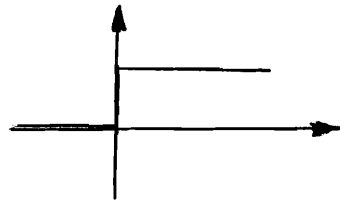


## Problem 1

a) signals of control technique

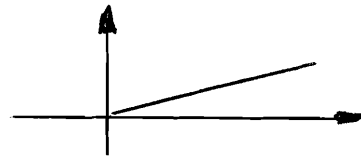
1. step



2. (dirac) impulse



3. ramp function



b) weighting function = answer / response of a system on a (dirac) impulse as input

c) eigenvalue of a system  $\lambda_i$

$$\Rightarrow \det(\lambda_i I - A) \stackrel{!}{=} 0$$

pole of a transfer function

e.g.  $G(s) = \frac{k}{T_1 s + 1} \Rightarrow \text{pole: } s = -\frac{1}{T_1}$

SISO - system:

A pole is a solution of a characteristic equation.

Each pole is an eigenvalue, but not each eigenvalue is a pole.

d) PDT<sub>2</sub> - System

Differential equation

$$\Rightarrow T_2 \ddot{y} + T_1 \dot{y} + y = k \cdot (u + T_D \dot{u})$$

Transfer function

$$\Rightarrow T_2 s^2 y + T_1 s y + y = k (u + T_D s u)$$

$$\Leftrightarrow (T_2 s^2 + T_1 s + 1) y = k (1 + T_D s) u$$

$$\Rightarrow G(s) = \frac{Y(s)}{U(s)} = \frac{k \cdot (1 + T_D s)}{T_2 s^2 + T_1 s + 1}$$

e) MIMO - System

$$i) \dot{X} = A X + B u$$

$$y = C X + D u$$

ii) Transfer function

$$X(s) = \begin{pmatrix} x_1(s) \\ \vdots \\ x_n(s) \end{pmatrix}$$

$$s X(s) = A \cdot X(s) + B \cdot u(s)$$

$$\Leftrightarrow (s I - A) X(s) = B \cdot u(s)$$

$$\Rightarrow X(s) = (s I - A)^{-1} B \cdot u(s)$$

$$\Rightarrow y(s) = C \left[ (s I - A)^{-1} B + D \right] u(s)$$

iii) eigenvalues and eigenvectors

EV  $\lambda_i$

$$\det(\lambda_i I - A) \stackrel{!}{=} 0$$

EV  $v_i$

$$(\lambda_i I - A) v_i \stackrel{!}{=} 0$$

## Problem 2

### a) PIT3 - behavior

differential equation PIT3 - system

$$\Rightarrow T_3 \ddot{y} + T_2 \ddot{y} + T_1 \dot{y} + y = K \cdot \left[ u + \frac{1}{T_I} \int_0^t u(\tau) d\tau \right]$$

transfer function

#### 1. Laplace transformation

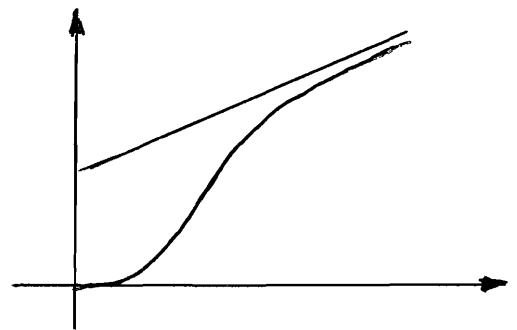
$$\Rightarrow T_3 s^3 y + T_2 s^2 y + T_1 s y + y = K \cdot \left[ u + \frac{1}{T_I} \cdot \frac{1}{s} u \right]$$

$$\Leftrightarrow (T_3 s^3 + T_2 s^2 + T_1 s + 1) y = \left( K + \frac{K}{T_I s} \right) u$$

#### 2. transfer function

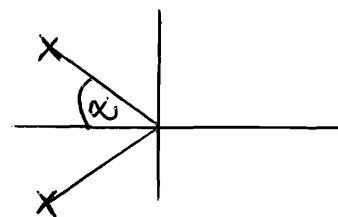
$$G(s) = \frac{Y(s)}{U(s)}$$

$$\Rightarrow G(s) = \frac{K + \frac{K}{T_I s}}{T_3 s^3 + T_2 s^2 + T_1 s + 1}$$



### b) damping

$$D = \cos \left( \arctan \frac{\operatorname{Im} \{ \lambda_i \}}{\operatorname{Re} \{ \lambda_i \}} \right)$$



oscillation between

$$0 \leq D < 1$$

### c) occurrence of the frequencies in the signal

d) Laplace transformation

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt$$

Dirac - Impulse

$$\delta(t) = \frac{1}{\alpha} [1(t) - 1(t-\alpha)] ; \alpha \rightarrow 0$$

$$\mathcal{L}\{\delta(t)\} = \int_{-0}^{\infty} \delta(t) e^{-st} dt$$

Dirac - impulse from  $[-0; +0]$

$$= \int_{-0}^{+0} \delta(t) e^{-st} dt + \int_{+0}^{\infty} \delta(t) e^{-st} dt$$

$$= \int_{-0}^{+0} \delta(t) dt$$

$$= \underline{\underline{1}}$$

$$\left[ \mathcal{L}\{\delta(t)\} = 1 \right]$$

e) PIT1 - transfer function

$$G_S(s) = K_1 \frac{1 + \frac{1}{T_I s}}{1 + T_1 s}$$

PI transfer function

$$G_R(s) = K_2 \left[ 1 + \frac{1}{T_I s} \right]$$

e) disturbance transfer function

$$G_2(s) = \frac{N_R N_S}{N_R N_S + Z_R Z_S} = \frac{G_S}{1 + G_R G_S} = \frac{G_S}{1 + G_0}$$

$$\Rightarrow G_2(s) = \frac{T_1 s + 1}{T_{I1} s \left[ (1 + T_1 s) + \left(1 + \frac{1}{T_{I1} s}\right) K_2 \left(1 + \frac{1}{T_{I2} s}\right) \right]}$$

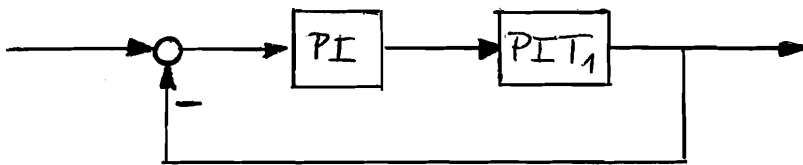
guidance transfer function

$$\begin{aligned} \Rightarrow G_W(s) &= \frac{Z_R Z_S}{N_S N_R + Z_S Z_R} = \frac{G_R G_S}{1 + G_R G_S} = \frac{G_0}{1 + G_0} \\ &= \frac{K_2 \left(1 + \frac{1}{T_{I2} s}\right) \left(1 + \frac{1}{T_{I1} s}\right)}{\left(1 + T_1 s\right) + K_2 \left(1 + \frac{1}{T_{I2} s}\right) \left(1 + \frac{1}{T_{I1} s}\right)} \end{aligned}$$

→ The system is semi-stable.

(Pole with  $s = 0$ )

block diagram.



### Problem 3

a)

$$G(s) = \frac{K_1}{T_1 s + 1}$$

$$K_1 = 1$$

$$T_1 = 1$$

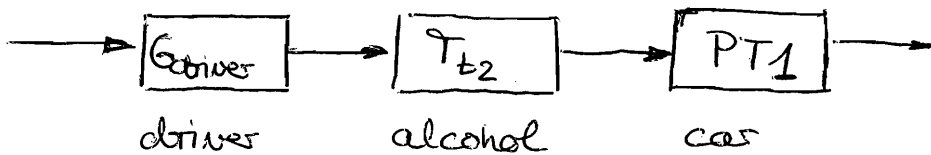
$$\Rightarrow G(s) = \frac{1}{s+1}$$

$$G_{\text{driver}_1} = \frac{K_1}{1 + T_1 s} e^{-T_{t1} s}$$

$$K_1 = 10$$

$$T_1 = 1$$

$$T_{t1} = 0.6$$



$$G_{\text{driver}_2} = \frac{K}{1 + T_1 s} e^{-(T_{t1} + T_{t2}) s}$$

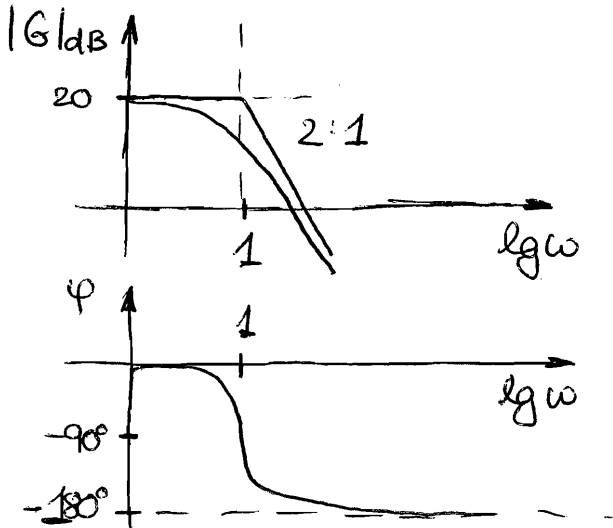
$$T_{t2} = 1$$

$$= \frac{10}{1 + s} e^{-1.6 s}$$

$$G_o(s) = \frac{1}{(1+s)(1+s)} e^{-1.6 s}$$

a)

Bode - Diagram



Stability:

Dead time,

because it causes a phase shift which causes a loss of stability

b)

$$10 \ddot{y} + ky = u(t)$$

$$\Rightarrow \ddot{y} + \frac{k}{10} y = \frac{u(t)}{10}$$

$$\dot{x} = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{10} & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{10} & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{10} \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Eigenvalues  $\lambda_{1,2}$

$$\det(\lambda_i I - A) \stackrel{!}{=} 0$$

$$\Rightarrow \lambda_{1,2} = \pm \sqrt{-\frac{k}{10}}$$

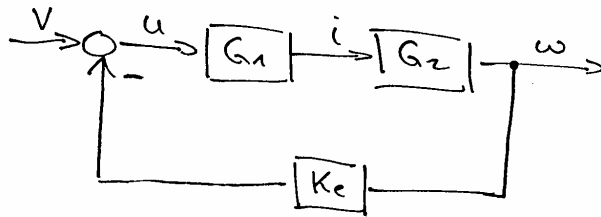
$$\Rightarrow \lambda^2 + \frac{k}{10} = 0$$

$$= \pm j \sqrt{\frac{k}{10}}$$

Because of  $\operatorname{Re}\{\lambda_i\}$  for  $\lambda_{1,2} = 0$ :

System is not asymptotically stable.

4 a)



$$u = V - K_e \cdot \omega$$

$$i = G_1 \cdot u = G_1 \cdot V - G_1 \cdot K_e \cdot \omega$$

$$\omega = G_2 \cdot i = G_2 \cdot G_1 \cdot V - G_2 \cdot G_1 \cdot K_e \cdot \omega \Rightarrow [\text{rearranging}] \Rightarrow$$

$$\Rightarrow \omega(1 + G_2 G_1 K_e) = G_2 G_1 \cdot V \Rightarrow$$

$$\Rightarrow \frac{\omega}{V} = G_{V\omega}(s) = \frac{G_1 G_2}{1 + G_1 G_2 K_e} = [\text{insertion}] =$$

$$= \frac{\frac{K_t}{(Ls+R)(Js+b)}}{1 + \frac{K_t \cdot K_e}{(Ls+R)(Js+b)}} = \frac{K_t}{(Ls+R)(Js+b) + K_t K_e}$$



b)

$$G_{vw}(s) = \frac{1}{(s+3)(s+2) + K_e} = \frac{1}{s^2 + 5s + 6 + K_e}$$

Characteristic equation:

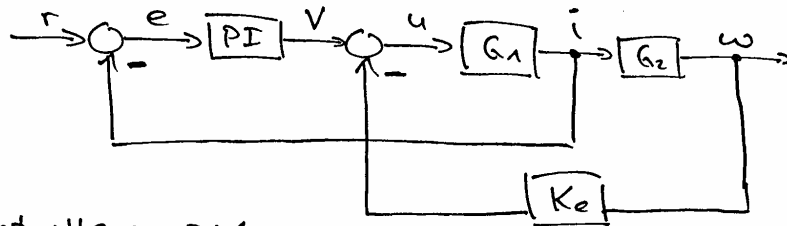
$$s^2 + 5s + 6 + K_e = 0$$

$$s = -\frac{5}{2} \pm \sqrt{\frac{25}{4} - 6 - K_e} = -\frac{5}{2} \pm \frac{1}{2} \sqrt{1 - 4K_e}$$

$\Rightarrow$  Oscillations principally occur, when complex solutions are possible, i.e.

$$\text{when } \underline{\underline{K_e > \frac{1}{4}}}$$

c)



$$\text{PI-controller} = \frac{s+1}{s}$$

$$e = r - i$$

$$V = \frac{s+1}{s} \cdot e = \frac{s+1}{s} \cdot r - \frac{s+1}{s} \cdot i$$

$$u = V - K_e \cdot w = \frac{s+1}{s} \cdot r - \frac{s+1}{s} \cdot i - K_e \cdot w$$

$$i = G_1 \cdot u = \frac{s+1}{s} \cdot G_1 \cdot r - \frac{s+1}{s} \cdot G_1 \cdot i - G_1 \cdot K_e \cdot w$$

[rearranging]:

$$i + \frac{s+1}{s} \cdot G_1 \cdot i = \frac{s+1}{s} \cdot G_1 \cdot r - G_1 \cdot K_e \cdot w \Rightarrow$$

$$i = \frac{\frac{s+1}{s} \cdot G_1 \cdot r - G_1 \cdot K_e \cdot w}{1 + \frac{s+1}{s} \cdot G_1} = \frac{(s+1)G_1 r - G_1 \cdot K_e \cdot s \cdot w}{s + G_1 s + G_1}$$

$$w = G_2 \cdot i = \frac{(s+1)G_1 G_2 r - G_1 G_2 K_e \cdot s \cdot w}{G_1 \cdot s + s + G_1} = [\text{rearranging}] \Rightarrow$$

$$\Rightarrow w = \frac{G_1 G_2 \cdot s + G_1 G_2}{s(G_1 + G_1 G_2 K_e + 1) + G_1} = \dots$$

....c)

$$\omega = \frac{G_1 G_2 s \cdot r + G_1 G_2 r}{G_1 s + s + G_1} - \frac{G_1 G_2 K_e \cdot s}{G_1 s + s + G_1} \cdot \omega$$

$$\omega \left( 1 + \frac{G_1 G_2 K_e \cdot s}{G_1 s + s + G_1} \right) = \frac{G_1 G_2 s + G_1 G_2}{G_1 s + s + G_1} \cdot r$$

$$\omega = \frac{G_1 G_2 s + G_1 G_2}{G_1 s + s + G_1 + G_1 G_2 K_e s} \cdot r = [\text{insertion}]$$

$$\frac{\frac{K_t \cdot s}{(L+sR)(J+s+b)} + \frac{K_t}{(L+sR)(J+s+b)}}{=}$$

$$\frac{\frac{K_t \cdot s}{L+sR} + s + \frac{K_t}{L+sR} + \frac{K_t K_e \cdot s}{(L+sR)(J+s+b)}}{=}$$

$$= \frac{K_t \cdot s + K_t}{K_t \cdot s(J+s+b) + s(L+sR)(J+s+b) + K_t(J+s+b) + K_t K_e \cdot s} =$$

$$= \frac{s+1}{s(s+2) + s(s+3)(s+2) + 1(s+2) + K_e \cdot s} =$$

$$= \frac{s+1}{s^2+2s + s(s^2+5s+6) + s+2 + K_e \cdot s} =$$

$$= \frac{s+1}{\cancel{s^3} + \cancel{5s^2} + \cancel{6s} + \cancel{s^2} + 2s + s+2 + \cancel{K_e \cdot s}}{=}$$

$$= \frac{s+1}{s^3 + 6s^2 + s(9+K_e) + 2}$$

d)

$$G(s) = \frac{s+1}{s^3 + 6s^2 + 10s + 2}$$

$$\left\{ \text{Initial value theorem: } \lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} s \cdot F(s) \right\}$$

$$\lim_{s \rightarrow \infty} s \cdot G(s) \cdot \underset{\substack{\uparrow \\ \text{step input}}}{\frac{1}{s}} = \lim_{s \rightarrow \infty} \frac{s+1}{s^3 + 6s^2 + 10s + 2} = \underline{\underline{0}}$$

$$\left\{ \text{Final value theorem: } \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \cdot F(s) \right\}$$

$$\lim_{s \rightarrow 0} s \cdot G(s) \cdot \underset{\substack{\uparrow \\ \text{step input}}}{\frac{1}{s}} = \lim_{s \rightarrow 0} \frac{s+1}{s^3 + 6s^2 + 10s + 2} = \underline{\underline{\frac{1}{2}}}$$

Gradients:

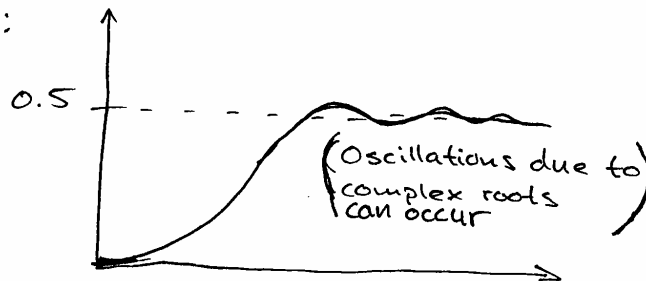
Initial value:

$$\lim_{s \rightarrow \infty} s \cdot G(s) \cdot \underset{\substack{\uparrow \\ \text{derivative}}}{s} \cdot \underset{\substack{\uparrow \\ \text{step input}}}{\frac{1}{s}} = \lim_{s \rightarrow \infty} \frac{s^2 + s}{s^3 + 6s^2 + 10s + 2} = \underline{\underline{0}}$$

Final value:

$$\lim_{s \rightarrow 0} s \cdot G(s) \cdot \underset{\substack{\uparrow \\ \text{derivative}}}{s} \cdot \underset{\substack{\uparrow \\ \text{step input}}}{\frac{1}{s}} = \lim_{s \rightarrow 0} \frac{s^2 + s}{s^3 + 6s^2 + 10s + 2} = \underline{\underline{0}}$$

Step response:



# Problem 5:

(1)

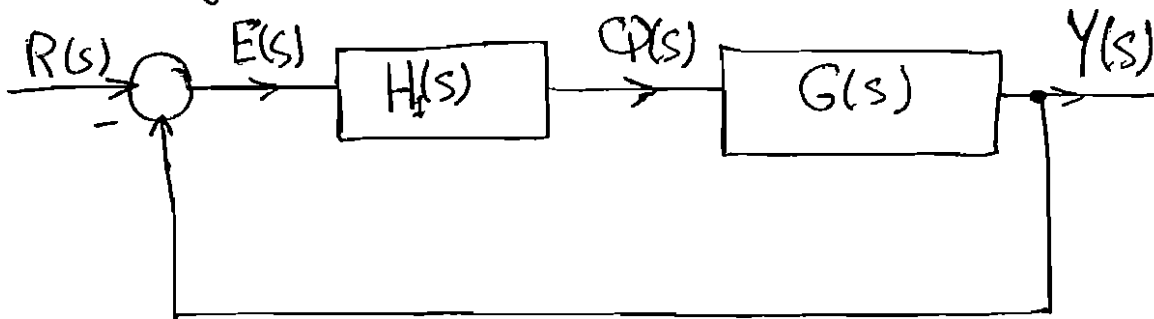
a)  $\dot{y}(t) = \frac{1}{A} q(t)$  (1)

Laplace transform of (1)

$$Y(s) = \frac{1}{A} \cdot \frac{1}{s} Q(s) \Rightarrow G(s) = \frac{Y(s)}{Q(s)} = \frac{1}{A \cdot s}$$

b) 1. PITi-element:  $H_I(s) = K \frac{(1 + \frac{1}{T_I s})}{(1 + T_I s)} = K \frac{(1 + \frac{1}{s})}{(1 + s)}$

\* Negative Feedback:



Open-loop system:  $H_I(s) \cdot G(s) = C_{O1}(s)$

$$C_{O1}(s) = \frac{K(1 + \frac{1}{s})}{(1 + s)} \cdot \frac{1}{As} = \frac{K(\frac{s+1}{s})}{As(s+1)} = \frac{K(s+1)}{As^2(s+1)}$$

$$C_{O1}(s) = \frac{K}{As^2}$$

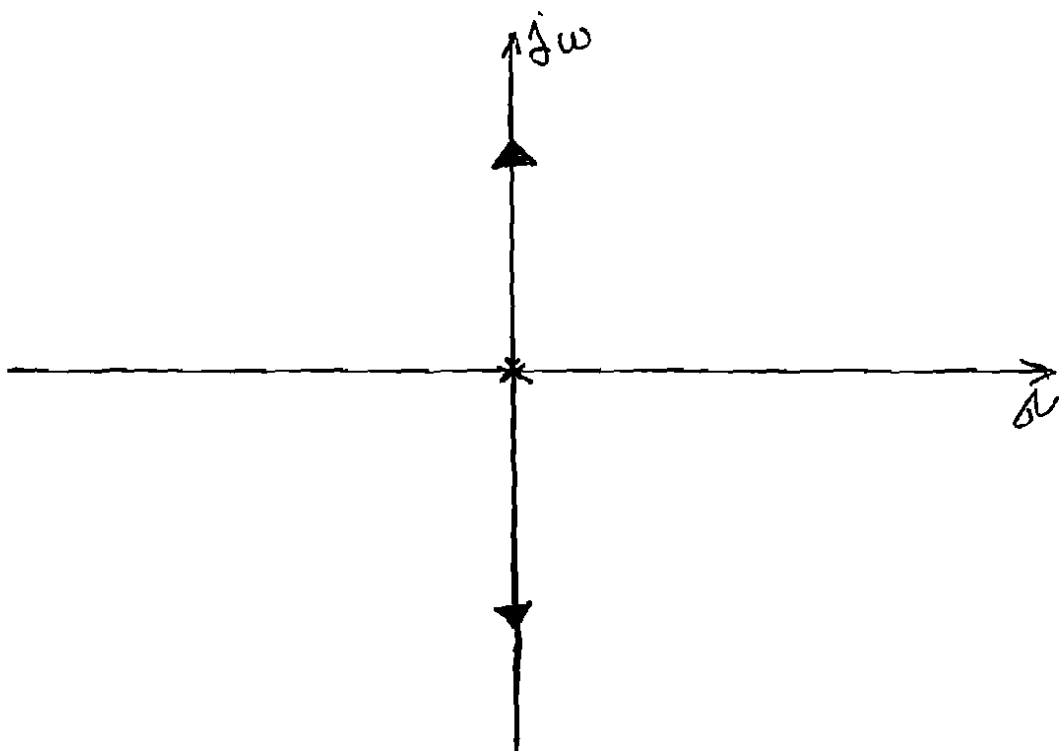
- Root locus of  $C_{O1}(s)$

\* poles:  $P_{1,2} = 0 \Rightarrow n=2$

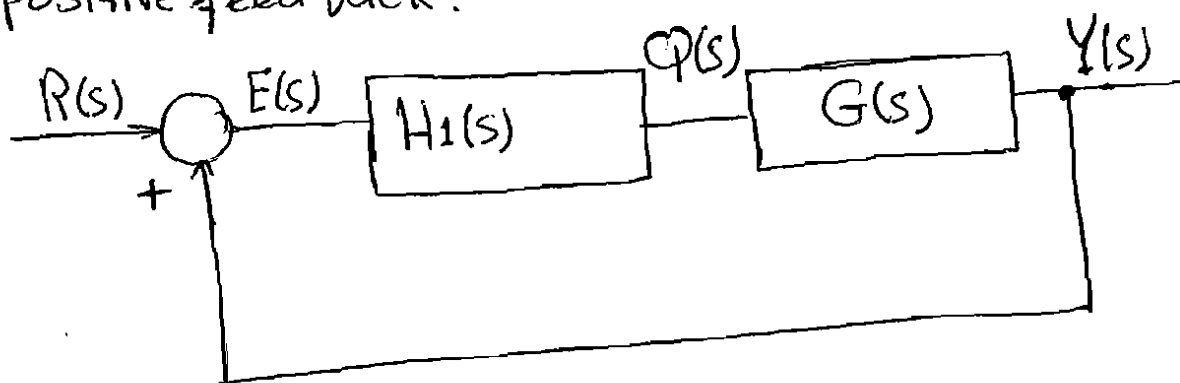
\* there are no zeros  $\Rightarrow m=0$

$$* \varphi_{\eta} = \frac{(2\eta-1)\pi}{n-m} = \frac{(2\eta-1)\pi}{2} ; \eta = 1, 2$$

$$\varphi_{\eta} = \mp \frac{\pi}{2}$$



(\*) Positive feedback:



Open-loop system:

$$C_{o2}(s) = \frac{k(1 + \frac{1}{s})}{(1+s)} \cdot \frac{1}{As} = \frac{k}{As^2}$$

Root locus of  $C_{o2}(s)$ :

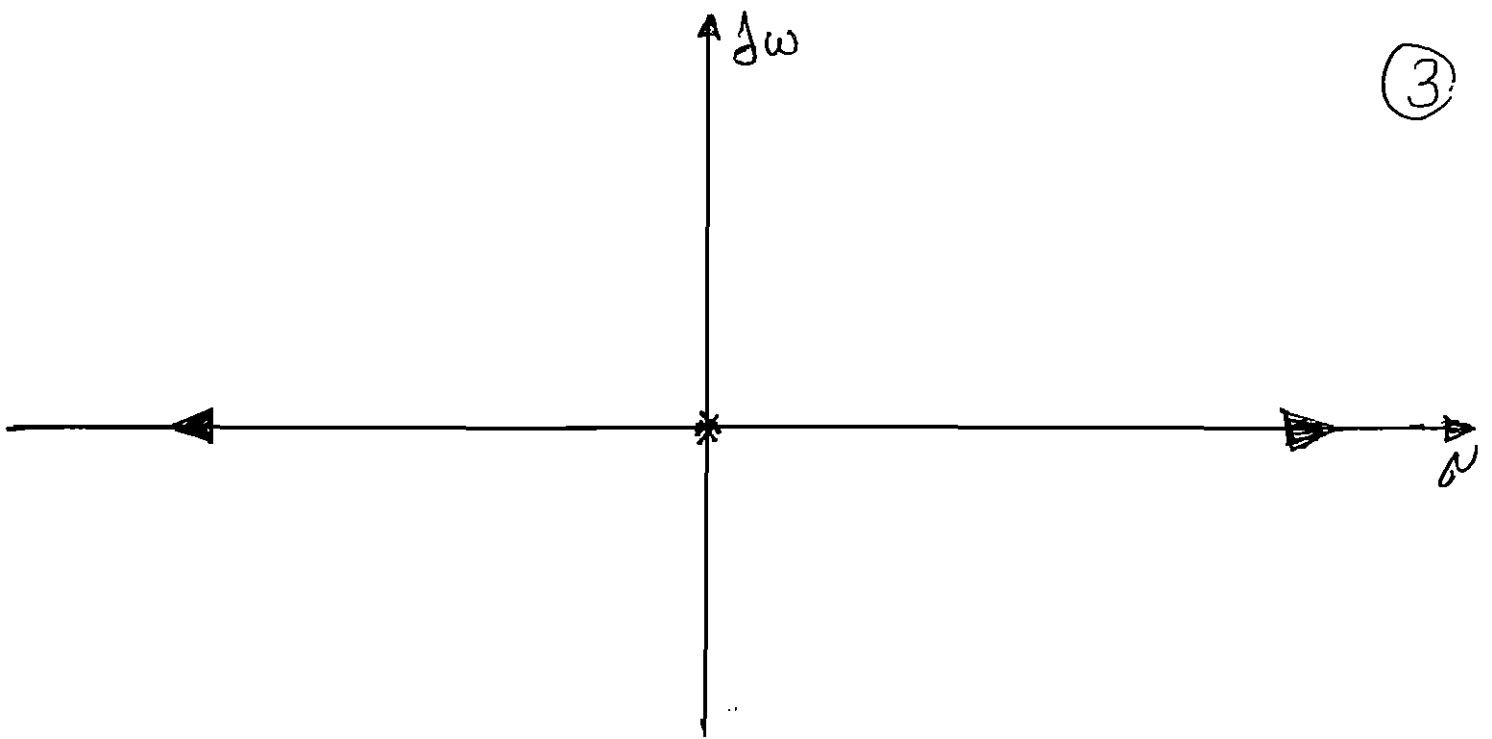
\* - Poles:  $P_{1,2} = 0 \Rightarrow n = 2$ .

\* - Zeros: There are no zeros  $\Rightarrow m = 0$ .

$$\phi_{\eta} = \frac{2\eta\pi}{n-m} = \frac{2\eta\pi}{2} = \eta\pi, \quad \eta = 1, 2$$

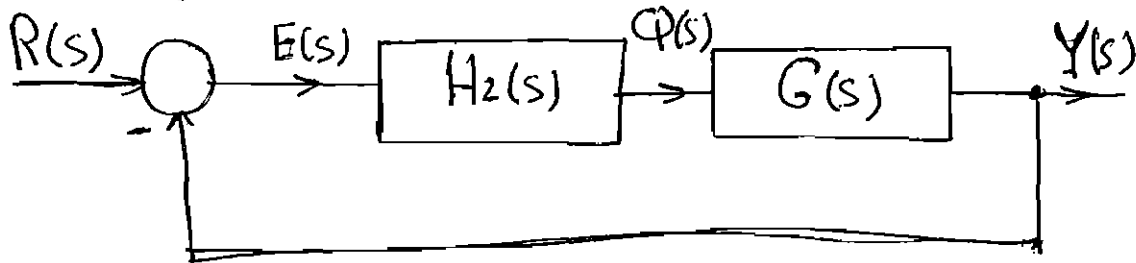
$$\phi_{\eta} = \pm\pi$$

(3)



2. PD<sub>T</sub>I - element:  $H_2(s) = k \frac{(1+T_D s)}{(1+T_I s)} \cdot k \frac{(1+2s)}{(1+s)}$

⊗ Negative feedback:



Open-loop system is:  $C_{O3} = H_2(s) \cdot G(s) = k \frac{(1+T_D s)}{(1+T_I s)} \cdot \frac{1}{As}$

$$C_{O3} = \frac{k(1+2s)}{As(1+s)} = \frac{k(1+2s)}{A(s^2+s)}$$

Root locus of  $C_{O3}$ :

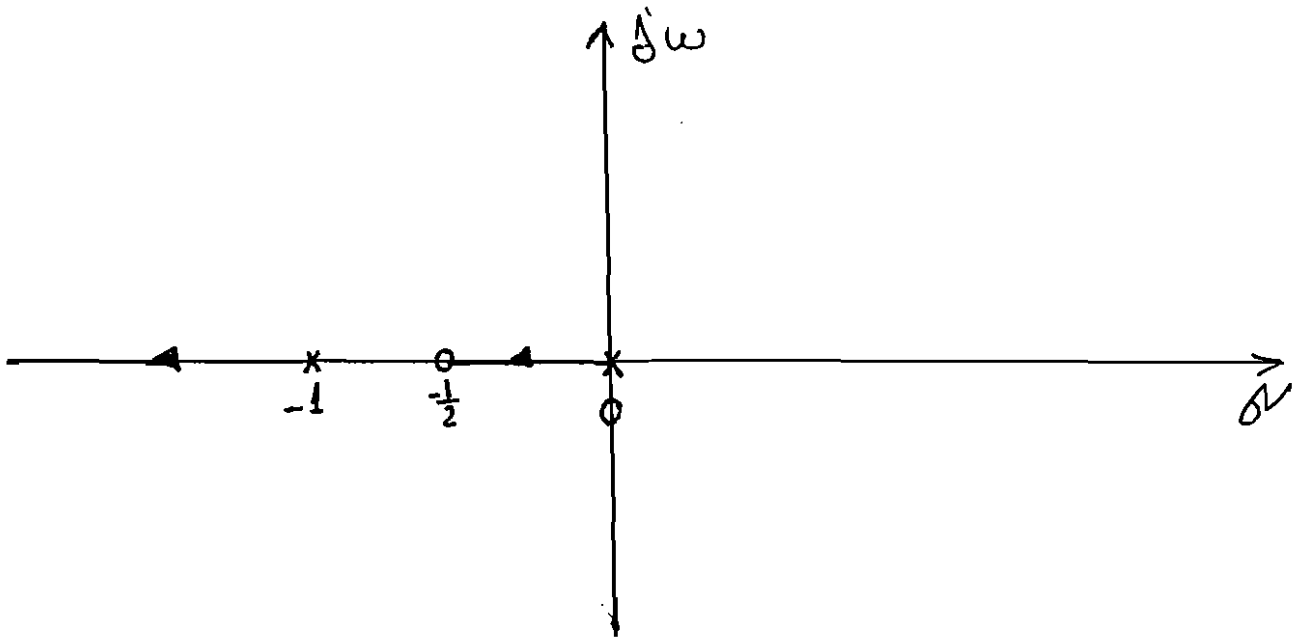
\* Zeros:  $z_1 = -1/2 \Rightarrow m = 1$ .

\* Poles:  $As(s+1) = 0 \Rightarrow P_1 = 0$  &  $P_2 = -1 \Rightarrow n = 2$ .

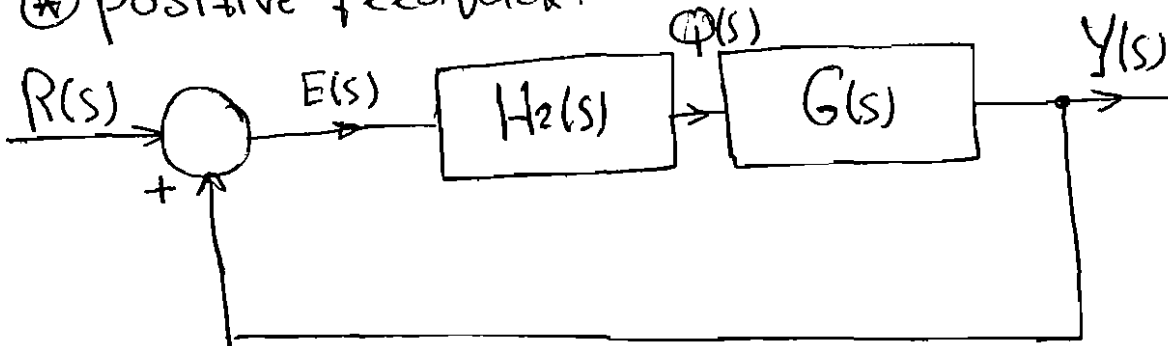
\*  $\varphi_{\eta} = \frac{(2\eta-1)\pi}{n-m} = \frac{(2\eta-1)\pi}{2-1} = (2\eta-1)\pi$  ;  $\eta = 1$

$$\varphi_{\eta} = \pi$$

(4)



\* positive feedback:



Open-loop system:  $C_{o4} = H_2(s) \cdot G(s) = k \frac{(1+7Ds)}{(1+Tis)} \cdot \frac{1}{As}$

$$C_{o4} = \frac{k(1+2s)}{A(s^2+s)}$$

Root-locus of  $C_{o4}$ :

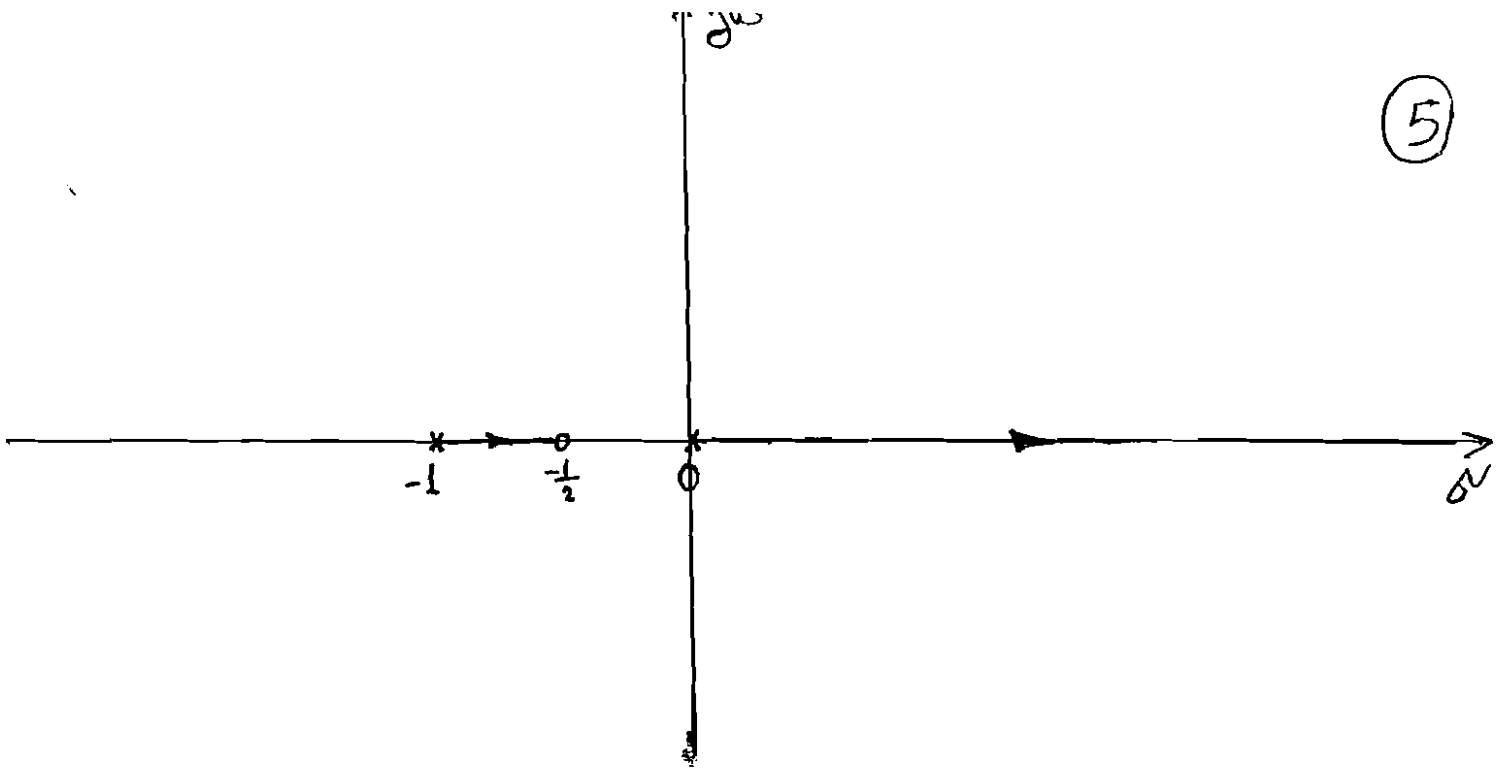
\* zeros:  $z_1 = -1/2 \Rightarrow n=1$ .

\* poles:  $s^2+s=0 \Rightarrow s(s+1)=0 \Rightarrow p_1=0 \ \& \ p_2=-1 \Rightarrow n=2$ .

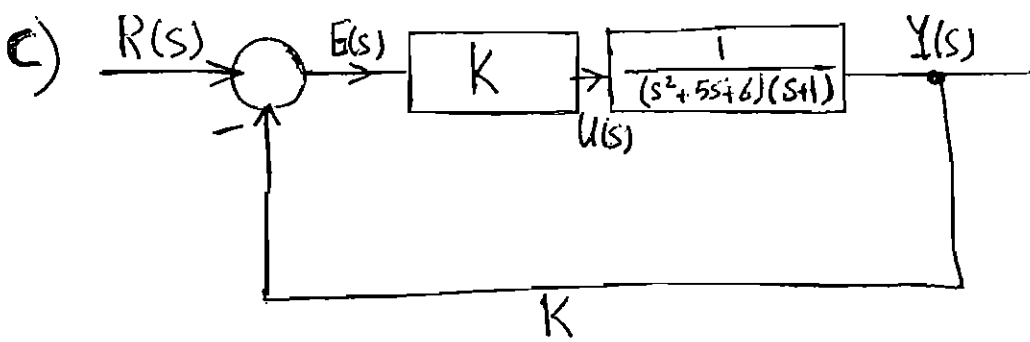
$$* \varphi_{91} = \frac{291\pi}{n-m} = 291\pi; \ n=1 \Rightarrow \varphi_{91} = 2\pi$$



5



The system can be asymptotic only with  $PDT_i$ -element  
and negative feedback for  $|k| > 0$



6

$$C(s) = \frac{\frac{K}{(s^2+5s+6)(s+1)}}{1 + \frac{K}{(s^2+5s+6)(s+1)}}$$

$$C(s) = \frac{\frac{K}{(s^2+5s+6)(s+1)}}{\frac{(s^2+5s+6)(s+1) + K}{(s^2+5s+6)(s+1)}}$$

$$C(s) = \frac{K}{s^3 + s^2 + 5s^2 + 6s + 6 + K + 5s}$$

$$C(s) = \frac{K}{s^3 + 6s^2 + 11s + 6 + K}$$

Characteristic equation:

$$s^3 + 6s^2 + 11s + 6 + K = 0 \Rightarrow a_0s^3 + a_1s^2 + a_2s + a_3 = 0$$

$$a_0 = 1 \quad \& \quad a_1 = 6 \quad \& \quad a_2 = 11 \quad \& \quad a_3 = 6 + K \Rightarrow \boxed{K > -6}$$

$$\Delta_1 = |a_1| = 1 > 0$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} > 0 \Rightarrow \begin{vmatrix} 6 & 6+K \\ 1 & 11 \end{vmatrix} > 0$$

$$66 - 6 - K > 0 \Rightarrow 60 - K > 0 \Rightarrow \boxed{K < 60}$$

The system is stable for

$$\boxed{-6 < K < 60}$$

d) Open-loop system:

$$C(s) = \frac{K}{(s+1)(s+2)(s+3)} \quad ; K=6$$

(7)

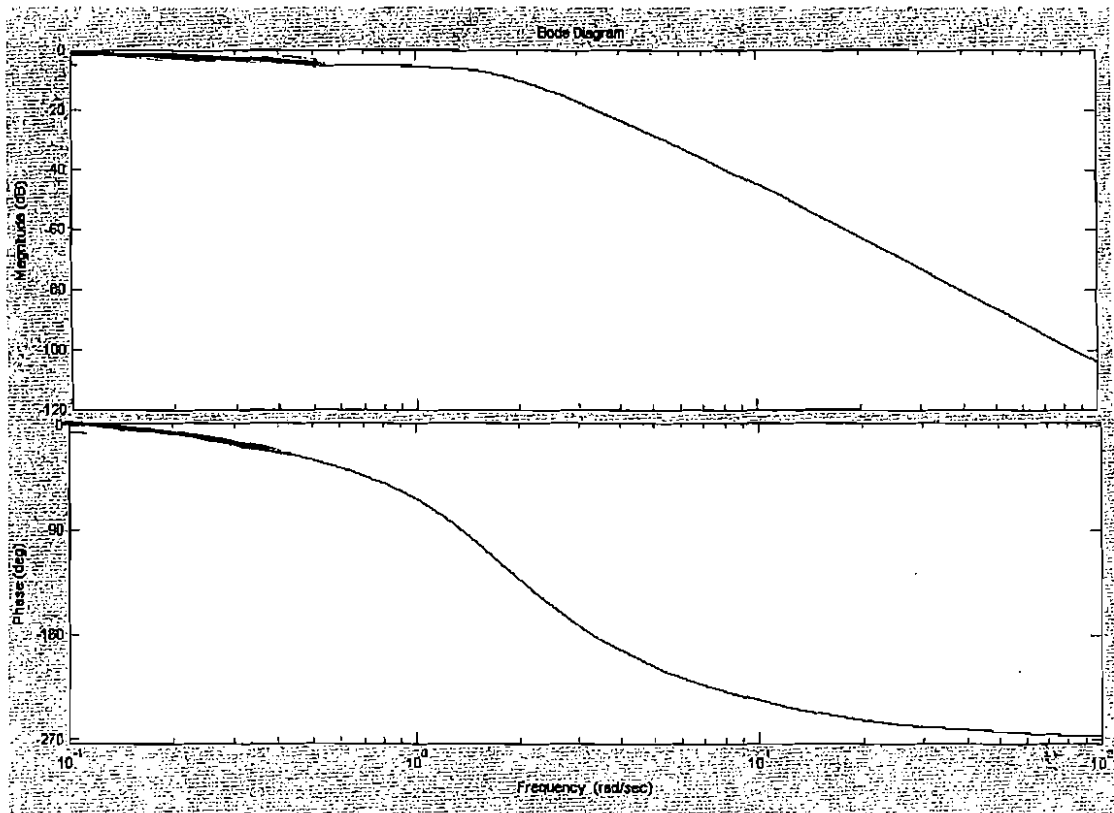
$$C(s) = \frac{6}{(s+1)(s+2)(s+3)}$$

No zeros & Poles:  $P_1 = -1$ ,  $P_2 = -2$ ,  $P_3 = -3$ .

$$C(s) = \frac{6}{6 \left(1 + \frac{s}{1}\right) \left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{3}\right)}$$

$$C(s) = \frac{1}{\left(1 + \frac{s}{1}\right) \left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{3}\right)}$$

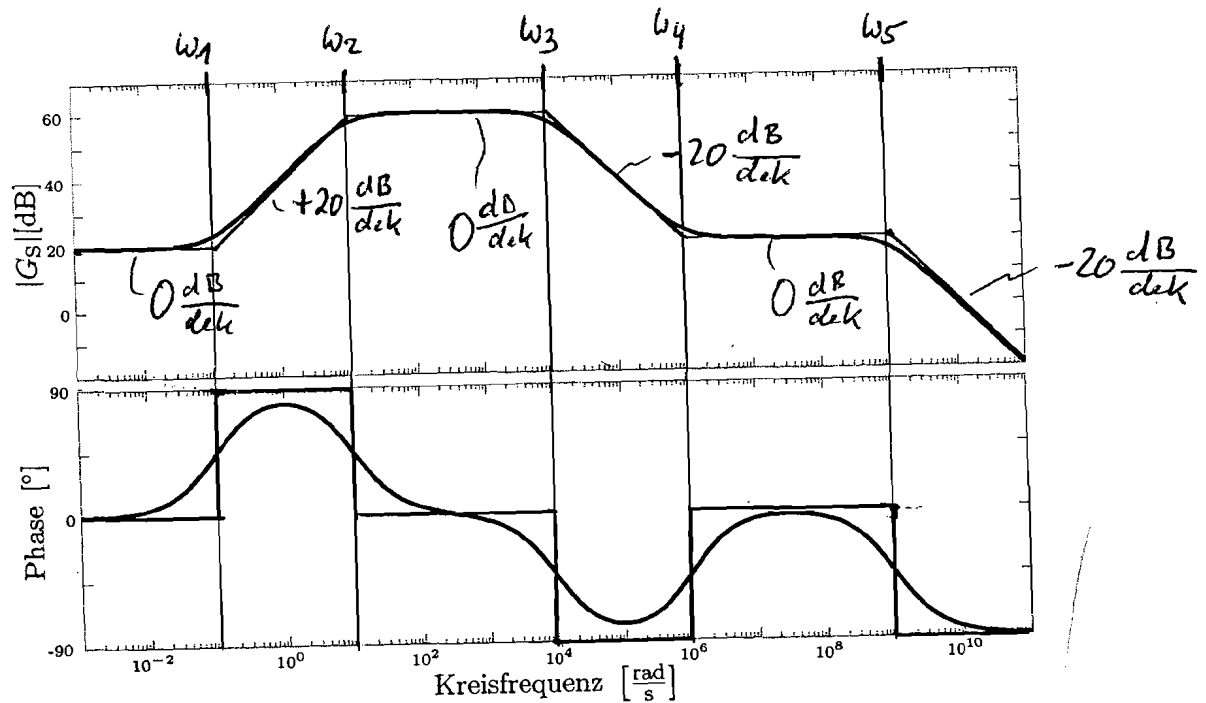
$\omega_{01} = 1 \text{ rad/sec}$  &  $\omega_{02} = 2 \text{ rad/sec}$  &  $\omega_{03} = 3 \text{ rad/sec}$



# Aufgabe 6

15.02.'08

a)



$$b) \quad G_S(s) = k_S \cdot \frac{\left(\frac{1}{\omega_1} s + 1\right) \left(\frac{1}{\omega_4} s + 1\right)}{\left(\frac{1}{\omega_2} s + 1\right) \left(\frac{1}{\omega_3} s + 1\right) \left(\frac{1}{\omega_5} s + 1\right)}$$

$$k_S = 20 \text{ dB} \hat{=} 10$$

OR

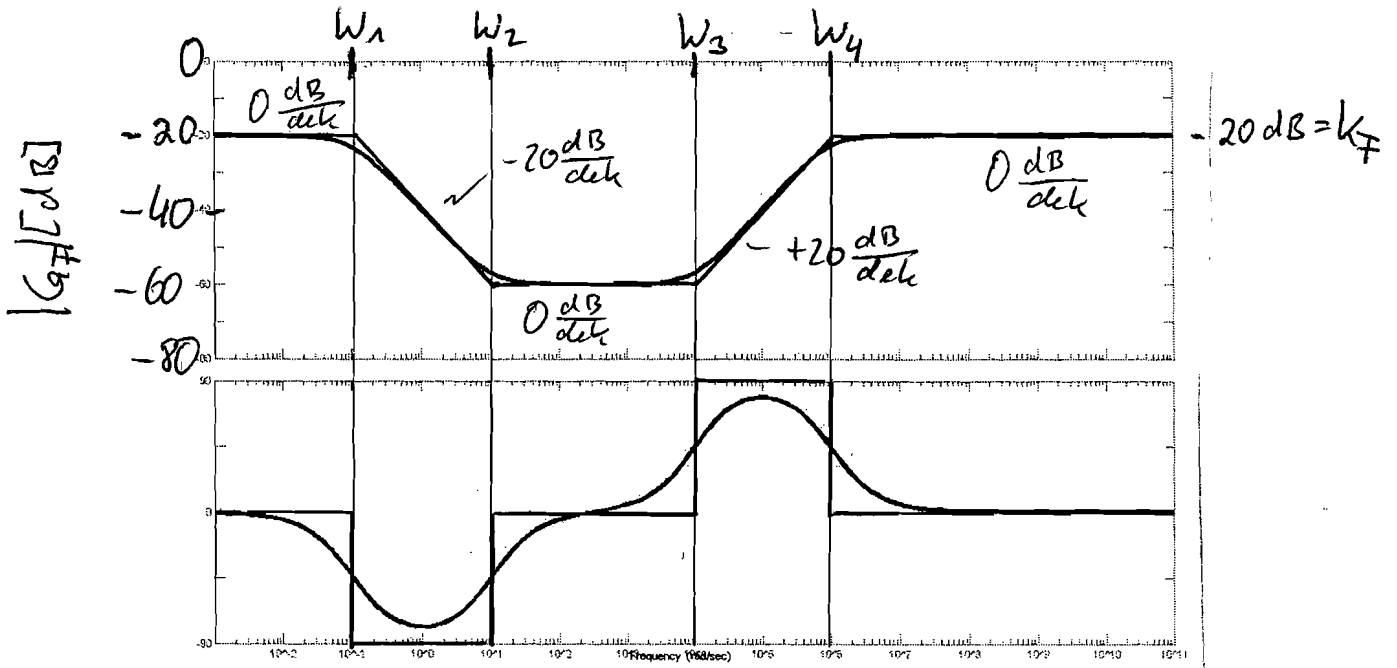
$$G_S(s) = k_S \cdot \frac{(s + \omega_1)(s + \omega_4)}{(s + \omega_2)(s + \omega_3)(s + \omega_5)}, \quad k_S = 10 \cdot \frac{\omega_2 \omega_3 \omega_5}{\omega_1 \omega_4}$$

$$c) \quad G_F = k_F \cdot \frac{\left(\frac{1}{\omega_2} s + 1\right) \left(\frac{1}{\omega_3} s + 1\right)}{\left(\frac{1}{\omega_1} s + 1\right) \left(\frac{1}{\omega_4} s + 1\right)}$$

$$k_F = -20 \text{ dB} \hat{=} 0.1$$

# Aufgabe 6

d)



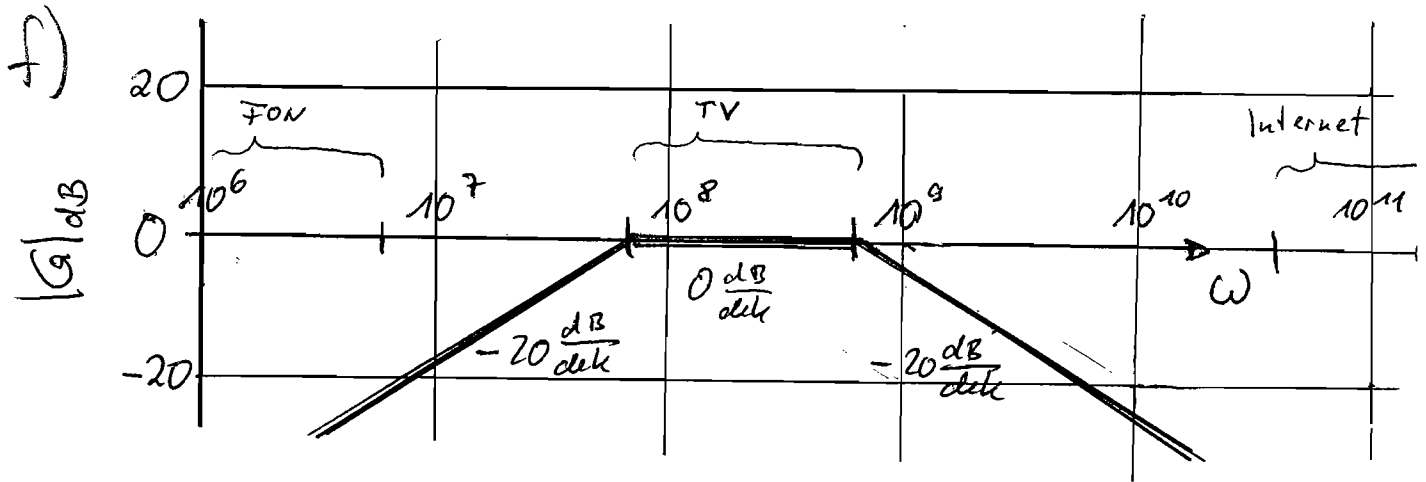
e) Yes, because signals with  $f \approx 50 \text{ GHz}$  are damped with  $-34 \text{ dB}$  ( $\approx 0.02$ )

OR

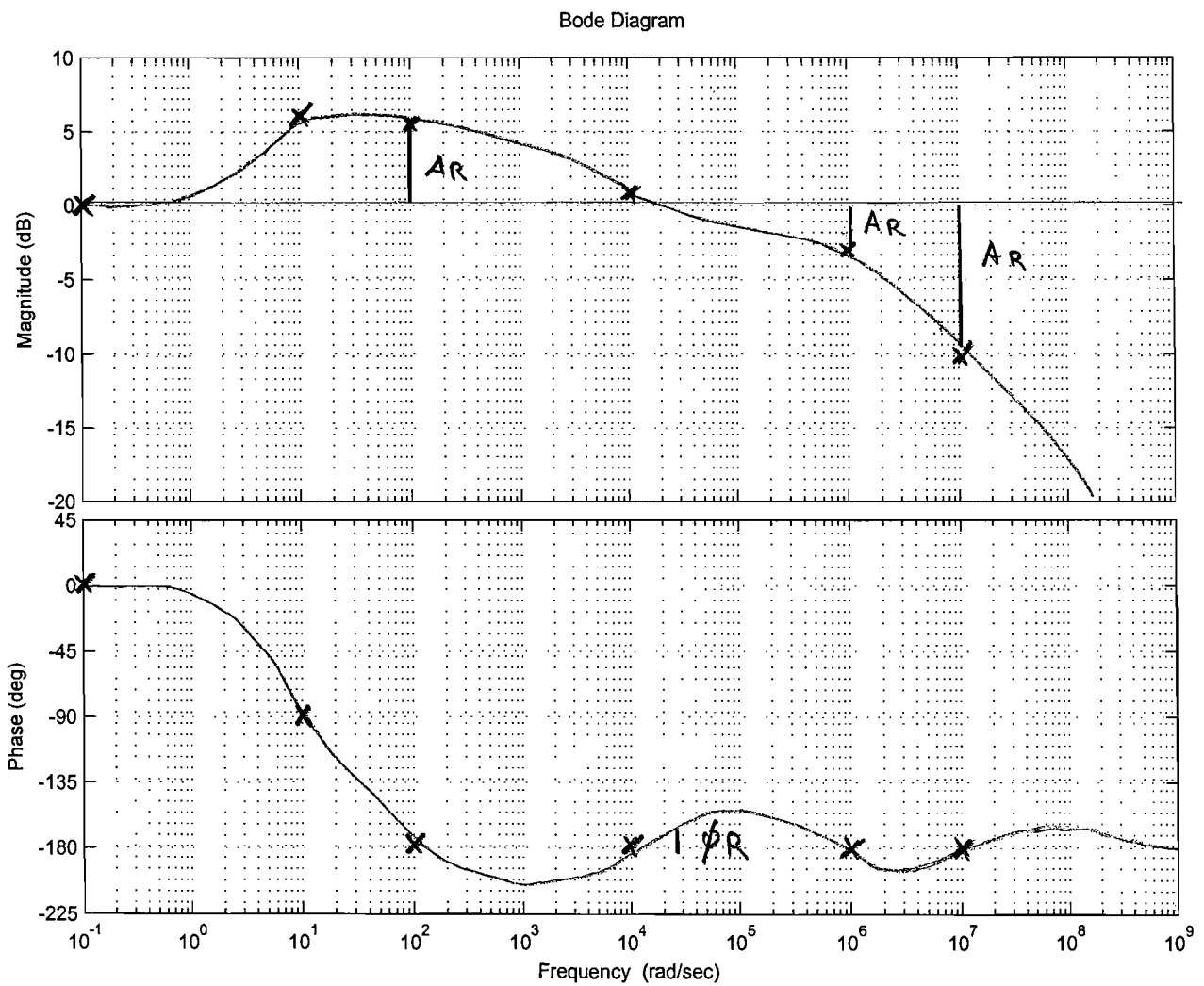
Yes, because signals with  $f \approx 10 \text{ GHz}$  are damped with  $-20 \text{ dB}$  ( $\approx 0.1$ )

$10 \text{ GHz} < 50 \text{ GHz}$  (even more damped)

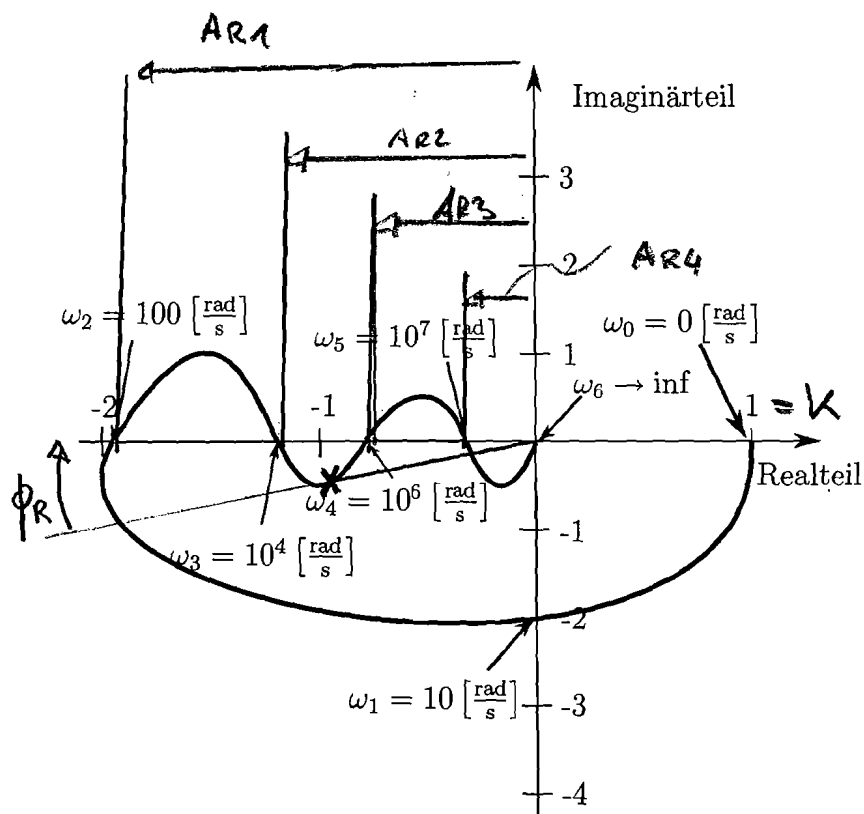
# Aufgabe 6



g)



# Aufgabe 6



h) Yes, it is stable;  $(-1/0j)$  is on the left side of the curve (spec. Nyquist).

Stable for:  $k < \frac{1}{AR_1}$  or

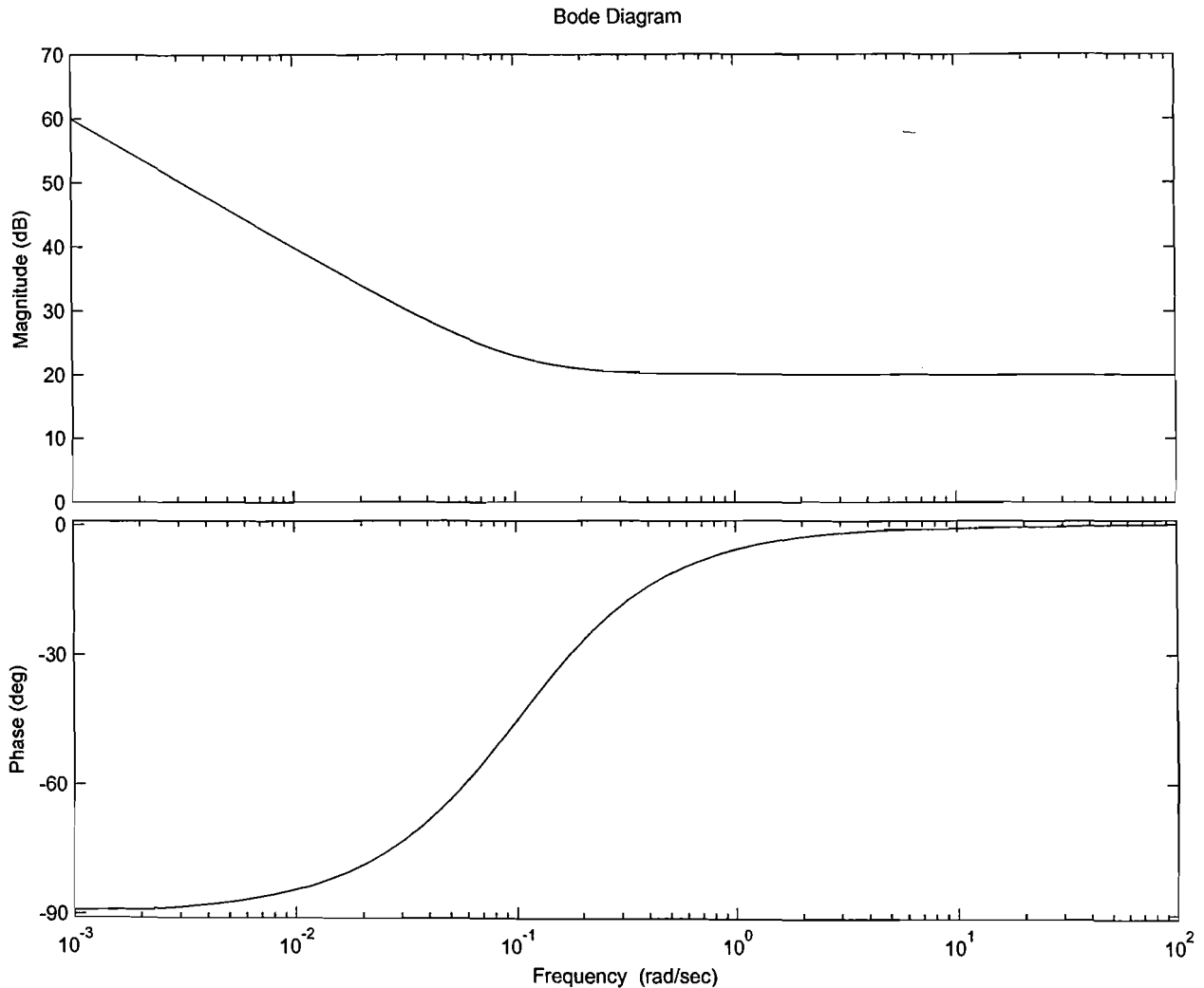
$\frac{1}{AR_2} < k < \frac{1}{AR_3}$  or

$k > \frac{1}{AR_4}$

# Aufgabe 6

i) PI,  $K = 10$ ,  $T_I = 10$

ii)



NO  $A_R$ , no  $\phi_R$

k) The corner frequencies do not change if the magnitude doubled / halved.



# Aufgabe 6

e) The phase is changing; the magnitude stays the same.

Example:

