

problem 1) a) usage in time domain

differential equations \Rightarrow stationary + not stationary

usage in frequency domain

transfer functions \Rightarrow stationary

b) validity of the principle of superposition

c) step function:

$$1(t) = \begin{cases} 0 & \forall t < 0 \\ 1 & \forall t \geq 0 \end{cases} \quad F(s) = \frac{1}{s}$$

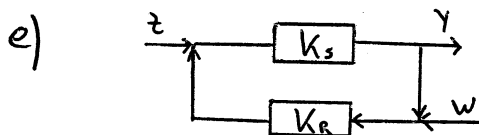
Dirac (Impulse) function:

$$u(t) = \left\{ \frac{1}{\alpha} \cdot [1(t) - 1(t - \alpha)] \right\} \quad F(s) = 1$$

ramp function

$$u(t) = \int_{-\infty}^t 1(\tau) d\tau = 1(t) t \quad F(s) = \frac{1}{s^2}$$

d) Laplace Transform of the differential equation
graphical description of the transfer behavior by
the phase plot and frequency response



disturbance transfer function

$$G_z(s) = \frac{k_s}{1 + k_s k_R}$$

difference:

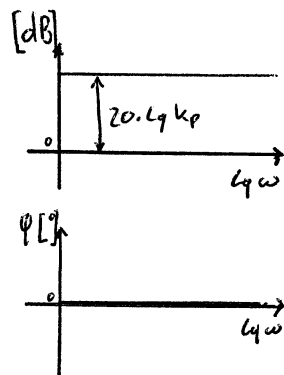
$$k_R \rightarrow \infty \Rightarrow \begin{aligned} G_z(s) &\rightarrow 0 \\ G_w(s) &\rightarrow 1 \end{aligned}$$

reference transfer function

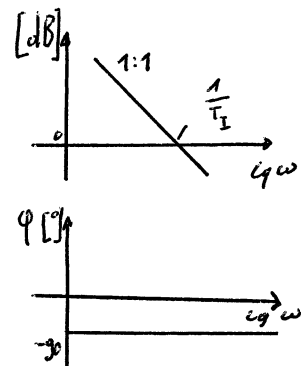
$$G_w(s) = \frac{k_s k_R}{1 + k_s k_R} = \frac{k_s}{k_s + \frac{1}{k_R}}$$

Problem 2

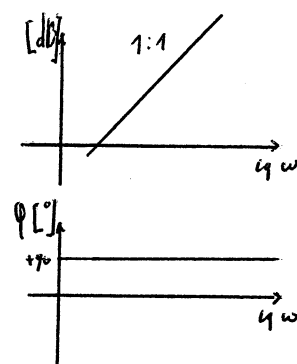
a) Proportional



integrating



differentiating

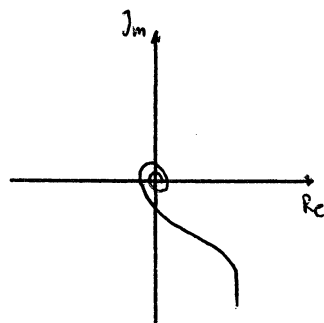


- b) checking of the eigen values of the matrix A
 checking of the poles of the transfer function

stability: A system is stable, if it returns after a deflection to the equilibrium position or doesn't leave it with increasing amplitude

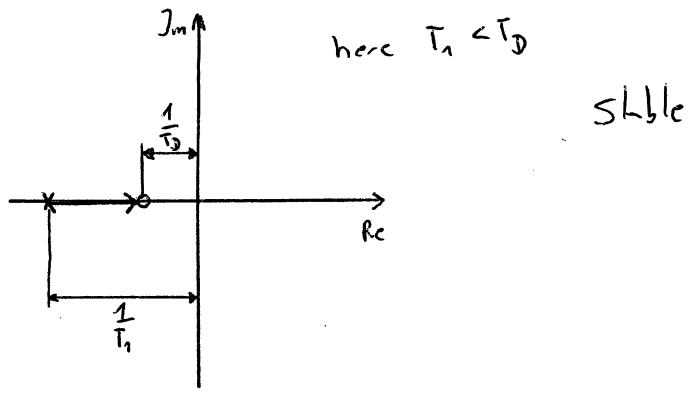
$$c) T_2^2 \ddot{x}_a(t) + T_1 \dot{x}_a(t) + x_a(t) = \frac{1}{T_I} \int x_c(t - T_d) dt + k_p x_c(t - T_d)$$

poles plot:

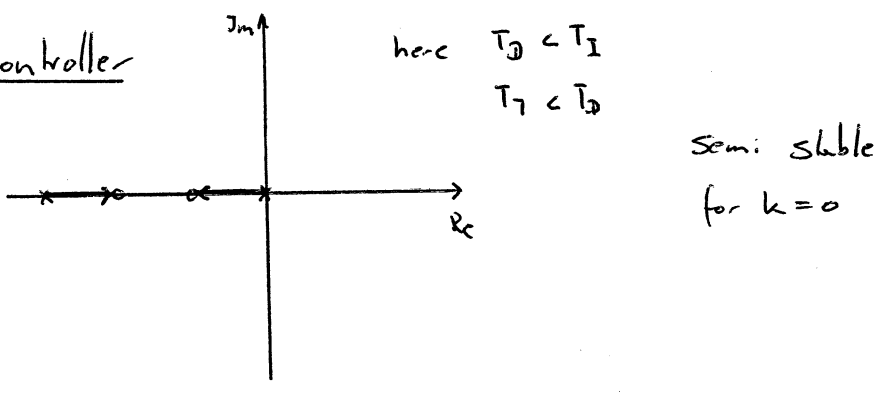


Problem 2

d) with P controller

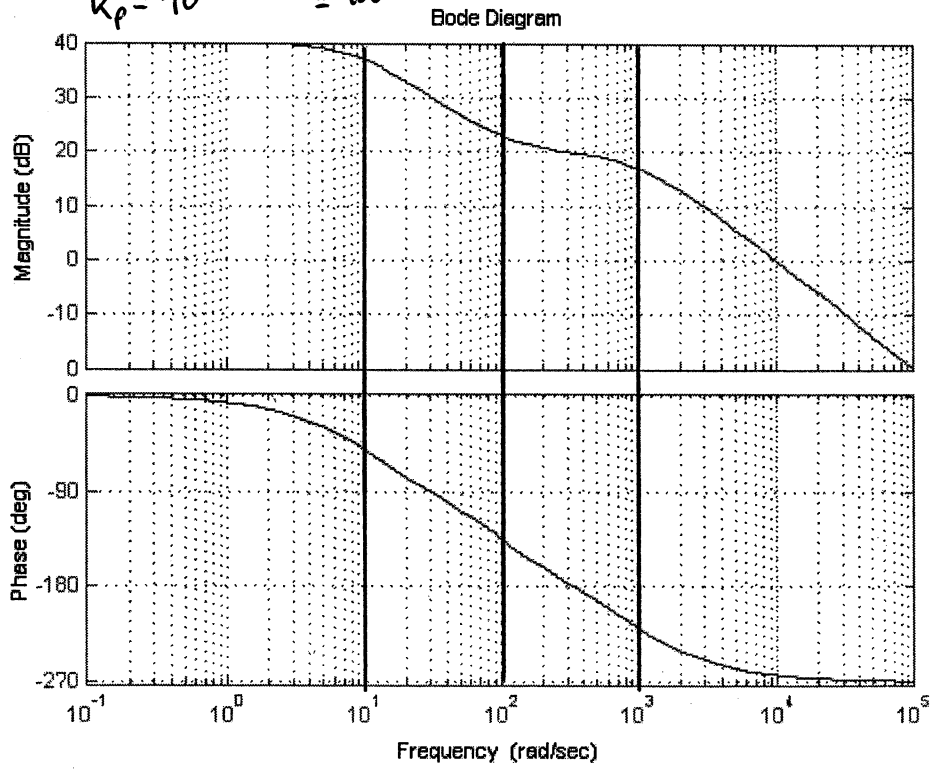


with PI controller



e)

$$k_p = 10 \lg\left(\frac{40}{20}\right) = 100$$



$$\Rightarrow G(s) = 100 \frac{s \cdot 0,01 - 1}{(s \cdot 0,1 + 1)(s \cdot 0,001 + 1)}$$

Problem 3

$$a) \quad x = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t)$$

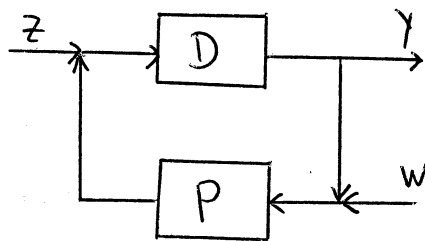
$$\dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} \quad c = [0 \ 1] \quad , \quad d = 0$$

$$\det(A - \lambda E) \stackrel{!}{=} 0$$

$$\lambda^2 + \frac{d}{m} \lambda + \frac{k}{m} = 0$$

Stabilität: $m > 0 \rightarrow d, k > 0$ für Stabilität

b)



disturbance transfer function

$\textcircled{DT_1}$

$$G_z(s) = \frac{s \cdot k_D}{1 - s \cdot k_D \cdot k_P} \quad \xrightarrow{\text{Stabilität}} \quad k_D \cdot k_P < 0$$

reference transfer function

$\textcircled{DT_1}$

$$G_w(s) = \frac{s \cdot k_D \cdot k_D}{1 - s \cdot k_D \cdot k_P} \quad \xrightarrow{\text{Stabilität}} \quad k_D \cdot k_P < 0$$

Problem 3)

c) characteristic equation

$$(s+1)(s-k_2)(s^2+k_1s+s+k_1) = 0$$

$$s_{1,2} = -\frac{k_1+1}{2} \pm \sqrt{\frac{(k_1+1)^2}{4} - k_1}$$

$$= -\frac{k_1+1}{2} \pm \sqrt{\frac{k_1^2 - 2k_1 + 1}{4}}$$

$$= -\frac{k_1+1}{2} \pm \frac{k_1-1}{2}$$

$$s_1 = -\frac{k_1}{2} + \frac{k_1}{2} - \frac{1}{2} - \frac{1}{2} = -1$$

$$s_2 = -\frac{k_1}{2} - \frac{k_1}{2} - \frac{1}{2} + \frac{1}{2} = -k_1 \Rightarrow -k_1 < 0$$

$$k_1 > 0$$

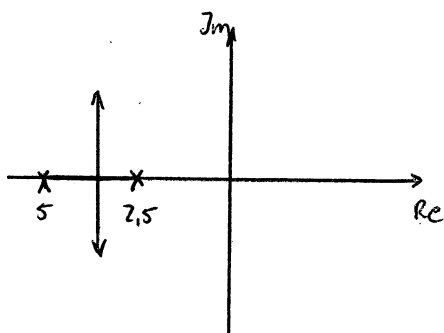
constraints for stability

$$k_2 < 0 \wedge k_1 > 0$$

d) assumptions for the special nyquist criterion

- negative feedback/gain $k > 0$
- no pole/zero cancellation
- G_0 stable
- may two poles in origin
- order of numerator $<$ order of denominator

e)



The system is always stable

A critical gain k_{crit} doesn't exist

Problem 4

a) PT1 ①: $G_1 = \frac{8}{0.2s+1}$
 PT1 ②: $G_2 = \frac{0.5}{s+1}$
 I ③: $G_3 = \frac{2}{s}$

$$G_T(s) = \frac{G_1}{1+G_1G_2} \cdot G_3 \Rightarrow G_T(s) = \frac{\frac{8}{0.2s+1}}{1 + \frac{8 \cdot 0.5}{0.2s+1} \cdot \frac{2}{s}} \cdot \frac{2}{s}$$

$$= \frac{16(s+1)}{s(0.2s+1)(s+1) + 4s}$$

$$= \frac{16(s+1)}{s(0.2s^2 + 1.2s + 5)}$$

poles: $s_1 = 0$ $s_{2,3} = \frac{-1.2 \pm \sqrt{1.2^2 - 4}}{0.4} = \frac{-1.2 \pm 1.6j}{0.4} = -3 \pm 4j$

zero: $s_{0,1} = -1$

$s_1 = 0 \Rightarrow$ at least one of the eigenvalues has not negative real part.

\Rightarrow The system is not asymptotically stable.

b) $G_T(s) = \frac{Y(s)}{U(s)} = \frac{16(s+1)}{s(0.2s^2 + 1.2s + 5)}$

\Rightarrow Differential equation:

$$0.2\ddot{y} + 1.2\dot{y} + 5y = 16\dot{u} + 16u \quad \otimes$$

Integrate the equation until there is explicit output y .

$$0.2\dot{y} + 1.2y + 5\int y = 16u + 16\int u$$

\Rightarrow The system is a PIT2 system.

b) Use equation \otimes

$$\Rightarrow \ddot{y} + 6\dot{y} + 25y + 0 \cdot y = 80\dot{u} + 80u$$

State space representation

$$\dot{x} = Ax + bu \quad y = cx$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -25 & -6 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad c = [80 \quad 80 \quad 0]$$

c) Eigenvalues:

$$\det(\lambda I - A) = 0$$

$$\begin{vmatrix} \lambda + 0.5 & 0 \\ -1 & \lambda + 1 \end{vmatrix} = (\lambda + 0.5)(\lambda + 1) = 0 \quad \lambda_1 = -0.5 \quad \lambda_2 = -1$$

for $\lambda_1 = -0.5$

$$Av_1 = \lambda_1 v_1$$

$$\begin{bmatrix} -0.5 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = -0.5 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

$$-0.5v_{11} = -0.5v_{11} \quad ; \quad v_{11} - 0.5v_{12} = 0$$

$$v_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} v_{11} \\ 2v_{11} \end{bmatrix}$$

$$\text{let } v_{11} = 1 \quad \underline{v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$

for $\lambda_2 = -1$ $Av_2 = \lambda_2 v_2$

$$\begin{bmatrix} -0.5 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} -v_{21} \\ -v_{22} \end{bmatrix}$$

$$v_{21} = 0$$

$$-v_{22} = -v_{22}$$

$$v_2 = \begin{bmatrix} 0 \\ v_{22} \end{bmatrix}$$

$$\text{let } v_{22} = 1 \quad \underline{v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

d) Transformation Matrix $T=V = [v_1, v_2]$

$$\tilde{x} = T^{-1}x \quad T=V = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad T^{-1} = V^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\tilde{A} = V^{-1}AV = T^{-1}AT = \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\tilde{b} = V^{-1}b = T^{-1}b = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$$

$$\tilde{c} = cV = cT = [-1 \quad -1]$$

Canonical form:

$$\dot{\tilde{x}} = \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0.5 \\ -1 \end{bmatrix} u, \quad y = [-1 \quad -1] \tilde{x}$$

e) Equations of motion

$$\tilde{x} = e^{\tilde{A}t} \tilde{x}_0 + \int_0^t e^{\tilde{A}(t-\tau)} \tilde{b} u(\tau) d\tau$$

$$= \begin{bmatrix} e^{-0.5t} & 0 \\ 0 & e^{-t} \end{bmatrix} \tilde{x}_0 + \int_0^t \begin{bmatrix} e^{-0.5(t-\tau)} & 0 \\ 0 & e^{-(t-\tau)} \end{bmatrix} \begin{bmatrix} 0.5 \\ -1 \end{bmatrix} u(\tau) d\tau$$

$$\dots = \underbrace{\begin{bmatrix} e^{-0.5t} \cdot \tilde{x}_{10} \\ e^{-t} \cdot \tilde{x}_{20} \end{bmatrix}}_{\text{eigen motion}} + \underbrace{\int_0^t \begin{bmatrix} 0.5 \cdot e^{-0.5(t-\tau)} \\ -1 \cdot e^{-(t-\tau)} \end{bmatrix} u(\tau) d\tau}_{\text{excited motion}}$$

Problem 5

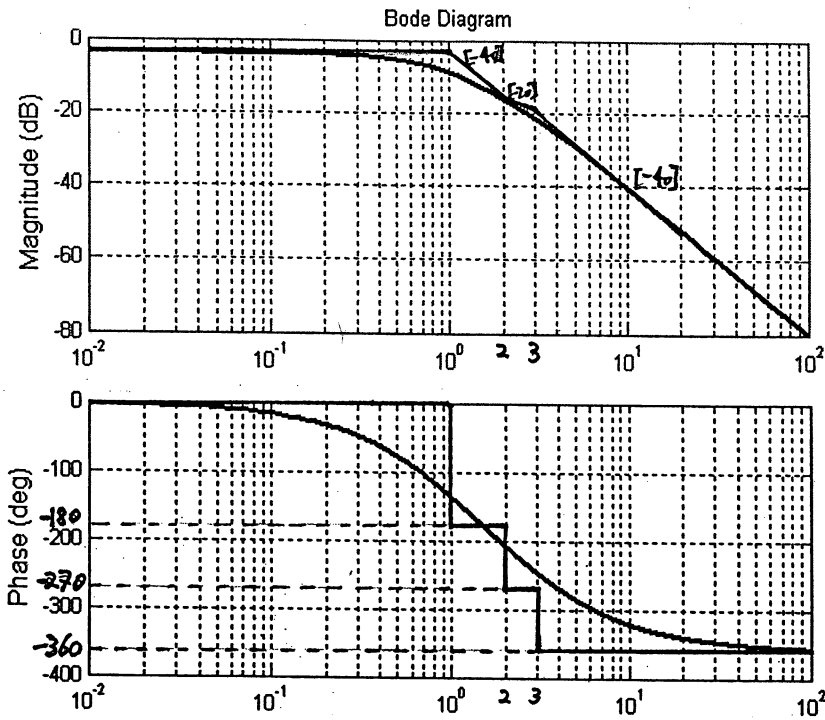
a) poles:

$$s_1 = -3$$

$$s_{2,3} = -1$$

zeros:

$$s_0 = 2$$



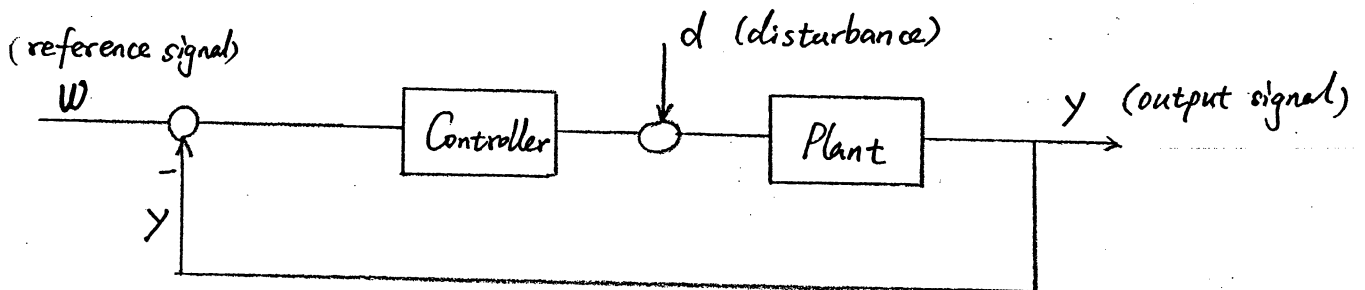
b) $s_1 = -3$, $s_{2,3} = -1$

$$\operatorname{Re}\{s_i\} < 0 \text{ for } i = 1, \dots, 3$$

\Rightarrow The system is I/O stable.

$$c) G_0 = K_1 \cdot \frac{s-2}{(s+3)(s^2+2s+1)}$$

$$G = \frac{G_0}{1+G_0} = \frac{K_1(s-2)}{(s+3)(s^2+2s+1) + K_1(s-2)} = \frac{K_1(s-2)}{s^3 + 5s^2 + (7+K_1)s + 3-2K_1}$$



c) Hurwitz - criterion :

$$a_3 = 1 > 0$$

$$a_2 = 5 > 0$$

$$a_1 = 7 + K_1 > 0 \Rightarrow K_1 > -7$$

$$a_0 = 3 - 2K_1 > 0 \Rightarrow K_1 < \frac{3}{2}$$

$$\left. \begin{array}{l} a_3 = 1 > 0 \\ a_2 = 5 > 0 \\ a_1 = 7 + K_1 > 0 \Rightarrow K_1 > -7 \\ a_0 = 3 - 2K_1 > 0 \Rightarrow K_1 < \frac{3}{2} \end{array} \right\} \Rightarrow -7 < K_1 < \frac{3}{2} \quad \textcircled{1}$$

$$\left| \begin{array}{ccc} 5 & 3-2K_1 & 0 \\ 1 & 7+K_1 & 0 \\ 0 & 5 & 3-2K_1 \end{array} \right| \quad |5| > 0$$

$$\left| \begin{array}{cc} 5 & 3-2K_1 \\ 1 & 7+K_1 \end{array} \right| = 35 + 5K_1 - 3 + 2K_1 = 7K_1 + 32 > 0$$

$$\left| \begin{array}{ccc} 5 & 3-2K_1 & 0 \\ 1 & 7+K_1 & 0 \\ 0 & 5 & 3-2K_1 \end{array} \right| = (3-2K_1) \cdot (7K_1 + 32) > 0$$

$$\Rightarrow \left. \begin{array}{l} (3-2K_1) > 0 \\ (7K_1 + 32) > 0 \end{array} \right\} \Rightarrow -\frac{32}{7} < K_1 < \frac{3}{2} \quad \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \Rightarrow -\frac{32}{7} < K_1 < \frac{3}{2}$$

\Rightarrow for $-\frac{32}{7} < K_1 < \frac{3}{2}$, the controlled system is stable.

d) $e(t) = w(t) - y(t) \quad E(s) = W(s) - Y(s)$

$$w(t) = 1(t) \quad W(s) = \frac{1}{s}$$

$$Y(s) = G(s) \cdot U(s) = G(s) \cdot W(s) = \frac{K_1(s-2)}{s^3 + 5s^2 + (7+K_1)s + 3-2K_1} \cdot \frac{1}{s}$$

$$E(s) = \frac{1}{s} \left[1 - \frac{s-2}{s^3 + 5s^2 + 8s + 1} \right]$$

remaining control error : $e(\infty)$

$$e(\infty) = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \cdot E(s) = \lim_{s \rightarrow 0} \left[1 - \frac{s-2}{s^3 + 5s^2 + 8s + 1} \right]$$

$$= 1 - \frac{-2}{1} = 3$$

$$e) \quad G_{new} = \frac{G_{new}}{1 + G_{new}} = \frac{(K_2 + \frac{K_3}{s}) G_s}{1 + (K_2 + \frac{K_3}{s}) G_s}$$

$$= \frac{(K_2 s + K_3)(s-2)}{s(s+3)(s^2+2s+1) + (K_2 s + K_3)(s-2)}$$

$$E(s) = W(s) - Y(s)$$

$$= W(s) - G_{new} \cdot W(s)$$

$$= \frac{1}{s} \left[1 - \frac{(K_2 s + K_3)(s-2)}{s(s+3)(s^2+2s+1) + (K_2 s + K_3)(s-2)} \right]$$

remaining control error

$$e(\infty) = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \cdot E(s)$$

$$= \lim_{s \rightarrow 0} s \cdot \frac{1}{s} \left[1 - \frac{(K_2 s + K_3)(s-2)}{s(s+3)(s^2+2s+1) + (K_2 s + K_3)(s-2)} \right]$$

$$= \left[1 - \frac{-2K_3}{-2K_3} \right]$$

$$= 1 - 1 = 0$$

for P-controller $e(\infty) = 3$

for PI-controller $e(\infty) = 0$, it is because that P-plant with I-controller leads automatically to zero stationary ~~or~~ control error.

$$a) \quad \underline{G_w} : z := 0$$

$$Y = G_1 \cdot W - G_2 \cdot \varepsilon$$

$$\varepsilon = W - G_3 \cdot G_2 \cdot \varepsilon \rightarrow \varepsilon = \frac{1}{1 + G_2 G_3} W$$

$$\Rightarrow Y = G_1 \cdot W - \frac{G_2}{1 + G_2 G_3} W$$

$$\underline{G_w} = \frac{Y}{W} = \frac{G_1 + G_1 G_2 G_3 - G_2}{1 + G_2 G_3}$$

$$G_z : W := 0$$

$$Y = -(z + G_2 \cdot \varepsilon)$$

$$\varepsilon = -G_3 \cdot (z + G_2 \varepsilon) \rightarrow \varepsilon = \frac{-G_3}{1 + G_2 G_3} z$$

$$\Rightarrow Y = -z - \frac{-G_2 G_3}{1 + G_2 G_3} z$$

$$\underline{G_z} = \frac{Y}{z} = - \frac{1}{1 + G_2 G_3}$$

$$b) \quad \gamma_2 = \log(\varepsilon + 1) - 2\varepsilon \quad \rightarrow \quad \gamma_2 = G_2(\varepsilon)$$

$$\text{Taylor: } \delta \gamma_2 = G_2(\varepsilon_0) + \left. \frac{\partial G_2(\varepsilon)}{\partial \varepsilon} \right|_{\varepsilon_0} \cdot (\varepsilon - \varepsilon_0)$$

$$\left[(\log_{10} x)' = \frac{1}{x \cdot \ln 10} \right]$$

$$\Rightarrow \delta \gamma_2 \approx \underbrace{-1,57}_{G_{2,lin}} \cdot \varepsilon$$

$$= G_{2,lin} := G_2$$

... b)

$$Y(s) = G_w(s) \cdot W(s) + G_z(s) \cdot Z(s)$$

$$= \frac{G_1 + G_1 G_2 G_3 - G_2}{1 + G_2 G_3} W(s) - \frac{1}{1 + G_2 G_3} Z(s)$$

With given transfer behavior:

$$Y(s) = \frac{k(s+T_3) + 1,57(s+T_3-k)}{s+T_3-1,57} W(s) - \frac{s+T_3}{s+T_3-1,57} Z(s)$$

c) $1(t) \rightarrow \frac{1}{s}$

$$\lim_{t \rightarrow \infty} y(t) \rightarrow \lim_{s \rightarrow 0} s \cdot F(s)$$

i) $W(s) = \frac{1}{s}, Z(s) = 0$

$$\lim_{s \rightarrow 0} s \cdot \left[G_w(s) \cdot \frac{1}{s} + G_z(s) \cdot 0 \right]$$

$$\lim_{s \rightarrow 0} s \cdot \frac{1}{s} \cdot \frac{(k-1)(s+T)}{s+T+1}$$

$$= \frac{T(k-1)}{T+1}$$

ii) $W(s) = 0, Z(s) = \frac{1}{s}$

$$\lim_{s \rightarrow 0} s \cdot \frac{1}{s} \cdot \frac{-(s+T)}{s+T+1} = \frac{-T}{T+1}$$

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... c) iii) $Z(s) = W(s) = \frac{1}{s}$

From linearity:

$$\lim_{s \rightarrow 0} s \cdot \left[G_{W1}(s) \cdot \frac{1}{s} + G_{W2}(s) \cdot \frac{1}{s} \right]$$

$$= \frac{T(K-2)}{T+1}$$

d) For direct feedthrough: $\lim_{s \rightarrow \infty} \left[s \cdot \frac{1}{s} G(s) \right] \neq 0$

here: $\lim_{s \rightarrow \infty} \frac{(k-1)(s+T)}{s+T+1} = \frac{s \left[k + \frac{kT}{s} - 1 - \frac{T}{s} \right]}{s \left[1 + \frac{T}{s} + \frac{1}{s} \right]} = k-1$

If $k \neq +1 \Rightarrow$ system has a direct throughput!

- e)
- $\omega_1 = 0 \frac{\text{rad}}{s} \rightarrow -20 \frac{\text{dB}}{\text{dek}}, -90^\circ \Rightarrow \text{neg. pole}$
 - $\omega_2 = \frac{1}{10} \frac{\text{rad}}{s} \rightarrow +20 \frac{\text{dB}}{\text{dek}}, +90^\circ \Rightarrow \text{neg. zero}$
 - $\omega_3 = 50 \frac{\text{rad}}{s} \rightarrow -20 \frac{\text{dB}}{\text{dek}}, -90^\circ \Rightarrow \text{neg. pole}$
 - $\omega_4 = 10^3 \frac{\text{rad}}{s} \rightarrow +20 \frac{\text{dB}}{\text{dek}}, +90^\circ \Rightarrow \text{neg. zero}$
 - $k = 1$

$$G(s) = k \cdot \frac{(1+10s)(1+\frac{1}{10^3}s)}{(1+\frac{1}{50}s) \cdot s}$$

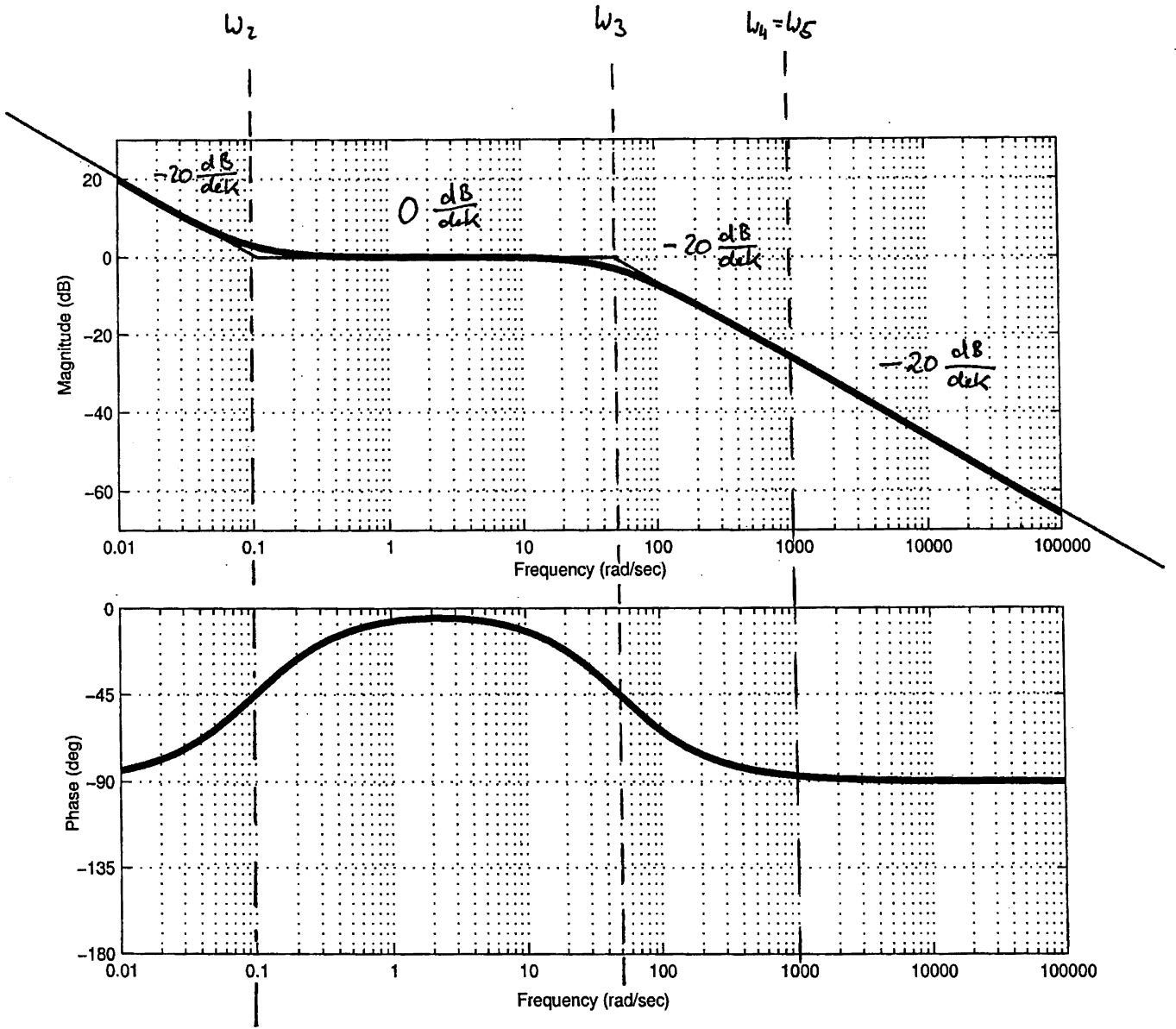
$$\Rightarrow (s+T_3 s^2) Y(s) = k \cdot [1 + (T_2+T_4)s + T_2 T_4 s^2] U(s)$$

$$\Rightarrow y(t) + T_3 \ddot{y}(t) = k \cdot \left[\int u(\tau) d\tau + (T_2+T_4)u(t) + T_2 T_4 \frac{du(t)}{dt} \right]$$

\Rightarrow PIDT₁ - Transfer element

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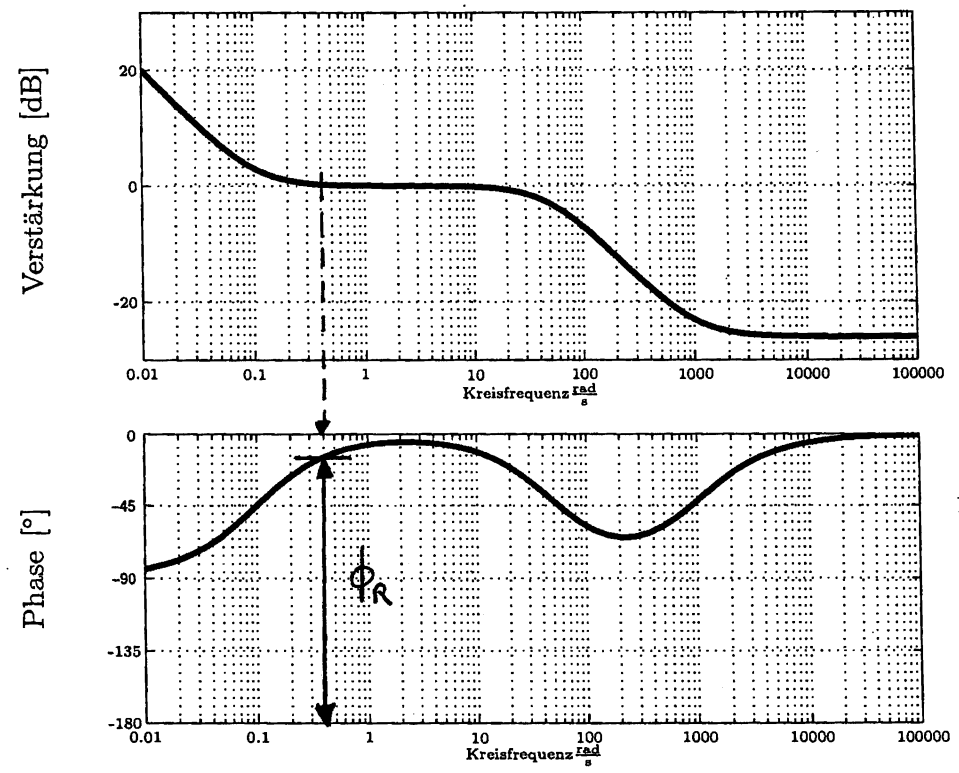
g) Yes, both are phase minimum systems:

e): 2 neg. poles, 2 neg. zeros \rightarrow all real parts ≤ 0 ✓

f): 3 " " , 2. " " \rightarrow " " " ≤ 0 ✓

\Rightarrow phase minimum systems

h)



$\phi_R \approx 165^\circ$

A_R — because no intersection with -180°

i) open loop: $G_0 = \frac{(1+10s)(1+\frac{1}{10^3}s)}{s \cdot (\frac{1}{50}s+1) \cdot (1+\frac{1}{10^3}s)}$

↑
Integral part, so automatically boundary stable, not asymptotical stable!

Alternative way: Also from Hurwitz criterion:

$$\underbrace{\frac{1}{50 \cdot 10^3}}_{a_3} s^3 + \underbrace{\frac{21}{10^3}}_{a_2} s^2 + \underbrace{1}_{a_1} s = 0$$

a_0 missing

...i) closed loop: $G = \frac{G_0}{1+G_0}$

$$\Rightarrow G = \frac{50(10s+1)}{s^2+550s+50}$$

-> Hurwitz: 1) all coefficients exist and have the same sign ✓

$$2) \begin{bmatrix} \overset{D_1}{\boxed{a_2}} & 0 \\ a_0 & a_1 \end{bmatrix}$$

$$\det D_1 = |a_2| > 0$$

$$= 1 > 0 \quad \checkmark$$

$$\det D_2 = \begin{vmatrix} 1 & 0 \\ 50 & 550 \end{vmatrix}$$

$$= 550 > 0 \quad \checkmark$$

\(\Rightarrow\) closed loop asymptotical stable

f) PT₃ transfer behavior

$$k) \quad \overset{\circ\circ}{y} + T_1 \overset{\circ}{y} + T_2 \dot{y} + T_3 y = k \cdot u$$

$$G = \frac{k}{s^3 + T_1 s^2 + T_2 s + T_3}$$

l) Canonical normal form:

$$n = 3, \quad m = 0$$

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... l)

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -T_3 & -T_2 & -T_1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_1 = [k \ 0 \ 0]$$

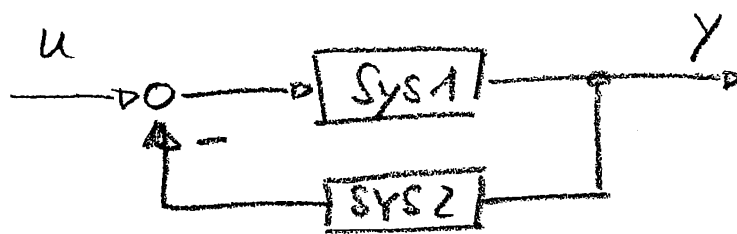
$$D_1 = 0$$

$$x_1 = \frac{1}{k} \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}$$

- A: state matrix ($n \times n$)
- B: input " ($n \times m$)
- C: output " ($r \times n$)
- D: direct transmission matrix ($r \times m$)
- x: state vector ($n \times 1$)
- u: input (SISO) (1×1)
- y: output (SISO) (1×1)

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m)



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$$A_2 = \begin{bmatrix} \frac{1}{T_R} \\ - \end{bmatrix}, \quad x_2 = [x]$$

$$B_2 = [1]$$

$$C_2 = \begin{bmatrix} \frac{1}{k T_R} \\ - \end{bmatrix}$$

$$D_2 = [0]$$

$$\Rightarrow \dot{x}_G = \underbrace{\begin{bmatrix} A_1 & -B_1 C_2 \\ B_2 C_1 & A_2 \end{bmatrix}}_{A_G} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x_G} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B_G} u$$

$$y_G = \underbrace{\begin{bmatrix} C_1 & 0 \end{bmatrix}}_{C_G} x + \underbrace{0}_{D_G} \cdot u$$

$$x_G = \begin{bmatrix} \frac{y}{k} \\ \frac{\dot{y}}{k} \\ \ddots \\ \frac{\dot{y}}{k} \\ x \end{bmatrix}, \quad A_G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -3 & -3 & \frac{1}{k T_R} \\ k & 0 & 0 & -\frac{1}{T_R} \end{bmatrix}$$

$$B_G = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad C_G = [k \ 0 \ 0 \ 0]$$

$$D_G = 0$$

... m) $\det |A_G - \lambda I| \stackrel{!}{=} 0$

$$\left| \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ -1 & -3 & 3-\lambda & \frac{1}{k T_R} \\ k & 0 & 0 & -\frac{1}{T_R} - \lambda \end{bmatrix} \right| \stackrel{!}{=} 0$$

$$\Rightarrow -\lambda \cdot \left| \begin{array}{ccc|c} -2 & 1 & 0 & 1 \\ -3 & 3-\lambda & \frac{1}{k T_R} & -1 \\ 0 & 0 & -\frac{1}{T_R} - \lambda & - \end{array} \right| = 0$$

$$\Rightarrow \lambda \cdot \left[(\lambda^3 - 3\lambda^2 + 3\lambda + 1) + \frac{1}{T_R} (\lambda^2 - 3\lambda + 3) \right] \stackrel{!}{=} 0$$



There is always one eigenvalue with $\lambda = 0$

\Rightarrow system cannot be asymptotical stable!

Independent from T_R : system boundary stable.