LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Attention:

- Give your answer to problem 1 and problem 2 directly below the questions in the exam question sheet.
- This exam 'Control Technique' is taken by me as a
  ☐ Pflichtfach/prerequisite subject
  ☐ Wahlfach/elective subject.
  (Cross ONE option according to your own situation.)

Problem 1

(each 2 points)

a) Give a short definition of the terms 'plant' and 'controller'. From the view of system theory, what do they have in common, what are the differences?

☐

b) What is the Laplace transform $U(s)$ of the function

$$u(t) = 1(t-1) + 2(t-2) - 3(t-3)?$$

☐

c) Determine for the system given by $\ddot{y} + 2\dot{y} + y = u$ the state space representation as well as the transfer function and calculate the eigenvalues and the poles.

☐

d) Describe the Input/Output behavior of a $PIDT_1T_t$ system with a differential equation. Give a sketch of the step response $h(t)$ and mark the parameters K, T_1, T_D, T_t etc.

☐

e) Describe the Input/Output behavior of the system given in the following state space representation

$$A = \begin{bmatrix} 0 & 1 \\ -T_2 & -T_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \text{and} \quad \mathrm{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

☐

Problem 2

(each 2 points)

a) A system with a PDT_1 transfer behavior has the coefficients $K = 2$, $T_1 = 3$, and $T_D = 4$. What is the initial, what the final value of the step response of the system?

☐

b) How can the damping of an eigenvalue be calculated from the eigenvalue? (Sketch the related values in Figure 2.1.)
Illustrate two different cases that occur in time domain.

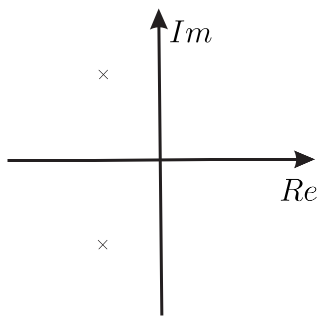
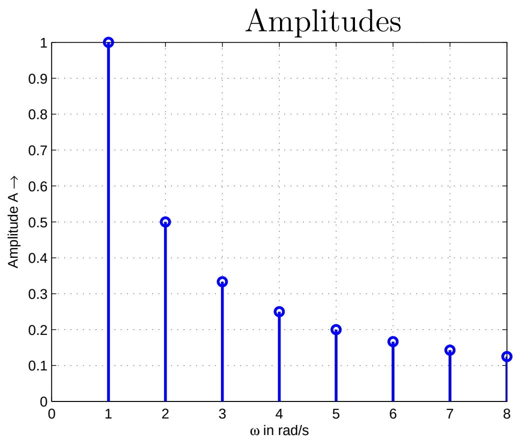


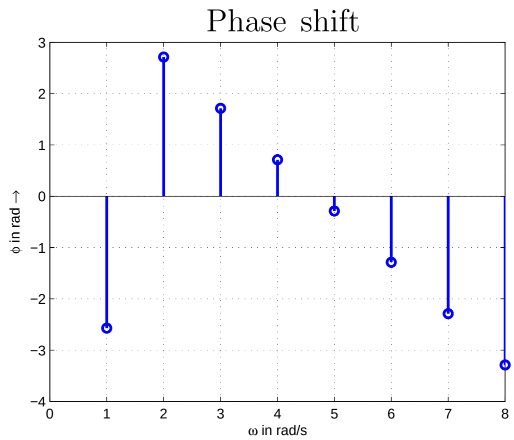
Figure 2.1: A pair of conjugate complex eigenvalues

☐

c) Sketch the differences resulting from the damping value in frequency domain. Use the step response behavior of a PT_2-system as example.

d) Read out the amplitudes and phase shifts from the figures given below and complete the transformed equation stated below.





$$y = \frac{\pi}{2} - \frac{4}{\pi}\Big[\,\underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;) + \underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;)$$

$$+\underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;) + \underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;)$$

$$+\;\underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;) + \underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;)$$

$$+\;\underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;) + \underline{\hspace{1.5cm}}\; sin(\underline{\hspace{1.5cm}}\; t + \underline{\hspace{1.5cm}}\;)\Big]$$

e) A PIT_1-system is controlled by a PD-controller with positive feedback. Determine the transfer function of the disturbance as well as the transfer function of the reference input. The disturbance acts between controller and plant.

□

Problem 3

(15 points)

The plant is described by the equation

$$10\frac{\mathrm{d}^2 p(t)}{\mathrm{d}\, t^2} + 7\frac{\mathrm{d}\, p(t)}{\mathrm{d}\, t} + p(t) = q(t),$$

where $q(t)$ is defined as

$$\frac{\mathrm{d}\, q(t)}{\mathrm{d}\, t} = u(t).$$

a) (1 point)

Determine the transfer function $\frac{P(s)}{U(s)}$ of the plant.

b) (3 points)

A P-controller with parameter K_p is utilized to realize the negative feedback. Calculate the phase shift and amplitude of the open-loop transfer function for $\omega \to 0$ and $\omega \to +\infty$. Draw the polar plot of the open-loop system for $K_p = 1$ qualitatively.

c) (4 points)

Use the special Nyquist criterion to determine the interval of K_p ($K_p > 0$) for stable behavior of the closed-loop system.

d) (2 points)

The closed-loop system has a phase margin $\phi_R = 135°$ for $K_p = 0.147$. Calculate the crossover frequency ω_s. (Hint: $\tan(45°) = 1$, $\tan(135°) = -1$)

Now add an actuator into the control loop. The block diagram of the system with an actuator is shown in Figure 3.1.

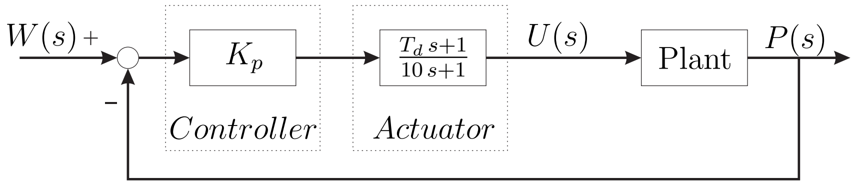


Figure 3.1: The control loop with actuator

e) (2 points)

Determine the poles and zeros of the open-loop system for $K_p = 0.1$ and $T_d = 0.5$ in Figure 3.1.

f) (3 points)

For $K_p = 0.1$ and $T_d = 0.5$ draw the approximated Bode-diagram of the open-loop system in e) in Figure 3.2 . Please note that the important frequencies and the approximated gradients of the curve have to be marked in the diagrams.

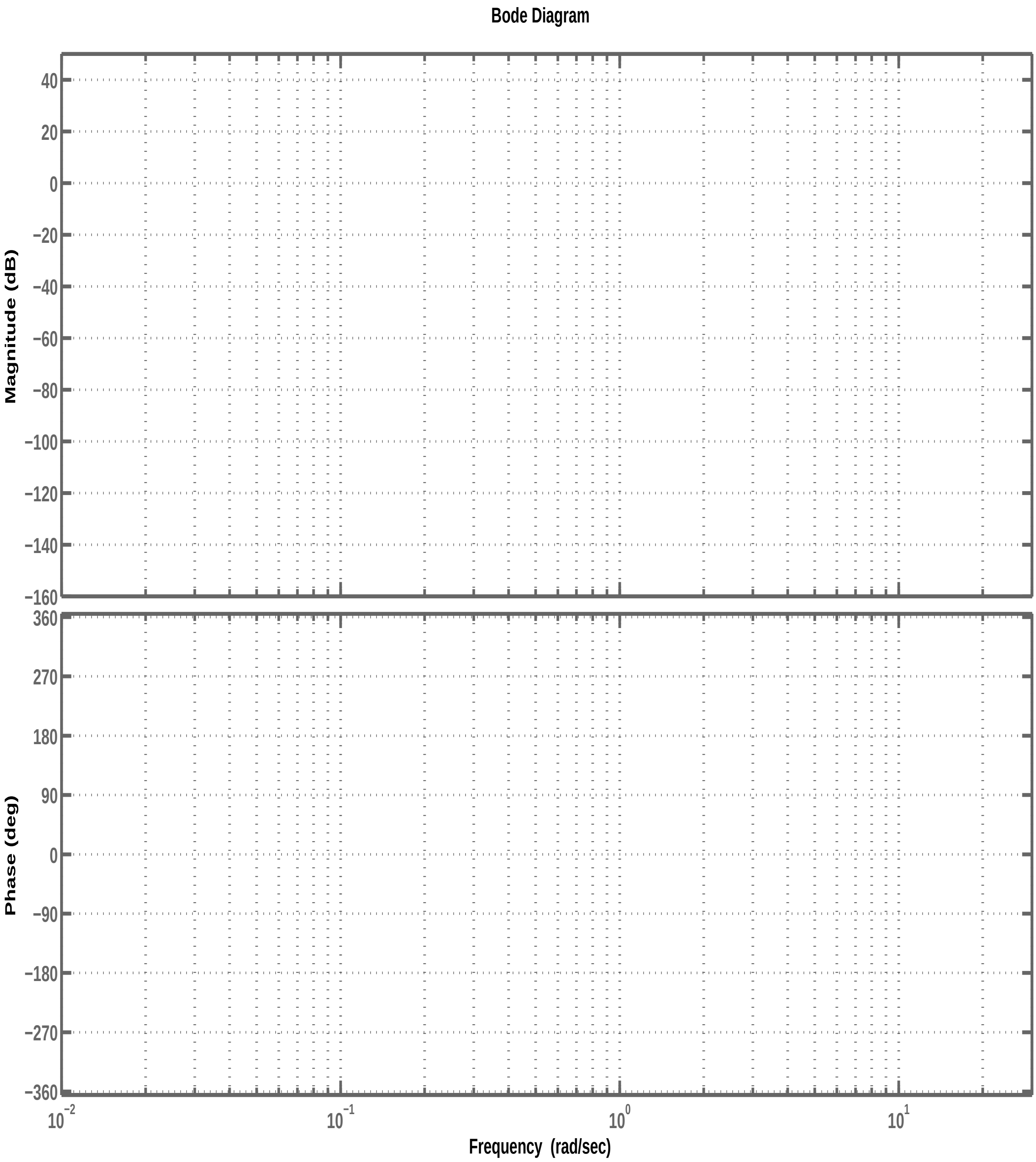


Figure 3.2: Bode Diagram

Problem 4

(15 points)

The transfer behavior $G_{U \to X_2}(s)$ of a system is dominated by the transfer elements $G_1(s)$, $G_2(s)$, and $G_3(s)$ (see Figure 4.1, S open).

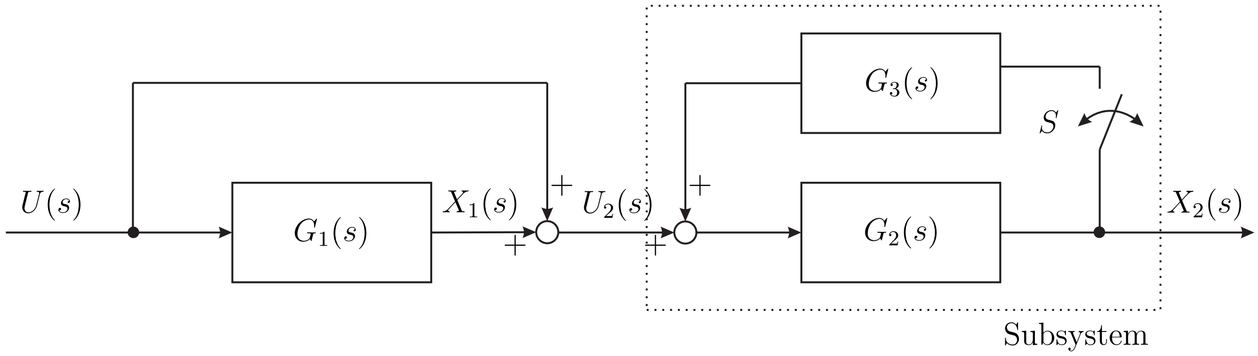


Figure 4.1: Block diagram

The individual transfer elements are described by

$$G_1(s) = \frac{1}{s+a}, \quad G_2(s) = \frac{1}{s(s+b)}, \quad \text{and} \quad G_3(s) = K_P \frac{1 + \frac{1}{T_3}s}{1 + \frac{1}{T_4}s}.$$

a) (2 points)
State the differential equations for the elements $G_1(s)$ and $G_2(s)$.

b) (4 points)
Set up the state space model for the transfer behavior $G_{U \to X_2}(s)$ (S open) with the state vector $x = [x_1 \;\; x_2 \;\; \dot{x}_2]^T$ and calculate the eigenvalues and poles.

c) (1 point)
Calculate the order of the system $G_{U \to X_2}(s)$ (S open).

In the following, the transfer function

$$G_{U \to X_2}(s) = \frac{s+8}{s^3 + 2s^2 - 35s}$$

is used to describe the transfer behavior of the system (S open).

d) (2 points)
Is the system described by $G_{U \to X_2}(s)$ asymptotically stable? State reason (e.g. by calculation).

e) (6 points)
Now the switch S in the subsystem is closed. Determine the transfer function $G_{U_2 \to X_2}(s)$ of the subsystem. Furthermore, determine the interval of T_4 for asymptotically stable behavior of $G_{U_2 \to X_2}(s)$ with $K_P = -\frac{1}{6}$, $T_3 = \frac{1}{60}$, and $b = -5$.

Problem 5

(16 points)

The transfer function of a plant is given by

$$G_P(s) = 15 \cdot \frac{2 + 2s + s^2}{(1-s)(4-s)(5-4s+s^2)}.$$

Beside the plant, two controllers are given by the transfer functions

$$G_{C1}(s) = K_{C1}$$

and

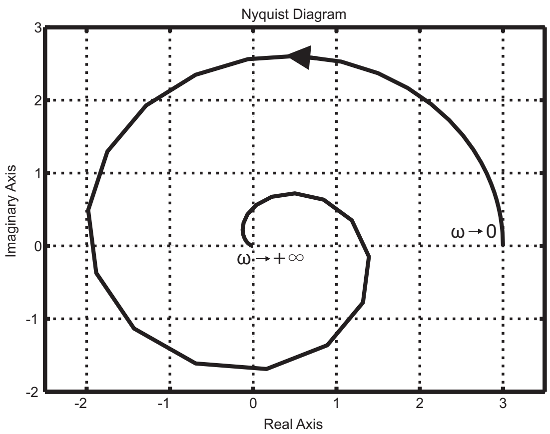
$$G_{C2}(s) = K_{C2} \cdot (1 + \frac{1}{2}s).$$

Both controllers are combined individually with the given plant to a closed-loop system with negative feedback.

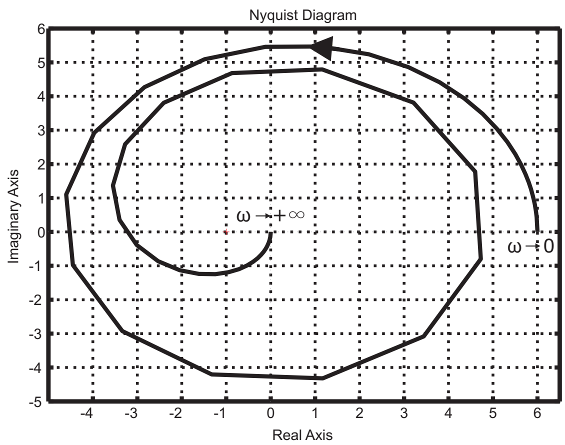
a) (2 points)
Determine the stability of the plant and give the transfer functions of both open loops.

b) (2 points)
Can the special/simplified Nyquist criterion be applied to determine the stability of both closed-loop systems? State reasons.

In Figure 5.1, the calculated polar plots of the two open-loop systems are visualized. In the following, the general Nyquist criterion should be applied to determine the stability of the closed-loop systems.



(a) Open-loop polar plot with G_{C1}



(b) Open-loop polar plot with G_{C2}

Figure 5.1: Polar plots of the open loops

c) (3 points)
Calculate the gain K_P of the plant and determine the gains K_{C1} and K_{C2} of the controllers with the help of the polar plots.

d) (3 points)
Use the general Nyquist criterion to evaluate the stability of both closed-loop systems.

Two new stable plants are given, which are both controlled by a transfer element with P character (negative feedback). The input/output behaviors of the two plants correspond to a PDT_3 (with $T_1 < T_D$) and an IT_2 transfer element respectively.

e) (3 points)
Draw the polar plots of both open-loop systems qualitatively.

f) (3 points)
Evaluate the stability of both closed-loop systems, depending on the gains, using the general Nyquist criterion.

Maximum achievable points:	**66**
Minimum points for the grade 1,0:	**95** %
Minimum points for the grade 4,0:	**50** %