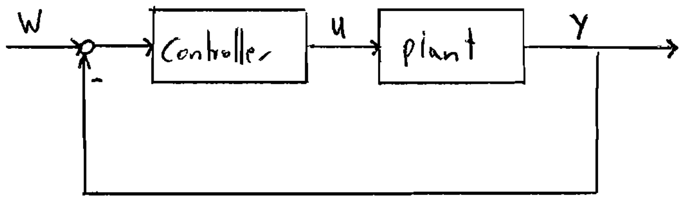
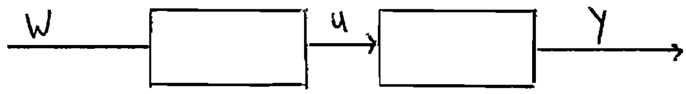
Problem 1)

a)




Closed loop control




open loop control

b) multivariable System:

more than one input or/and more than one output

c) $u(t) = k\,u_1(t) + L\,u_2(t)$

$y(t) = k\,y_1(t) + L\,y_2(t)$

$\longrightarrow$ Linearity

d) $f(+0) = \lim\limits_{t \to +0} f(t) = \lim\limits_{s \to \infty} s\,F(s)$

calculation of the initial value with the Laplace transformated expression

e) poles: roots of the denominator of a tranfer function

zeros: roots of the numerator of a transfer function

eigenvalues: roots of the characteristic polynomial

$(\det(A - \lambda I) \overset{!}{=} 0)$

Problem 2)

a) expressions without single s-expressions

$$P: \quad G(s) = k \cdot (\quad)$$

$$I: \quad G(s) = \frac{1}{s^i}(\quad) \qquad \text{with } i \geq 1$$

$$D: \quad G(s) = s^i(\quad)$$

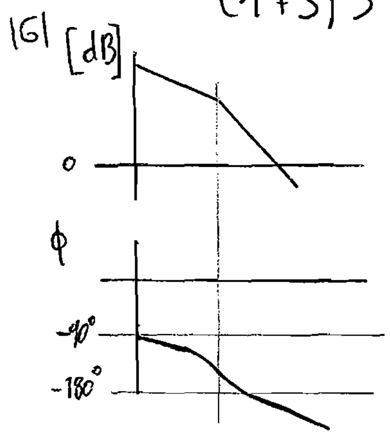
b) A system is state stable, if it returns after a deflection to the equilibrium position or does not leave with increasing amplitude.

- checking the eigenvalues of the matrix A
- checking the poles of the transfer function

c) $$T_3 \dddot{x}_a(t) + T_2 \ddot{x}_a(t) + T_1 \dot{x}_a(t) + x_a(t) = k\left[x_e(t) + \frac{1}{T_I}\int x_e(\cdot)\, d\tau\right]$$

$$G(s) = \frac{1 + \frac{1}{T_I \cdot s}}{s^3 T_3 + s^2 T_2 + s T_1 + 1} = \frac{T_I s + 1}{(T_3 s^3 + T_2 s^2 + T_1 s + 1)\, T_I s}$$

d) $$G_0 = \frac{k}{(1+s)s} \cdot e^{-s}$$




System without time delay:
→ Stable

System with time delay:
the system gets unstable with increasing time delay

e) $$G_z = \frac{G_s}{1 - G_s G_R} = \frac{T_I s + 1}{T_I s - (k_I \cdot s T_I + 1) \cdot k_D (1 + T_D s)}$$

$\Longrightarrow$ PD $T_2$

Problem 3)

a) $x_1 = y \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$x_2 = \dot{y}$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ -k & -0{,}5 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} [f(t) + g(t)]$$

$$y = [0 \;\; 1] \; x$$

$$\det(A - \lambda I) \overset{!}{=} 0$$

$$\Leftrightarrow \begin{vmatrix} -\lambda & 1 \\ -k & -0{,}5-\lambda \end{vmatrix} = \lambda^2 + 0{,}5\lambda + k \overset{!}{=} 0$$

$$\lambda_{1,2} = -0{,}25 \pm \sqrt{\frac{1}{16} - k}$$

z.B. Stodola: for $k > 0$, the system is stable

! The stability will not affected by different measurements.

b) Determination of input-output stability of a closed loop system by the nyquist plot of the corresponding open loop system.

problem 3)

c) $$Y = \frac{z\, G_s(s) + \omega\, G_k(s)\, G_s(s)}{1 + G_k(s)\, G_s(s)}$$

$$Y_z = \frac{k_p\,(1 + sT_1)}{\left[(1+sT_1) + (k_p k_D \cdot s)\right] \cdot s}$$

$$Y_\omega = \frac{k_p(1+sT_1)(s \cdot k_D)}{\left[(1+sT_1) + (k_p k_D \cdot s)\right] \cdot s}$$

$$E_z(s) = 0 - Y_z(s)$$

$$e_z(\infty) = \lim_{s \to 0} s\, E(s) = -k_p$$

$$E_w(s) = \omega(s) - Y_\omega(s)$$

$$e_\omega(\infty) = \lim_{s \to 0} s\, E(s) = 1$$

d) Asymptotic stability: $Re(\lambda_i) < 0$
(Ljapunov)

e) System 1: linear — the frequency of the input and the output signal is equal

system 2: not linear — the frequency of the input and the output signal is not equal

Problem 4

a) • $G(s) = \dfrac{3}{s^2(s^2+6s+13)}$

• Zeros : —

Poles : $p_1 = 0$
$p_2 = 0$
$p_3 = -3+2i$
$p_4 = -3-2i$

• $\phi_1 = 45°$
$\phi_2 = 135°$
$\phi_3 = 225°$
$\phi_4 = 315°$

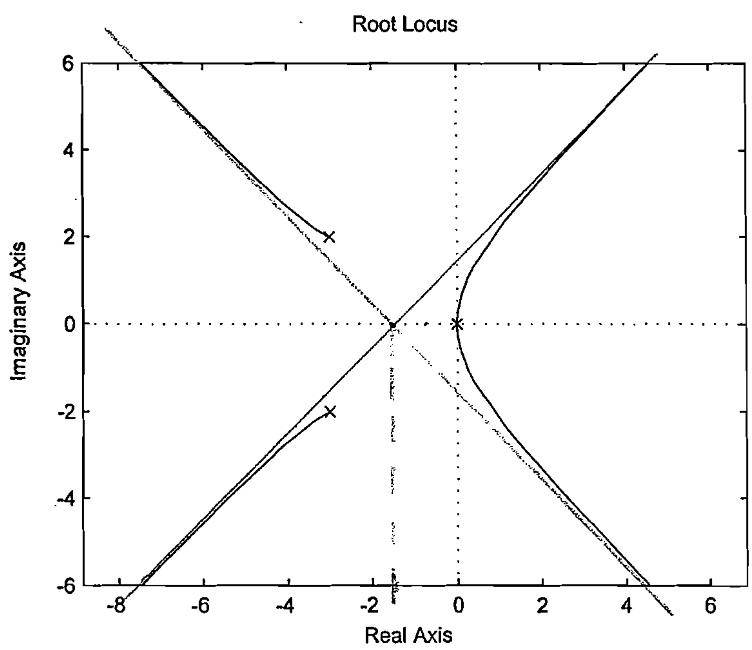
• $s_{Asymp} = -\frac{3}{2}$

• $s^2(s^2+6s+13) + 3K_p = 0$

$\Leftrightarrow s^4 + 6s^3 + 13s^2 + 3K_p = 0$

HURWITZ ⇒ System is always unstable for $K_p > 0$.
⇒ There is <u>no</u> $K_{krit}$

b)



Problem 4

c) $\frac{Y(s)}{W(s)} = \frac{3}{s^4+6s^3+13s^2+3K_p T_D s + 3K_p}$

Polsat $\pm 2i$ !

Insertion of one pole:

$16-48i-52+6K_p T_D i + 3K_p \overset{!}{=} 0$

$\Leftrightarrow -48i + 6K_p T_D i = 0$

$\wedge \; 16-52+3K_p = 0$

$\Leftrightarrow K_p = 12 \wedge T_D = \frac{2}{3}$

d) • $G(s) = \frac{3}{s^2(s^2+6s+13)} \cdot \left(1+\frac{2}{3}s\right)$

• Zeros: $P_{01} = -\frac{3}{2}$

Poles: $P_1 = 0;\ P_2 = 0;\ P_{3/4} = -3 \pm 2i$

• $\phi_1 = 60^0$

$\phi_2 = 180^0$

$\phi_3 = 300^0$

• $s_{Asymp} = -1{,}5$

• $P_{Imag,1} = +2i$ ; $P_{Imag,2} = -2i$ ; $K_{krit} = 12$

• Verzweigungspunkt (Bifurcation points)

$\frac{2(s+\frac{3}{2})}{(s+\frac{3}{2})^2} = \frac{2s}{s^2} + \frac{2s}{s^2} + 2 \cdot \frac{2(s+3)}{(s+3)^2+4}$

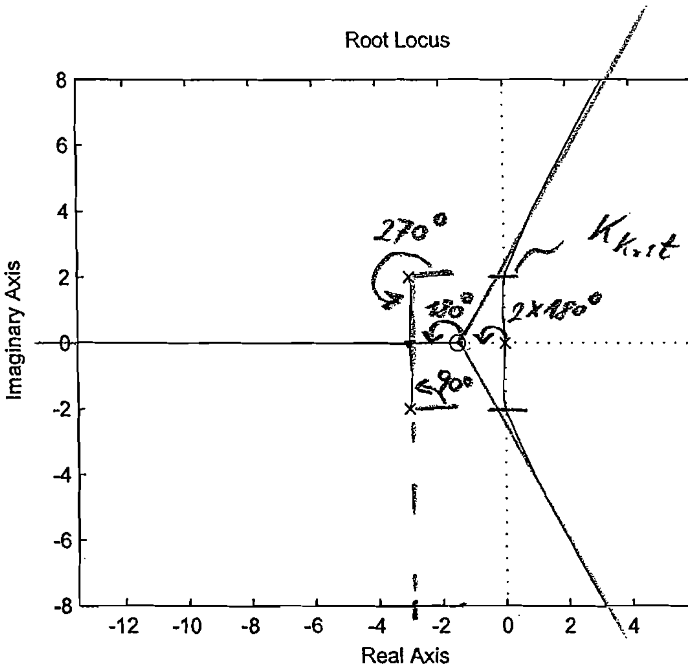
$\Leftrightarrow 3s^3 + 18s^2 + 40s + 39 = 0$

$\Leftrightarrow s_1 = -3$

$\left.\begin{matrix} s_2 \\ s_3 \end{matrix}\right\}$ complex! Not needed!

Problem 4

d)




Phasenbedingung (Phase condition): $180° - (270° + 90° + 2 \cdot 180°) = (2 \cdot l + 1)\, 180°$

$\Leftrightarrow l = -2$

$\Rightarrow s_1 = -3$ is a bifurcation point!

Problem 5

a) $F(s) = K_3 \cdot \dfrac{K_1 K_2 - 1 + K_1 T_I s}{(1+sT_1)(K_2 + sT_I)}$

b) $F(s) = -\dfrac{0{,}5 - T_I s}{0{,}5 + T_I s} \cdot \dfrac{K_3}{1+T_1 s}$

$$\left.\begin{aligned} &\text{Poles: } p_1 = -\frac{1}{2T_I} \;;\; p_2 = -\frac{1}{T_1} \\ &\text{Zeros: } p_{01} = \frac{1}{2T_I} \end{aligned}\right\}$$

All-pass + PT$_1$-System

=> no minimumphase system!

OR

zero in the right half of the s-plane

c) $F_a(s) = \dfrac{-Ks}{1+T_1 s}$

$F_b(s) = \dfrac{1}{s(1+T_0 s)}$

d) $F_1(s) = \dfrac{1+sT_1}{s((1+T_0 s)(1+T_1 s)+k)}$

$F_2(s) = \dfrac{-k}{(1+T_0 s)(1+T_1 s)+k}$

e) No, because of $\frac{1}{s}$ in $F_1$, denoting integrabel behavior.

f) Poles : $p_{1,2} = \frac{1}{T}\left(-1 \pm \sqrt{-k}\right)$

=> $\forall\, k > 0$ stable conjugated complex poles:

$p_{1,2} = \frac{1}{T}\left(-1 \pm i\sqrt{|k|}\right)$

=> $\forall\, K < -1$ one unstable pole $p = \frac{1}{T}\left(-1 + \sqrt{|k|}\right)$

# Aufgabe 6

a) Eigenvalues: $\det(\lambda I - A) = \begin{vmatrix} \lambda-4 & 2 \\ -5 & \lambda+3 \end{vmatrix} = (\lambda-4)(\lambda+3)+10$

$$= \lambda^2 - \lambda - 2 \overset{!}{=} 0$$

$\lambda_1 = -1 \quad \lambda_2 = 2$

b) Transfer function: $G_1(s) = \frac{1}{s+1}$

poles: $s+1 \overset{!}{=} 0 \qquad s = -1$

The system is not asymptotically stable, because one of the eigenvalues, $\lambda_2 = 2$, has a positive real part.

The system is however I/O stabl, because the only pole, $s=-1$, has negative real part.

c) Transfer function of the system

$$G_5(s) = G_1 \cdot (G_2 + G_3)$$

$$= \frac{1}{s+1} \left( \frac{2}{s+3} + \frac{2}{s-2} \right)$$

$$= \frac{2(s-2) + 2(s+3)}{(s+1)(s+3)(s-2)}$$

$$= \frac{4s+2}{(s+1)(s+3)(s-2)}$$

poles: $(s+1)(s+3)(s-2) \overset{!}{=} 0$

$s_1 = -1 \qquad s_2 = -3 \qquad s_3 = +2$

zeros: $4s_0 + 2 \overset{!}{=} 0$

$s_0 = -\frac{1}{2}$

The system is not a minimum-phase system, because the pole $s_3 = +2$ has a positive real part.

d) Transfer function of the closed-loop system with P-controller

$$G(s) = \frac{G_0(s)}{1+G_0(s)} = \frac{G_R(s)\,G_S(s)}{1+G_R(s)\,G_S(s)}$$

$$= \frac{K_1\,(4s+2)}{(s+1)(s+3)(s-2)+(4s+2)\,K_1}$$

$$= \frac{K_1\,(4s+2)}{s^3+2s^2+(4K_1-5)\,s+2K_1-6}$$

Hurwitz criterium

$a_3 = 1 \quad a_2 = 2 \quad a_1 = 4K_1-5 \quad a_0 = 2K_1-6$

i) all coefficients exist and have same sign for asymptotic stability.

$$\left.\begin{array}{l} a_3 = 1 > 0 \\ a_2 = 2 > 0 \\ a_1 = 4K_1-5 > 0 \\ a_0 = 2K_1-6 > 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} K_1 > \frac{5}{4} \\ K_1 > 3 \end{array}\right\} \Rightarrow K_1 > 3$$

ii)

$$H = \begin{vmatrix} 2 & 2K_1-6 & 0 \\ 1 & 4K_1-5 & 0 \\ 0 & 2 & 2K_1-6 \end{vmatrix}$$

$$\left.\begin{array}{l} H_1 = |2| > 0 \\ H_2 = \begin{vmatrix} 2 & 2K_1-6 \\ 1 & 4K_1-5 \end{vmatrix} \begin{array}{l} = 8K_1-10-2K_1+6 \\ = 6K_1-4 > 0 \end{array} \Rightarrow K_1 > \frac{4}{6} \\ H_3 = (2K_1-6)\cdot H_2 \Rightarrow 2K_1-6 > 0 \quad K_1 > 3 \end{array}\right\} \Rightarrow K_1 > 3$$

From i) and ii), the controlled system is asymptotically stable for $K_1 > 3$.

e) Remaining control error

$$e(t) = w(t) - y(t)$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,(W(s) - Y(s)) = \lim_{s\to 0} s \cdot W(s)\,(1 - G(s))$$

$$\left[1(t) \xrightarrow{\mathcal{L}} \frac{1}{s}\right] \quad = \lim_{s\to 0} s \cdot \frac{1}{s}\,(1 - G(s))$$

$$= \lim_{s\to 0} \left(1 - \frac{K_1(4s+2)}{s^3 + 2s^2 + (4K_1 - 5)s + 2K_1 - 6}\right)$$

for $K_1 = 4$ $$= 1 - \frac{2K_1}{2K_1 - 6} = 1 - \frac{8}{8-6} = -3$$

The output of the system is not equal to the reference value for $t \to \infty$, because the remaining control error is not zero.

f) $$G_{R2}(s) = K_2 \frac{1+6s}{6s} = K_2\left(1 + \frac{1}{6s}\right)$$

It is a PI-controller.

g) Remaining control error

$$e(t) = w(t) - y(t)$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s(W(s) - Y(s)) = \lim_{s\to 0} s\,(W(s) - G_W(s)W(s) - G_D(s)D(s))$$

Transfer function of the controlled system according to reference

$$G_W(s) = \frac{G_0(s)}{1 + G_0(s)}$$

$$= \frac{K_2(6s+1)(4s+2)}{6s(s+1)(s+3)(s-2) + K_2(6s+1)(4s+2)}$$

Transfer function from the disturbance to the output

$$G_D(s) = \frac{1}{1 + G_0(s)}$$

$$= \frac{6s(s+1)(s+3)(s-2)}{6s(s+1)(s+3)(s-2) + (4s+2)K_2(1+6s)}$$

g) $w(t) = 1(t)$ $\qquad d(t) = \delta(t)$

$\Rightarrow W(s) = \frac{1}{s} \qquad D(s) = 1$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot \left[ \frac{1}{s} - \frac{K_2(6s+1)(4s+2)}{6s(s+1)(s+3)(s-2) + K_2(6s+1)(4s+2)} \cdot \frac{1}{s} - \frac{6s(s+1)(s+3)(s-2)}{6s(s+1)(s+3)(s-2) + (4s+2)K_2(1+6s)} \cdot 1 \right]$$

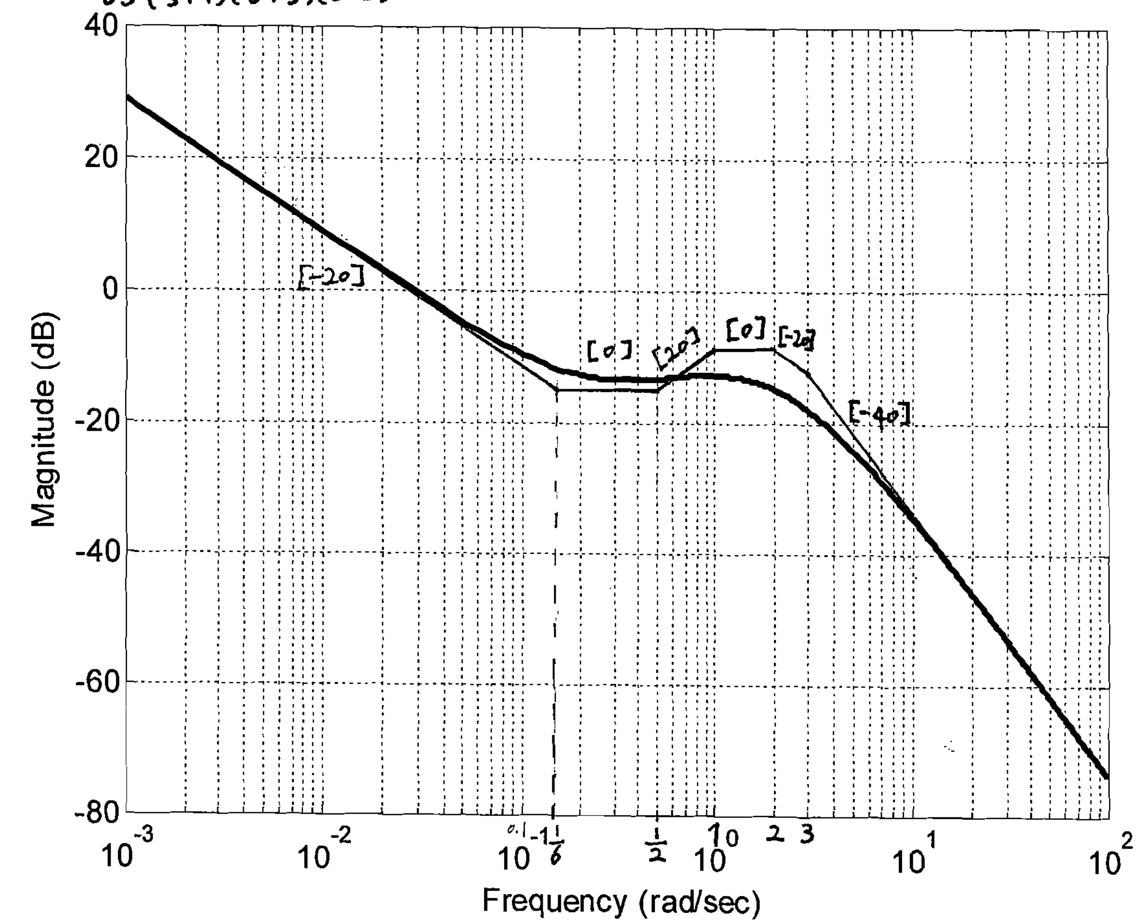
$$= 1 - \frac{2K_2}{2K_2} - 0$$

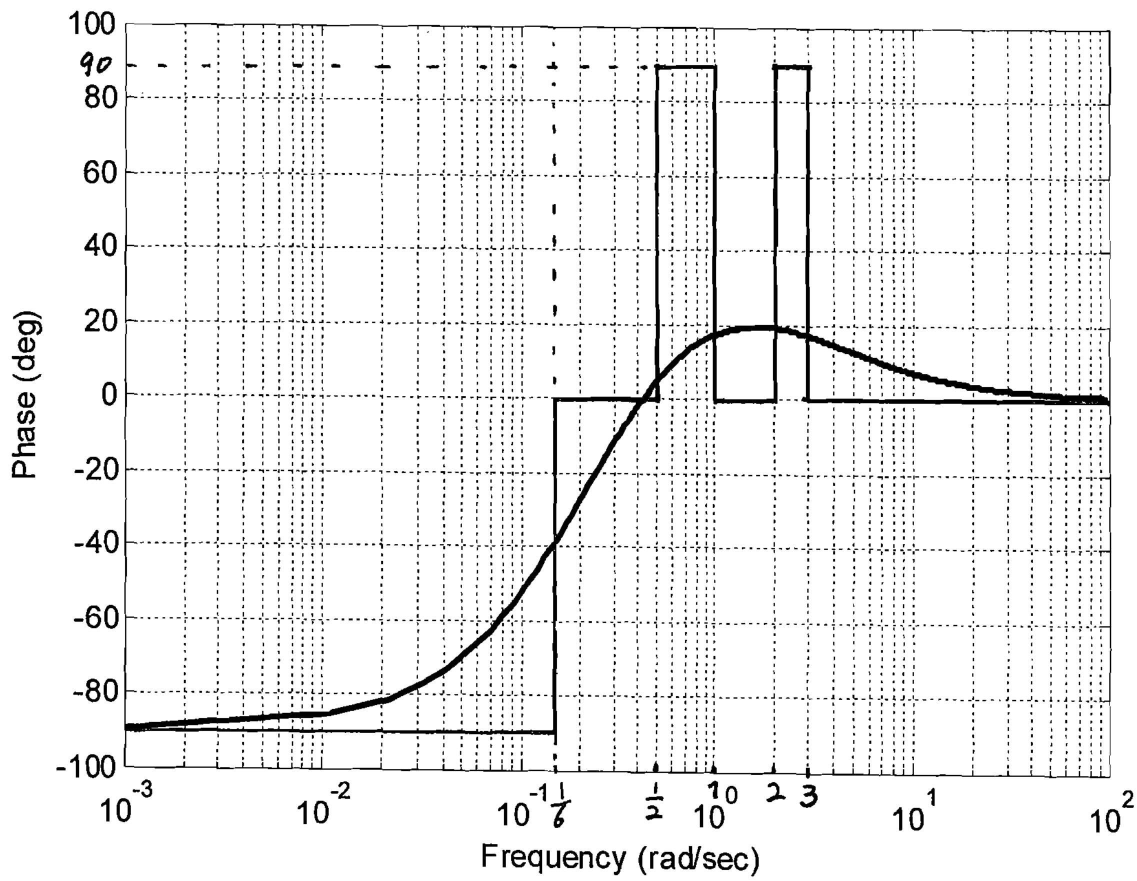
$$= 0$$

The system output will finally be equal to the reference value, because the remaining control error is zero.

h) Open-loop system: poles: $s_1 = 0$ $s_2 = -1$ $s_3 = -3$ $s_4 = 2$

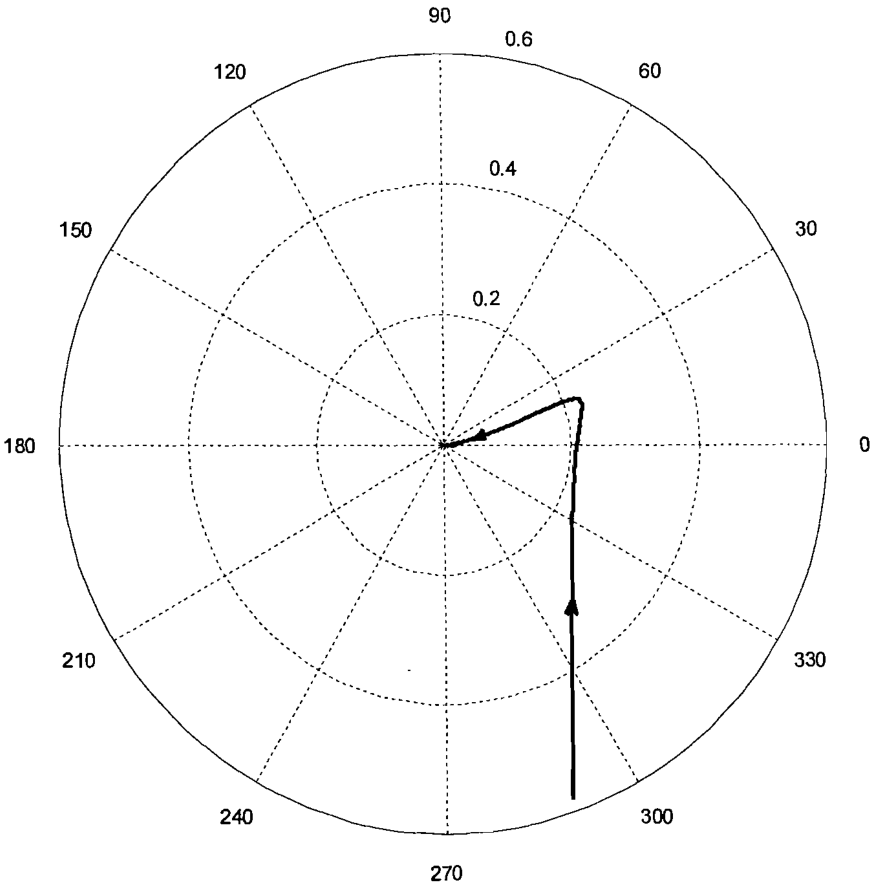
$G_o(s) = \frac{k_2(6s+1)(4s+2)}{6s(s+1)(s+3)(s-2)}$ zeros: $s_{o1} = -\frac{1}{6}$ $s_{o2} = -\frac{1}{2}$



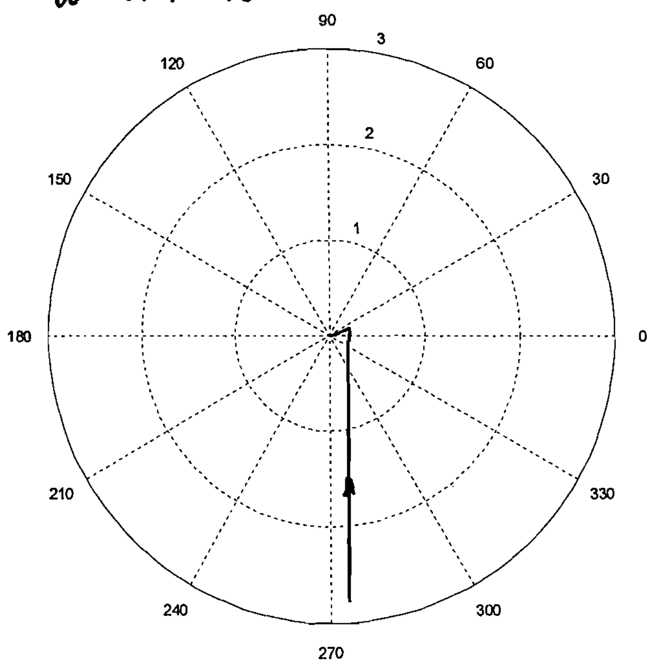



i)

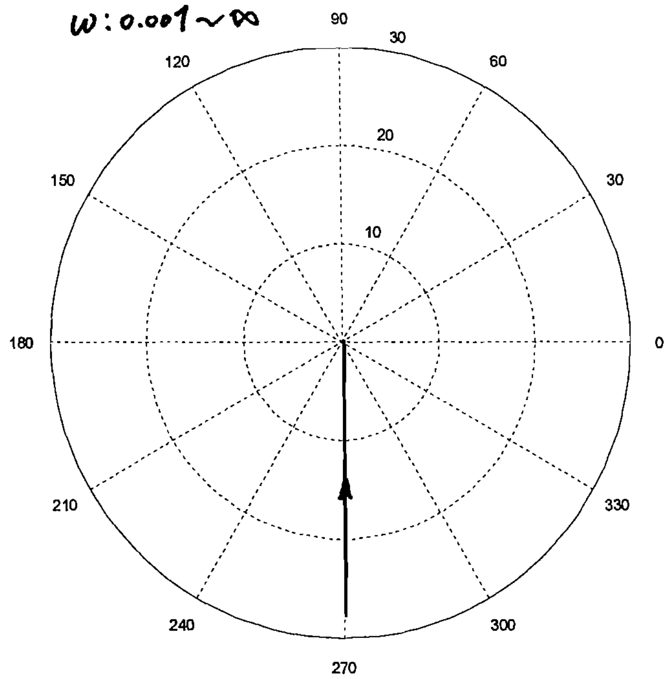
$w: 0.1 \sim \infty$




$w: 0.01 \sim \infty$




$w: 0.001 \sim \infty$




The special Nyquist criterium can not be applied here, because the open-loop system is unstable according to the unstable pole, $s_4 = 2$.