LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(each 2 points)

a) Define the terms 'input variable' and 'desired variable'.

b) Define the function $u(t) = 1(t-1) + 2(t-2) - 3(t-3)$ graphically.

c) Define the physical meaning of the terms 'pole' and 'zero' of a transfer function. How does a so called double pole look like in a Bode-diagram (use a case distinction if necessary)?

d) Give the I/O-relationship of a PIDT_1-system by its differential equation and transfer function.

e) Define mathematically as well with an adequate drawing the amplitude and phase margin of a controlled system. What is the underlying transfer function representing with respect to controller and plant?

Problem 2

(each 2 points)

a) A stable transfer system shows a PT_2-transfer behavior. Derive the Laplace transformation of the weighting function of the system and define mathematically the stationary final value.

b) A system has the eigenvalues $-2 \pm j2$ as well as $-4 \pm j2$. Which eigenvalue has the smallest damping coefficient? Answer mathematically or with a drawing.

c) A Fourier transformation is given by

$$y = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos(x) + \frac{\cos(3x)}{3^2} + \frac{\cos(5x)}{5^2} + \frac{7x}{7^2} + \dots\right).$$

Draw the concerning discrete amplitude spectrum in the given coordinate frame and complete the related axes nomenclatures.

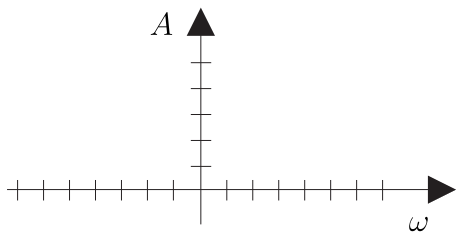

Figure 2.1: Coordinate system

d) Given is the Laplace transformation of a function with $T_1, T_2 > 0$

$$f_a(s) = \frac{1}{T_1T_2s^3 + (T_1+T_2)s^2 + s}.$$

Assume that the function $f_a(s)$ is the output function of a system with a PT_2-transfer behavior

$$G(s) = \frac{K}{T_1T_2s^2 + (T_1+T_2)s + 1}$$

(with $K = 1, T_1, T_2 > 0$), which input $f_e(t)$ was given to the system?

e) A transfer system with PDT_1-behavior is controlled by a transfer system with P-behavior with negative feedback. Define the disturbance transfer function and the reference transfer function. Is the closed loop stable?

Problem 3

(15 points)

A system is modeled by using $G_1(s)$ to $G_6(s)$ according to Figure 3.1.

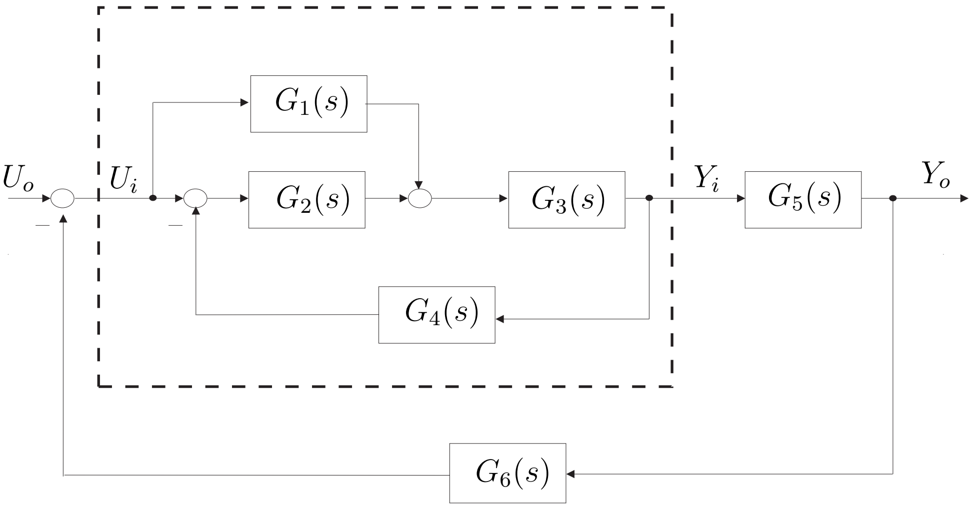

Figure 3.1: Block diagram of a technical system

a) (4 points)
Derive the transfer function $G_i(s) = Y_i(s)/U_i(s)$ of the marked inner system.

b) (3 points)
Derive the transfer function $G_o(s) = Y_o(s)/U_o(s)$ of the whole system.

A new system is given by the differential equation

$$2\ddot{y}(t) - 4\dot{u}(t) = -10\dot{y}(t) - 12y(t) + 2\int u(t)dt + 12u(t).$$

c) (3 points)
Derive the state space description of the given system.

d) (2 points)
Calculate the transfer function of the system.

e) (3 points)
Define the stability of the system from the derived transfer function.

Problem 4

(15 points)

A plant is described by the transfer function

$$G_p(s) = \frac{5 \times 10^7 (s+5)(s+10)}{(s+1)(s+50)(s+100)(s+500)(s+1000)}.$$

Two different controllers will be applied to control the plant with negative feedback.
The first one is a P-controller with control gain $K_p = 1$,
the second one is a PD-controller with gain $K_p = 800$ and the time constant $T_d = \frac{1}{800}$.

a) (6 points)
Determine the transfer functions of the P-controller $G_1(s)$ and the PD-controller $G_2(s)$. Draw the two Bode-diagrams of the corresponding open-loop systems qualitatively.

b) (2 points)
Calculate the phase margin of the control loop with the P-controller
(*Hint: for* $\omega \geq 50$ *rad/s use* $|G_p(s)| < 1$) and the amplitude margin of the control loop with the PD-controller.

c) (2 point)
Is it possible to apply the special Nyquist criterion here? State reason.

d) (2 points)
Are the close-loop systems stable? State reason with Nyquist criterion.

e) (3 points)
Sketch the polar plots of the two open loops.

Problem 5

(16 points)

The transfer function of a plant is given by

$$G_p(s) = \frac{(s+3)(s-1)(s-2)}{s(s+0.5)(s+2)(s+4)(s^2+2s+2)}.$$

a) (2 points)
Calculate the poles and zeros of the plant. Give a statement about the stability of the plant and state reason.

b) (3 points)
Determine the damping and eigenfrequencies of the plant's poles.

Beside the plant, two controllers are given by the differential equations

$$y(t) = K_p u(t)$$

and

$$\frac{1}{2}\ddot{y}(t) - \frac{3}{2}\dot{y}(t) + y(t) = K_d \dot{u}(t).$$

Both are combined individually with the given plant to a closed loop with negative feedback. In the following, the root locus method has to be applied to analyze the dynamics of both closed-loop systems consisting of plant and controller in each case.

c) (3 points)
Classify both controllers and determine the transfer function as well as the poles and zeros. (*Hint:* $1.5^2 = 2.25$)

d) (2 points)
Give the transfer function of both open loops and calculate the corresponding poles and zeros.

e) (2 points)
Calculate the number of separate branches and the number of branches going to infinity in both root loci.

f) (2 points)
Calculate the angles of the asymptotes and the center of the root loci.

g) (2 points)
Sketch the root loci of both systems with their asymptotes and mark the center of the root loci σ_w as well as the critical gains K_{crit}.

Maximum achievable points:	**66**
Minimum points for the grade 1,0:	**95 %**
Minimum points for the grade 4,0:	**50 %**