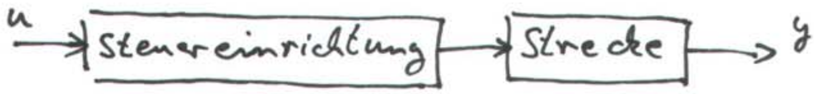
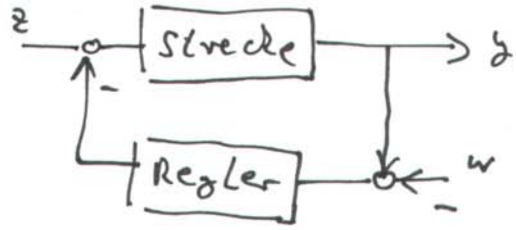
A1

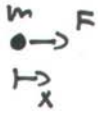
a) Steuerung :



Regelung :



b) Analytische Modellbildung : z.B. Impulssatz



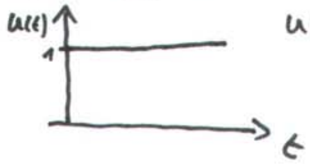
$m\ddot{x} = F$

Experimentelle Modellbildung : z.B. Modalanalyse

c) Impulsfunktion

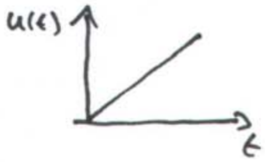
$u(t) = \delta(t)$

Sprungfunktion



$u(t) = 1(t)$

Rampenfunktion



$u(t) = t$

d)

$u_1(t) \to y_1(t)$ $\quad$ $u(t) = k u_1(t) + l u_2(t)$

$u_2(t) \to y_2(t)$ $\quad$ $y(t) = k y_1(t) + l y_2(t)$

e) Die Frequenz hat sich geändert.

A2

a) $$y(t) = \int_0^t g(t-\tau)\, u(\tau)\, d\tau$$

b) $$f(t) = 2(t) - 1(t-1)$$

$$\mathcal{L}\{f(t)\} = \frac{2}{s} - e^{-s} \cdot \frac{1}{s}$$

c) Pole: $s_1 = 0$

$s_2 = -\frac{1}{T_2}$ $\qquad \mathrm{Re}\{s_i\} \leq 0 \rightarrow$ zustandsstabil

d) PDT_2: $$T_1 \ddot{y} + T_2 \dot{y} + y = K(u + T_D \dot{u})$$

T_1, T_2: Verzögerungszeitkonst.

K: Verstärkung

T_D: Differenziationszeitkonst. (Differenzierkonstante)

e) $G_R = K$

$G_s = \frac{1}{s}$ $$G_w = \frac{K\frac{1}{s}}{1 + \frac{K}{s}} = K \frac{1}{s+K} \qquad PT_1$$

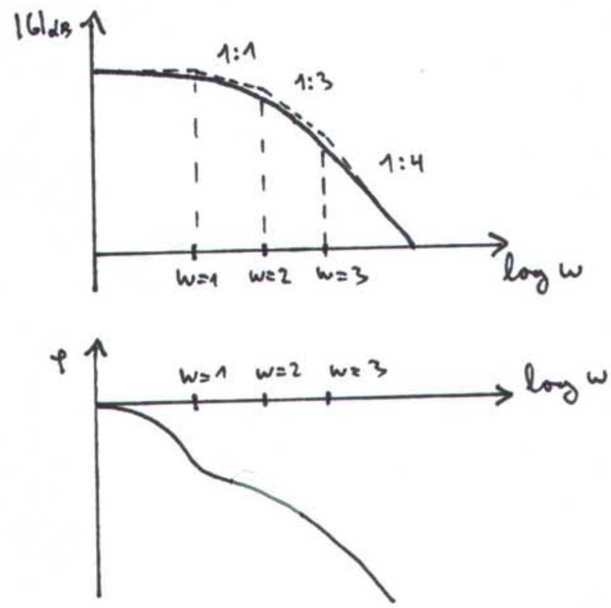
A3

a) Das Nyquistverfahren lässt sich zur Prüfung des Stabilitätsverhaltens des geschlossenen Regelkreises anwenden.

Aussage : Wie ist der Gesamtverstärkungsfaktor zu wählen, so dass der geschlossene Kreis stabil ist.

Es muss bekannt sein, ob in Mit- oder Gegenkopplung geregelt wird und wieviel Pole im Ursprung vorhanden sind.

b)

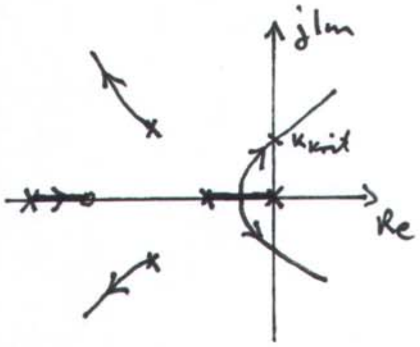


c) Charakt. Gleichung : Polynom 2er Ordnung

$K_1^2 > 0$ Stabil für $K_1 \neq 0$

$4 + K_2 > 0$ $K_2 > -4$

d)



stabilisierbar für $K < K_{krit}$

e)

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{K+K_R}{m} & -\frac{d_1+d_2}{m} \end{bmatrix}}_{\text{Systemmatrix } A} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{\text{Eingangsmatrix } B} u$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{\text{Ausgangsm. } C} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

Allg.

$\dot{x} = Ax + Bu$

$y = Cx$

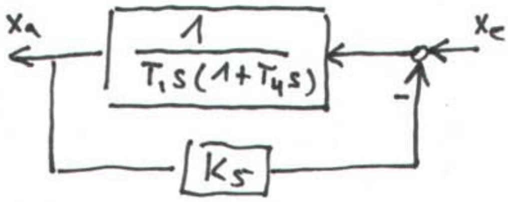
A4

a)
$$y(s) = G(s)\,u(s) = \frac{1}{1+Ts}\,\frac{1}{s} = \frac{1}{s(1+Ts)}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,y(s) = \lim_{s\to 0}\frac{1}{1+Ts} = 1$$

$$\lim_{t\to 0} y(t) = \lim_{s\to\infty} s\,y(s) = \lim_{s\to\infty}\frac{1}{1+Ts} = 0$$

b) Regler



$$(x_e - K_5\,x_a)\,\frac{1}{T_1 s(1+T_4 s)} = x_a$$

$$G_R = \frac{x_a}{x_e} = \frac{1}{T_1 T_4 s^2 + T_1 s + K_5}$$

$$G_s = \frac{K_1 K_2 K_3}{(T_1 s+1)(T_3 s+1)}$$

$$G_w = \frac{K_1 K_2 K_3}{(T_1 s+1)(T_3 s+1)(T_1 T_4 s^2 + T_1 s + K_5) + K_1 K_2 K_3}$$
$$= \frac{K_1 K_2 K_3}{(T_1 T_3 s^2 + (T_1+T_3)s + 1)(T_1 T_4 s^2 + T_1 s + K_5) + K_1 K_2 K_3}$$

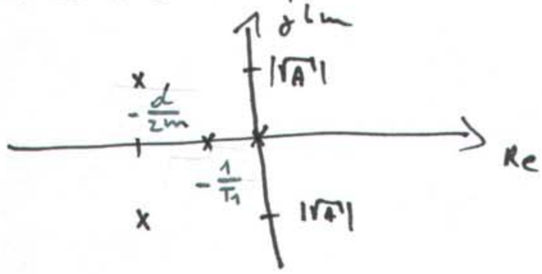
c)
$$G_1 = \frac{1}{ms^2 + ds + k} \qquad G_R = \frac{K_R}{(1+T_1 s)s}$$

$$G_0 = \frac{K_R}{(ms^2+ds+k)(1+T_1 s)s}$$

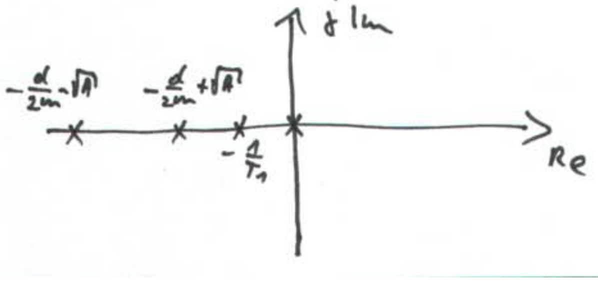
Nullst. – Polstelle:

$$s_1 = 0$$
$$s_2 = -\frac{1}{T_1}$$
$$s_{3,4} = -\frac{d}{2m} \pm \sqrt{\underbrace{-\frac{k}{m} + \left(\frac{d}{2m}\right)^2}_{A}}$$

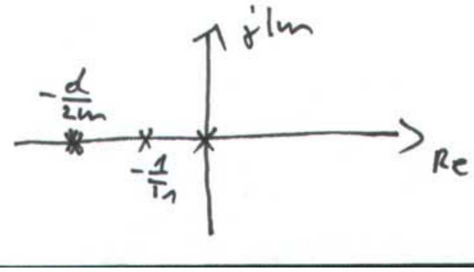
wenn $A < 0$



wenn $A > 0$



wenn $A = 0$



d) $G_0(s) = \frac{1}{(s^2+s+1)s}$

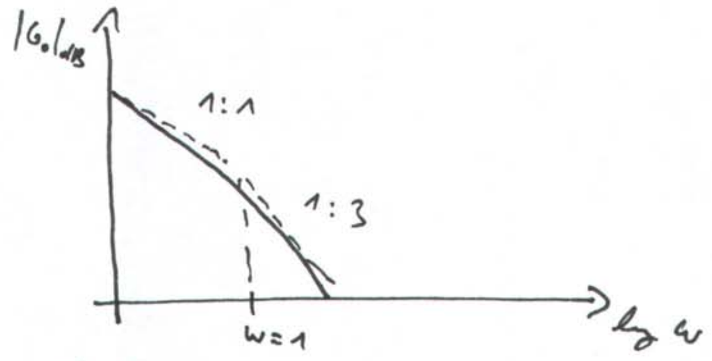
Pole: $s_1 = 0$

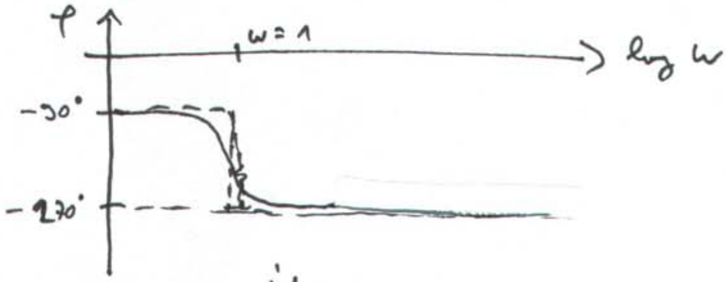
$s_{2,3} = -\frac{1}{2} \pm j\sqrt{\frac{3}{4}}$

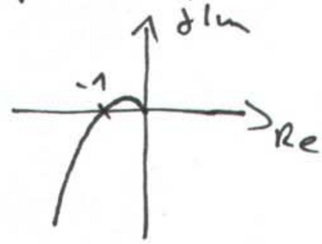
Allg. PT_2 ch. gleichung

$$\frac{1}{\omega_0^2}s^2 + \frac{2D}{\omega_0}s + 1 = 0$$

$\Rightarrow \omega_0 = 1 \quad ; \quad D = \frac{1}{2}$







$$G(j\omega) = \frac{1}{j\omega}\,\frac{1}{-\omega^2 + j\omega + 1}$$

$$= \frac{-\omega^2}{\omega^4(\omega-\omega^3)^2} - j\,\frac{\omega-\omega^3}{\omega^4+(\omega-\omega^3)^2}$$

$\mathrm{Im}\{G(j\omega)\} = 0$

$\Rightarrow \omega - \omega^3 = 0$

$\Rightarrow \omega = 1$

für $\omega = 1$: $\mathrm{Re}\{G(j\omega)\} = -1$

$\Rightarrow$ geschlossener Kreis wäre Grenzstabil

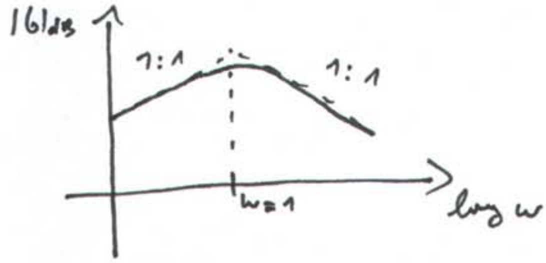
Stabilitätskrit. für das spezielle Nyquistkrit. zur Überprüfung im Bodediag.

$\phi_R > 0$

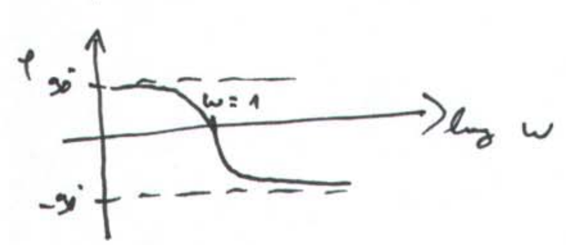
$A_R > 1$

e)

$$G_0 = \frac{s}{s^2+s+1}$$



$\phi_R > 0$ immer erfüllt



A5

a) G_{R1} : P Übertragungsverhalten

G_{R2} : PID Übertragungsverhalten

b)

i) $G_0 = \frac{K_{R1}}{(s+1)(s+4)(s-1)}$

Pole: $s_1 = -1$, $s_2 = -4$, $s_3 = 1$ $\quad n = 3$

Nullst. – $\quad q = 0$

Anzahl der Asymptoten: $n - q = 3$

Wurzelschwerpunkt: $\sigma_w = \frac{1}{3}(-1-4+1) = -\frac{4}{3} = -1,\overline{3}$

Winkel der Asymptoten: $\phi_{Asymp} = \frac{180° + l\,360}{n-q}$ $\quad l = 0, 1, \ldots$

$l = 0 \quad \phi_1 = 60°$

$l = 1 \quad \phi_2 = 180°$

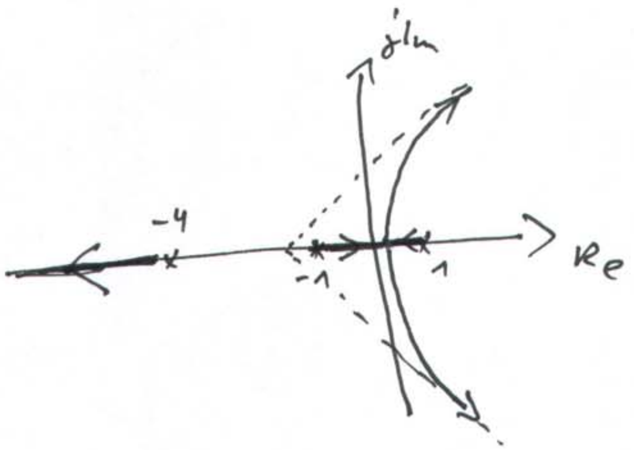
$l = 2 \quad \phi_3 = -60°$

Verzweigungspunkt: $\sum_{j=1}^{n} \frac{1}{a - s_j} = \underbrace{\sum_{i=1}^{q} \frac{1}{a - s_{0i}}}_{\text{hier } 0}$

$$\frac{1}{a+1} + \frac{1}{a+4} + \frac{1}{a-1} = 0$$

$\Rightarrow a = 0{,}1196$

$(a = -2{,}7863)$ kein Wurzelort



P-Regler ist nicht in der Lage den Regelkreis zu stabilisieren!

ii) $G_0 = K_R \dfrac{s+1}{s(s+4)(s-1)}$

Pole: $s_1 = 0$ Nullst. $s_{01} = -1$

$s_2 = -4$

$s_3 = 1$ $n = 3$ $q = 1$

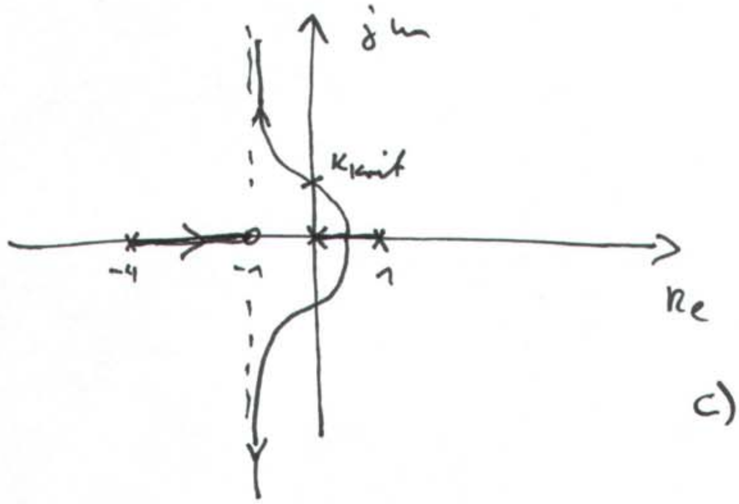
Anzahl der Asympt.: $n - q = 2$

Wurzelschwerpunkt: $\sigma_w = \frac{1}{2}(-4+1+1) = -1$

Winkel der Asymptoten: $\phi_{Asympt} = \dfrac{180° + l\,360°}{n-q}$ $l = 0, 1, \dots n-q$

$l = 0$ $\phi_1 = 90°$

$l = 1$ $\phi_2 = 270°$



c) PID Regler ist zu wählen

d) Ab $K > K_{krit}$ ist der geschlossene Kreis stabil.

$$K = \frac{\prod_{i=1}^{n} |s - s_i|}{\prod_{i=1}^{q} |s - s_{0i}|}$$

A6

a) $\left[u - K_2\left(y + \frac{y}{K_1}\right)\right] G_1 K_1 = y \qquad G(s) = \frac{K_1 G_1}{1 + G_1 K_1 K_2 + G_1 K_2}$

$$\frac{y}{u} = \frac{1}{s^2 + s + 4} \qquad PT_2$$

b) PID

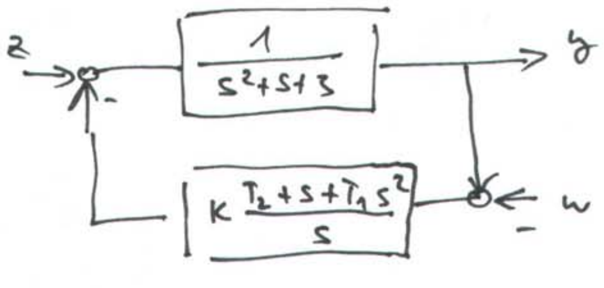
$$X_a = K\left(X_e + T_1 s X_e + T_2 \frac{1}{s} X_e\right)$$

$$G_R = \frac{X_a}{X_e} = K \frac{T_2 + s + T_1 s^2}{s}$$

$$G_0 = G_R G_S = \frac{1}{s^2 + s + 4} \, K \frac{T_1 s^2 + s + T_2}{s}$$

Pole: $s_{1,2} = -0{,}5 \pm \sqrt{-3{,}75} \qquad s_0 = 0$

$$s_{3,4} = -\frac{1}{2T_1} \pm \sqrt{-\frac{T_2}{T_1} + \left(\frac{1}{2T_1}\right)^2}$$



c)

$$G_w = \frac{K \frac{T_1 s^2 + s + T_2}{s(s^2+s+4)}}{1 + K \frac{T_1 s^2 + s + T_2}{s(s^2+s+4)}} = \frac{K(T_1 s^2 + s + T_2)}{s(s^2+s+4) + K(T_1 s^2 + s + T_2)}$$

charakt. Polynom.

$$s^3 + (KT_1 + 1)s^2 + (K+4)s + KT_2 = 0$$

Hurwitz. $s^3 + (T_1+1)s^2 + 5s + T_2 = 0$

Alle Koef. vorhanden und gleiches Vorzeichen:

$T_1 + 1 > 0$

$\Leftrightarrow T_1 > -1 \quad (T_1 > 0)$

$T_2 > 0$

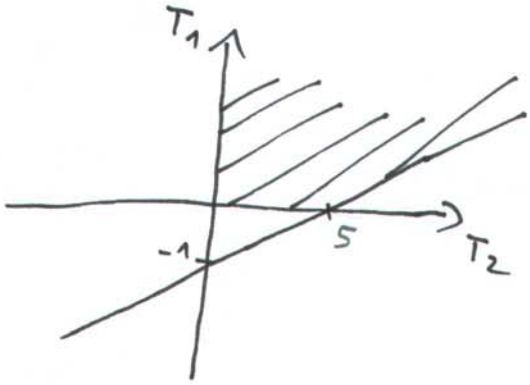
$$H = \begin{bmatrix} 1+T_1 & T_2 & 0 \\ 1 & 5 & 0 \\ 0 & 1+T_1 & T_2 \end{bmatrix}$$

$\det(H_1) = 1 + T_1 > 0$

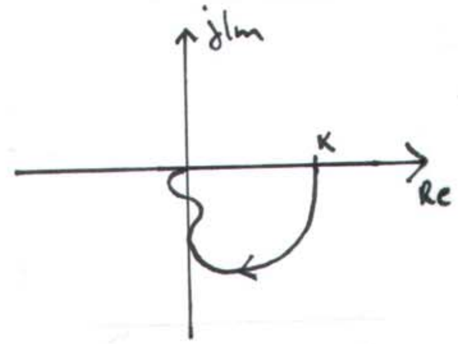
$\det(H_2) = 5(1+T_1) - T_2 > 0$

$\Leftrightarrow T_1 > -1 + \frac{1}{5}T_2$

$\det(H_3)$: $T_2 > 0$



d) $G(s) = \frac{K(s+5)}{(2+s)(100+s)^2}$ PDT_3



e) Für alle K stabil.