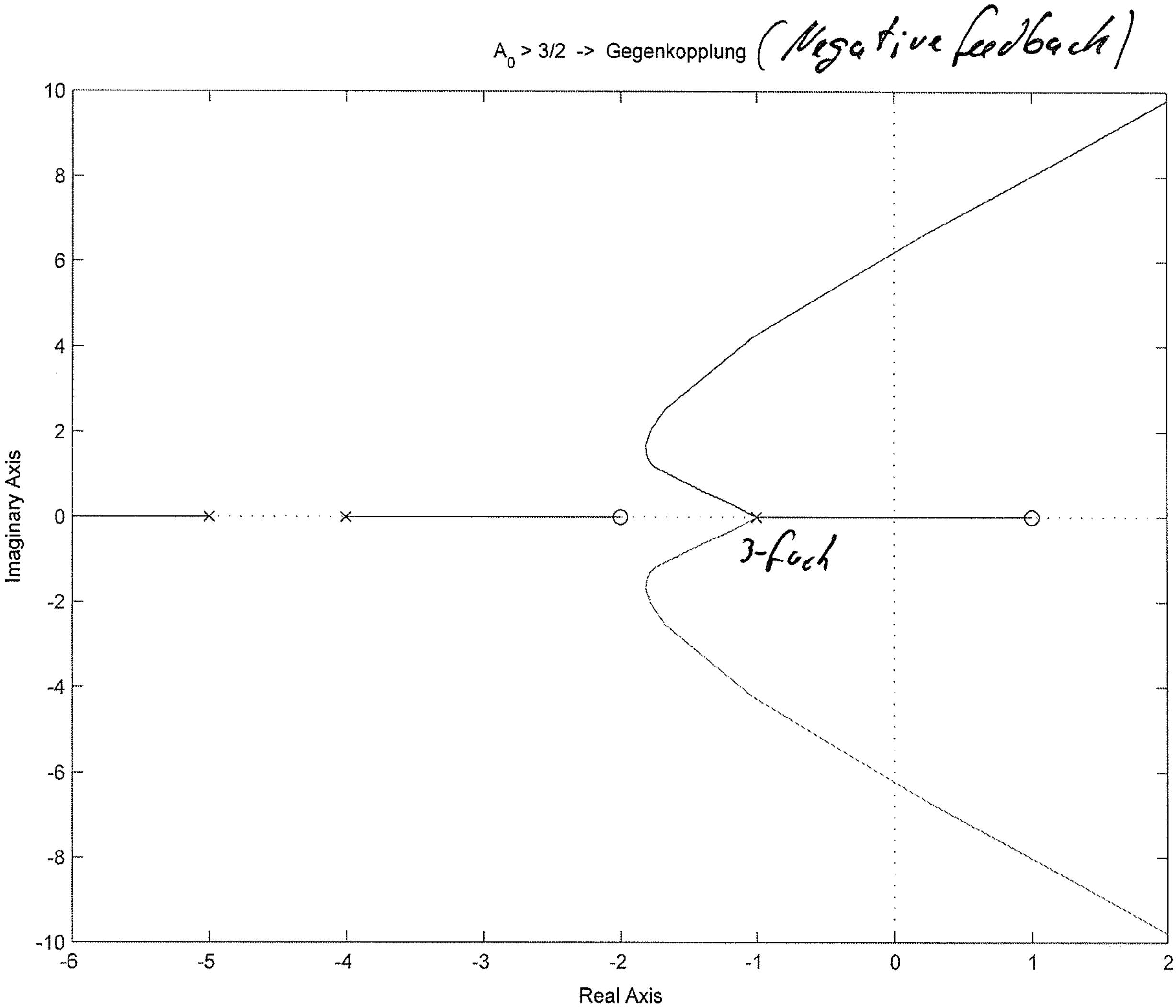
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -20 & -69 & -88 & -50 & -12 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Poles: $-5; -1; -1; -1; -4$

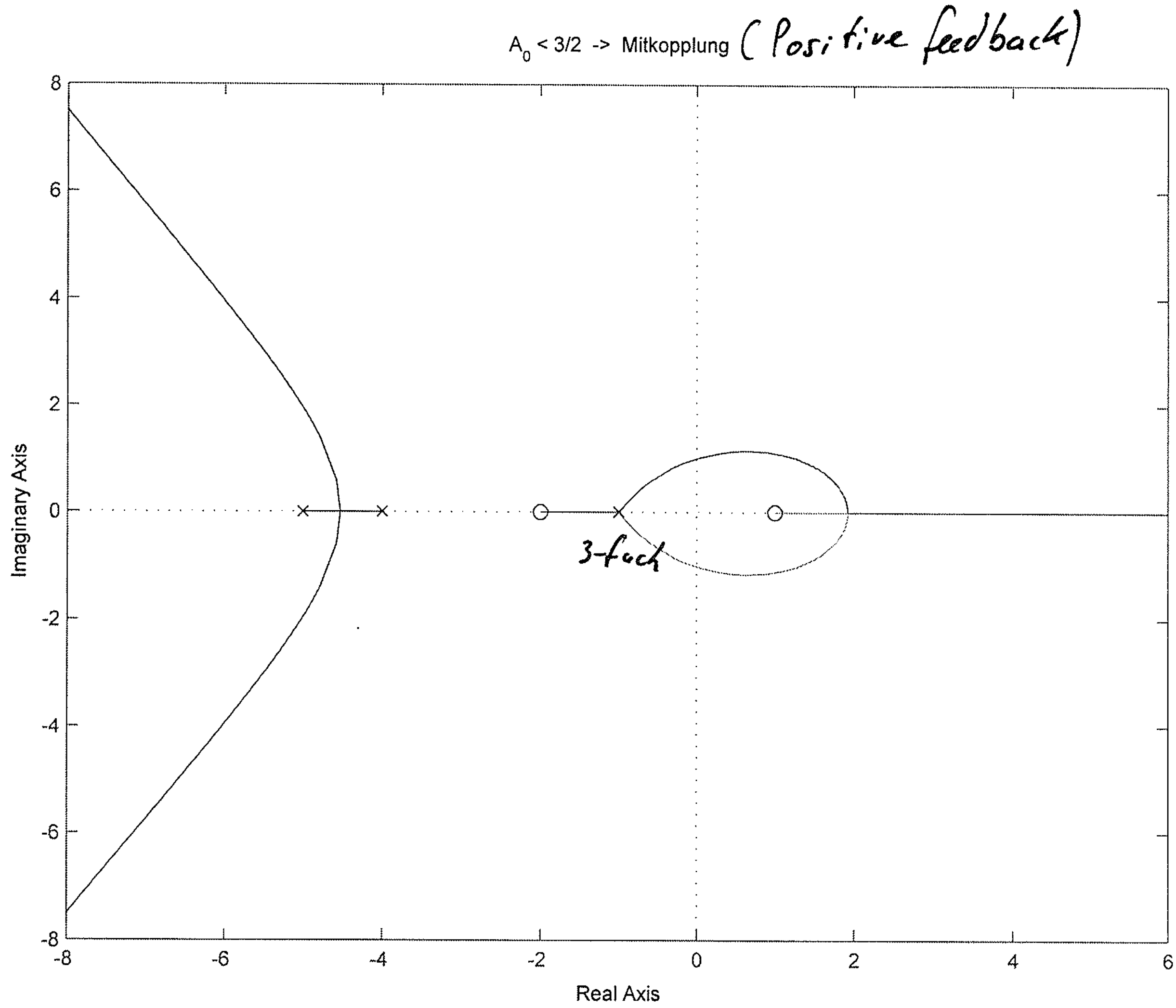
$\Rightarrow$ stable ($Re\{\lambda_i\} < 0$)

c)



$A_0 > 3/2$ -> Gegenkopplung (Negative feedback)
3-fach
Imaginary Axis
Real Axis

c)


$A_0 < 3/2$ -> Mitkopplung (Positive feedback)
Imaginary Axis
Real Axis
3-fach
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6

Lösung Aufgabe 6 (40 Punkte)

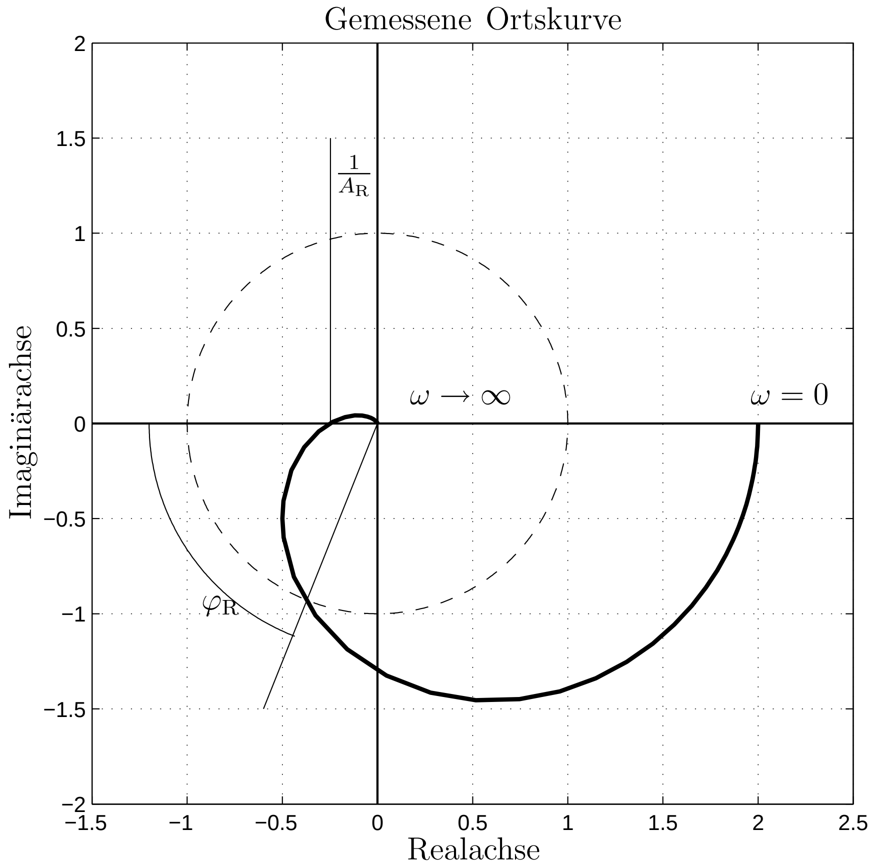


Abbildung 0.2: Gemessene Ortskurve

a) PT_3-Übertragungselement, da drei Quadranten durchlaufen werden.

b) Amplitudenrand $A_R = \frac{1}{|-0{,}2471|} \approx 4{,}047$
Phasenrand $\varphi_R \approx 74°$.

c) Ja, da gemessen.

d) $K_S = 2$ (abgelesen bei $\omega = 0$).
$T = T_1 = T_2 = T_3$

$$
\begin{aligned}
G_o &= G_R G_S && (0.1)\\
&= K_P \frac{K_S}{(1+sT)^3} && (0.2)\\
&= \frac{K_P K_S}{1-3\omega^2T^2 + j\omega(3T-\omega^2T^3)} && (0.3)\\
&= \frac{K_P K_S(1-3\omega^2T^2)}{(1-3\omega^2T^2)^2+(3T-\omega^2T^3)^2} - j\frac{K_P K_S(3T-\omega^2T^3)}{(1-3\omega^2T^2)^2+(3T-\omega^2T^3)^2} && (0.4)
\end{aligned}
$$

Kritischer Punkt (-1 / 0j)
Kritische Verstärkung K_P dann erreicht, wenn die Ortskurve durch diesen Punkt läuft, d. h.

- $\mathrm{Im}\{G_o\} \stackrel{!}{=} 0$ und
- $\mathrm{Re}\{G_o\} \stackrel{!}{=} -1.$

Für den Imaginärteil folgt:

$$
\begin{aligned}
\mathrm{Im}\{G_o\} &= K_P K_S(3T-\omega^2T^3) \stackrel{!}{=} 0\\
\Rightarrow \omega &= \frac{\sqrt{3}}{T} && (0.5)
\end{aligned}
$$

Für den Realteil folgt:

$$
\begin{aligned}
\mathrm{Re}\{G_o\} &= \frac{K_P K_S(1-3\omega^2T^2)}{(1-3\omega^2T^2)^2+(3T-\omega^2T^3)^2} \stackrel{!}{=} -1\\
\text{mit } \omega &= \frac{\sqrt{3}}{T} \text{ folgt}\\
K_P &= \frac{8}{K_S} && (0.6)\\
\text{hier } K_P &= 4 \text{ (abgelesen } \mathrm{K_S} = 2)
\end{aligned}
$$

Stabil für $0 < K_P < 4$.

Alternativ aus Amplitudenrand $A_R = K_P = 4,047$.

e) Ablesen aus Abbildung 0.3 für WOK:
Nullstellen m = 0
Polstellen n = 3
Verzweigungspunkt auf der Realachse bei $\delta_w = \frac{-6,5}{3} \approx -2,167$.
Anzahl der Asymptoten n-m = 3 bei 60°, 180° und 300°.

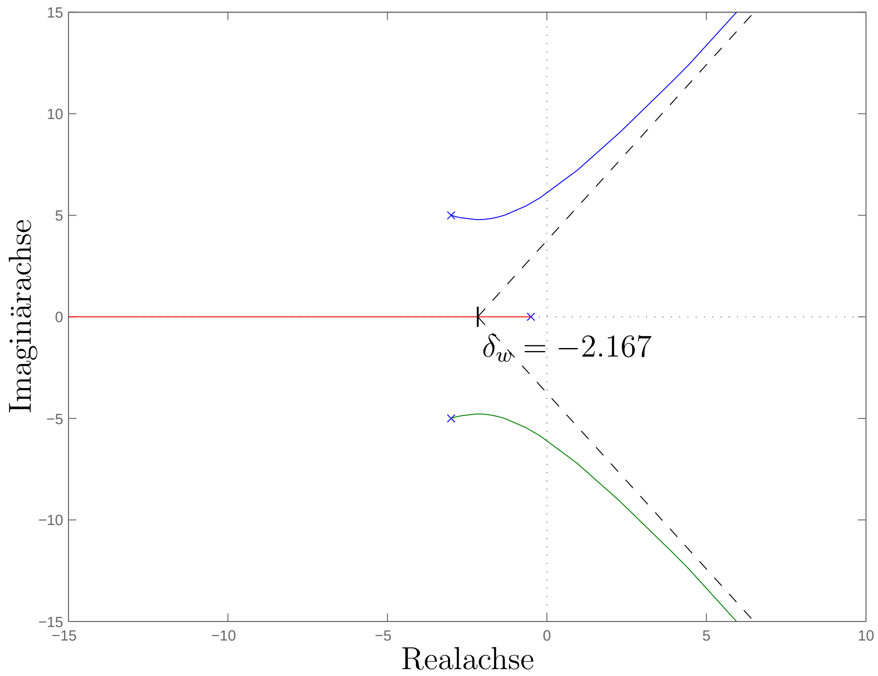


Abbildung 0.3: Wurzelortskurve zu Aufgabenteil e)

f) Ja, da Pole mit imaginärem Anteil.

g) Regler 1: PD-Übertragungselement
Regler 2: PI-Übertragungselement

h) Ablesen aus Abbildung 0.4 für WOK und Reglertyp:
Nullstellen m = 1
Polstellen n = 3
Verzweigungspunkt auf der Realachse bei $\delta_w = \frac{-1{,}5}{2} = -0,75$.
Anzahl der Asymptoten n-m = 2 bei 90°und 270°.

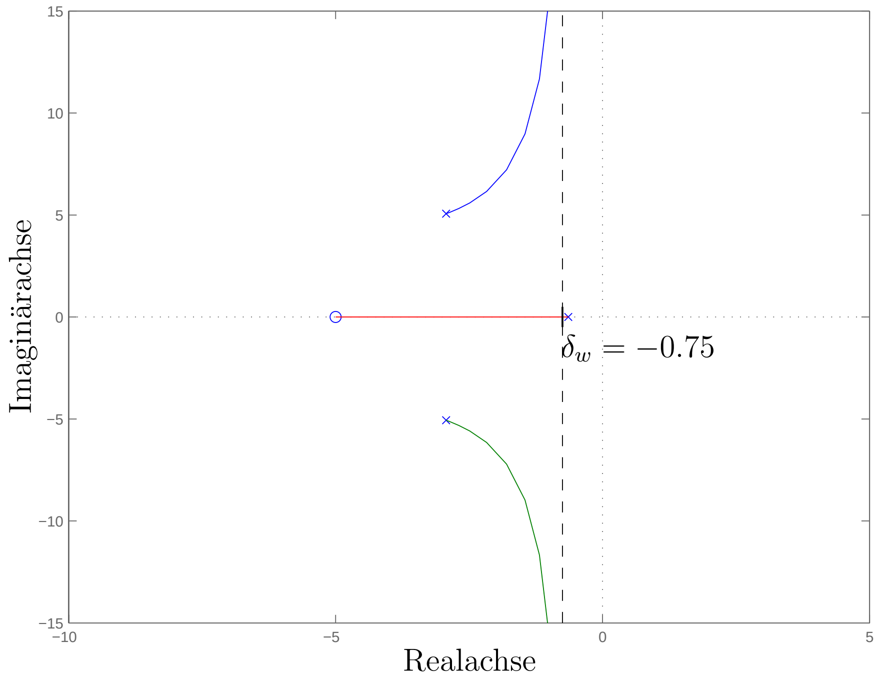


Abbildung 0.4: Wurzelortskurve zu Aufgabenteil h)

Das System ist stabil, da die WOK für alle K_{R1} nicht die Imaginärachse schneidet.

i)

$$
\begin{aligned}
G_\mathrm{o} &= G_\mathrm{S} G_{\mathrm{R2}} \\
&= K_{\mathrm{R2}} \frac{1+5s}{5s(s+0,5)(s+3+5i)(s+3-5i)}
\end{aligned}
$$

$$
\begin{aligned}
C(\lambda) &= 1 + G_\mathrm{o} \stackrel{!}{=} 0 \\
&= 1 + \frac{K_{\mathrm{R2}}(1+5\lambda)}{5\lambda(\lambda+0,5)(\lambda+3+5i)(\lambda+3-5i)} \stackrel{!}{=} 0 \\
\Rightarrow 0 &= 5\lambda(\lambda+0,5)(\lambda+3+5i)(\lambda+3-5i) + K_{\mathrm{R2}}(1+5\lambda) \\
&= \underbrace{5}_{a_4}\lambda^4 + \underbrace{\frac{65}{2}}_{a_3}\lambda^3 + \underbrace{185}_{a_2}\lambda^2 + \underbrace{(85+5K_{\mathrm{R2}})}_{a_1}\lambda + \underbrace{K_{\mathrm{R2}}}_{a_0}
\end{aligned}
$$

HURWITZ-Kriterium nach [Gantmacher] (notwendige und hinreichende Bedingung):

> *Das Polynom* $a_0 + a_1 s + \ldots + a_{n-1}s^{n-1} + s^n$ *ist dann und nur dann ein* HURWITZ-*Polynom, wenn alle* HURWITZ-*Determinanten* D_1, D_2,...,D_n *positiv sind.*

$$D_1 = \det \begin{vmatrix} a_{n-1} \end{vmatrix} = \tfrac{65}{10} \overset{!}{>} 0 \text{ , erfüllt.}$$

$$D_2 = \det \begin{vmatrix} a_{n-1} & a_{n-3} \\ 1 & a_{n-2} \end{vmatrix} = K_{\mathrm{R2}} \overset{!}{<} \tfrac{447}{2} \text{ , erfüllbar.}$$

$$D_3 = \det \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ 1 & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} = -\tfrac{4K_{\mathrm{R2}}^2 - 657K_{\mathrm{R2}} - 15198}{4} \overset{!}{>} 0$$

$$\Rightarrow K = -17.62 \overset{!}{<} K_{\mathrm{R2}} \overset{!}{<} 215.67 \text{ , erfüllbar.}$$

$$D_4 = \det \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} \\ 1 & a_{n-2} & a_{n-4} & a_{n-6} \\ 0 & a_{n-1} & a_{n-3} & a_{n-5} \\ 0 & 1 & a_{n-2} & a_{n-4} \end{vmatrix} = -\tfrac{K_{\mathrm{R2}}(20K_{\mathrm{R2}}^2 - 3961K_{\mathrm{R2}} - 75990)}{100} \overset{!}{>} 0$$

$$\Rightarrow K = -17.62 \overset{!}{<} 0 \overset{!}{<} K_{\mathrm{R2}} \overset{!}{<} 215.67 \text{ , erfüllbar.}$$

Für $0 \overset{!}{<} K_{\mathrm{R2}} \overset{!}{<} 215,67$ ist das System stabil.

Maximal erreichbare Punktzahl:	**100**
Mindestpunktzahl für die Note 1,0:	**95**
Mindestpunktzahl für die Note 4,0:	**50**

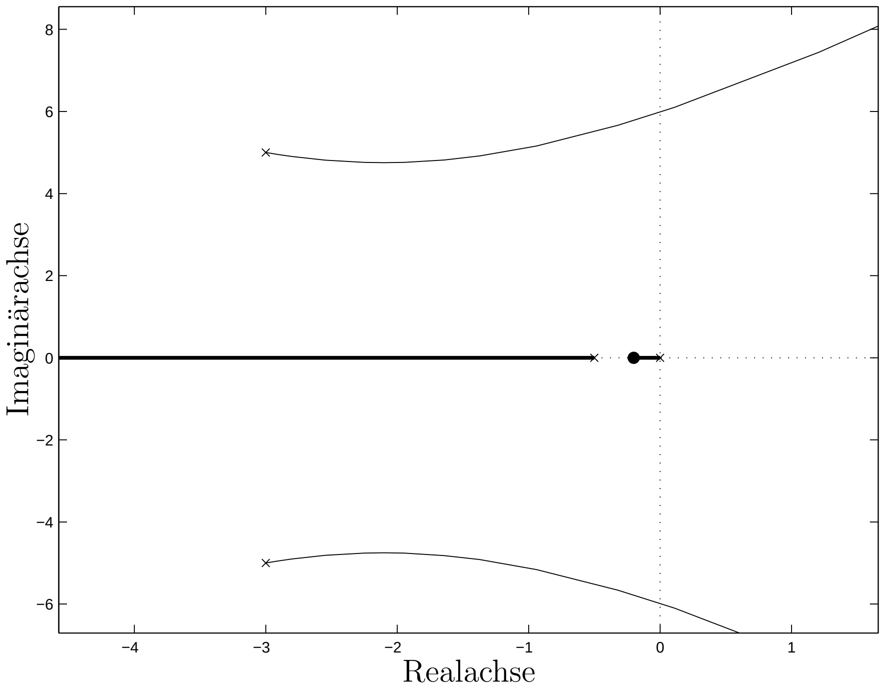


Abbildung 0.5: Wurzelortskurve zu Aufgabenteil i)