1a) plant: system to be controlled

controller: realises feedback and compares reference value and actual value.

⇒ common: in system theory: both are transfer element

⇒ differences: • function in control-loop

• plant is given, controller can be adapted

b) $u(s) = \frac{1}{s} e^{-s} + 2 \cdot \frac{1}{s} e^{-2s} - \frac{3}{s} \cdot e^{-3s}$

c) Eigenvalue: Solution of $\cancel{\det}(\lambda I - A) = 0$

Poles: zeros in denominator of transfer function

$$\ddot{y} = -2\dot{y} - y + u$$

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

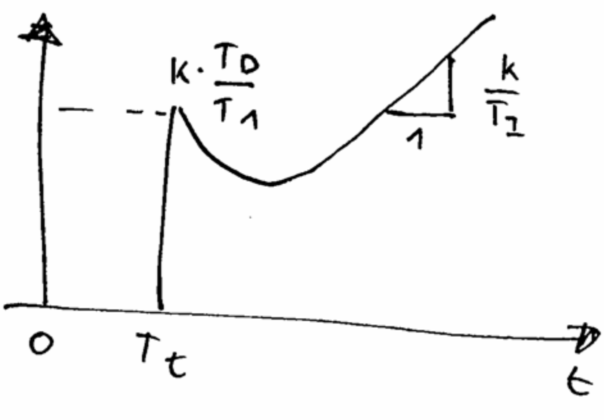
transfer function

$$G(s) = \frac{1}{s^2 + 2s + 1}$$

poles: $s_{1,2} = -1$

eigenvalues: $\lambda_{1,2} = -1$

d) $T_1 \dot{y} + y = k \left[u(t - T_t) + T_D \cdot \dot{u}(t - T_t) + \frac{1}{T_I} \cdot \int_0^t u(\tau - T_t) d\tau \right]$



$$\ddot{y} + T_1 \dot{y} + T_2 y = 2u$$ or

$$G(s) = \frac{2}{s^2 + T_1 s + T_2}$$

2 a) transfer function:

$$G(s) = \frac{2(1+4s)}{3s+1}$$

Step response:

$$h(s) = G(s)\cdot\frac{1}{s} = \frac{2\cdot(1+4s)}{3s^2+s}$$

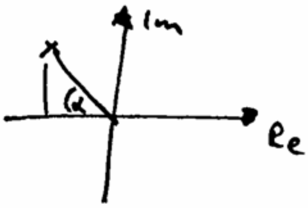
Initial value:

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} s\cdot\frac{1}{s}\cdot G(s) = \lim_{s\to\infty} s\cdot\frac{1}{s}\cdot 2\cdot\frac{1+4s}{1+3s} = \frac{8}{3}$$

stationary value:

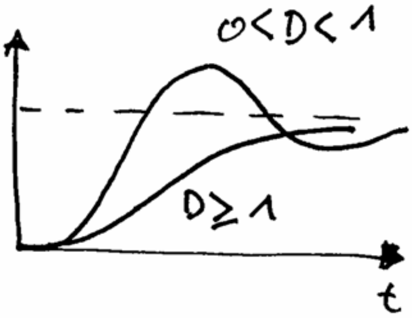
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot G(s) = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot 2\,\frac{1+4s}{1+3s} = 2$$

b)

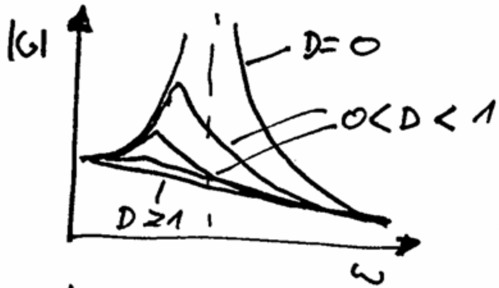


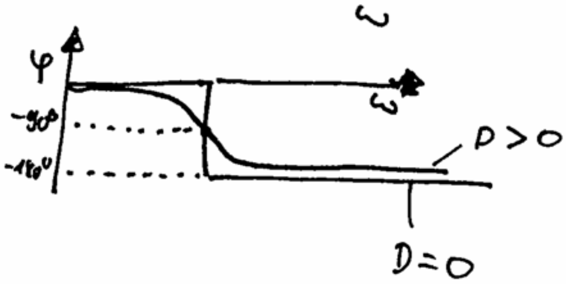
$D = \cos\alpha$

with $\tan\alpha = \frac{\mathrm{Im}\{x\}}{\mathrm{Re}\{x\}}$



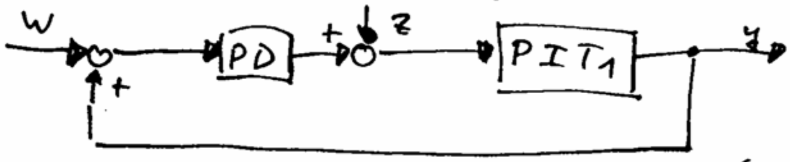
c)





d)
$$y = [\ldots] - \frac{4}{\pi}\Big[1\cdot\sin(t-2{,}7) + 0{,}5\cdot\sin(2t+2{,}7) + 0{,}33\cdot\sin(3\cdot t+1{,}8) + 0{,}25\cdot\sin(4t+0{,}7)\ldots\Big]$$

e)



$$G_{PD} = k_1\,(1+T_D\cdot s)$$

$$G_{PIT_1} = k_2\,\frac{1+\frac{1}{T_I\cdot s}}{1+T_1\cdot s}$$

$$G_0 = k_1\cdot k_2\,\frac{(1+T_D\cdot s)(1+\frac{1}{T_I\cdot s})}{1+T_1\cdot s}$$

$$G_z = \frac{G_{PIT_1}}{1-G_0}$$

$$G_w = \frac{G_0}{1-G_0}$$

3) a) $G_R G_S = \dfrac{k_R \cdot k_S \, (1+T_I \cdot s)}{s(1+T_1 \cdot s + T_2 \cdot s^2)(1+T_3 \cdot s)}$

b) poles: $s_1 = 0$; $s_2 = -\dfrac{1}{T_3}$; $s_{3,4} = -\dfrac{T_1}{2T_2} \pm \sqrt{\dfrac{T_1^2}{4T_2^2} - \dfrac{1}{T_2}}$

zeros: $s_{01} = -\dfrac{1}{T_I}$

c) Hurwitz matrix:

$$H = \begin{bmatrix} 1+T_1 & 1 & 0 & 0 \\ 1 & 1+T_1 & 10k & 0 \\ 0 & 1+T_1 & 1 & 0 \\ 0 & 1 & 1+T_1 & 10k \end{bmatrix}$$

Übertragungsfunktion des geschlossenen Regelkreises:

$$G(s) = \frac{G_0}{1+G_0} = \frac{10k}{s^4+(1+T_1)s^3+(1+T_1)s^2+s+10k}$$

$a_4 = 1$, $a_3 = 1+T_1$

$a_2 = 1+T_1$, $a_1 = 1$

$a_0 = 10k$

d) stationary accurate due to I-part in controller

e) $G_R G_S = 10K \, \dfrac{1}{s(1+T_1 s + s^2)(1+s)}$

poles: $n = 4$

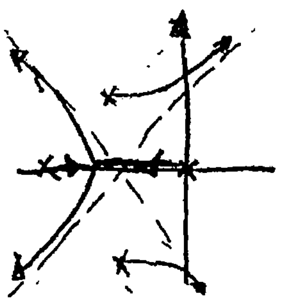
zeros: $q = 0$

4 asymptotes

angle: $\phi_1 = 45^\circ$; $\phi_2 = 135^\circ$; $\phi_3 = -135^\circ$; $\phi_4 = -45^\circ$

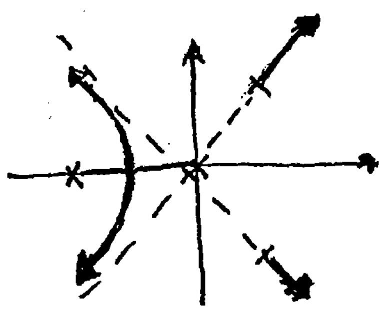
poles at: $s_1 = 0$; $s_2 = -1$; $s_{3,4} = -\dfrac{T_1}{2} \pm \sqrt{\dfrac{T_1^2}{4} - 1}$

$T_1 > 0$:



stable for $0 < K < K_{krit}$

$T_1 < 0$:



unstable

Problem 4

a). Using Laplace transformation

$$(10s^2+7s+1)\,P(s) = Q(s)$$

$$s\,Q(s) = U(s)$$

$$\Rightarrow \frac{P(s)}{U(s)} = \frac{1}{s(10s^2+7s+1)}$$

b). Open-loop transfer function is

$$G_0(s) = \frac{K_P}{s(5s+1)(2s+1)}$$

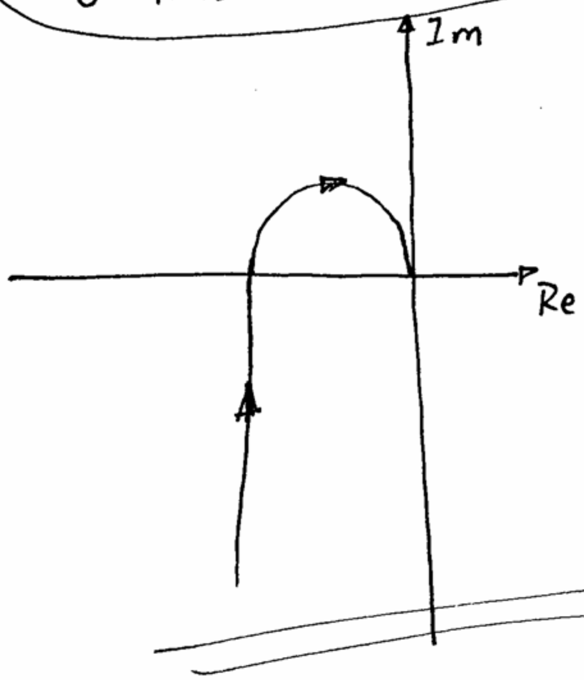
$$\Rightarrow G_0(j\omega) = \frac{K_P}{j\omega\,(j\cdot 5\omega+1)\,(j\cdot 2\omega+1)}$$

$\omega \to 0$ $\quad |G_0|_{\omega\to 0} = \infty$

$\arg G_{0\,\omega\to 0} = -90^\circ$

$\omega \to +\infty$ $\quad |G_0|_{\omega\to+\infty} = 0$

$\arg G_{0\,\omega\to+\infty} = -270^\circ$



C). From the polar plot of the open-loop system and special Nyquist criterion, it can be known that the system is stable, when

$$\mathrm{Re}\left(G_0(j\omega)\right)\Big|_{\mathrm{Im}(G_0(j\omega))=0} > -1 .$$

$$G_0(j\omega) = \frac{-7K_p - j\,\dfrac{K_p(1-10\omega^2)}{\omega}}{1+29\omega^2+100\omega^4}$$

$$\mathrm{Im}\left(G_0(j\omega)\right) = 0 \quad \Rightarrow \quad \omega^2 = \frac{1}{10}$$

$$\mathrm{Re}\left(G_0(j\omega)\right)\Big|_{\omega=\sqrt{\frac{1}{10}}} > -1$$

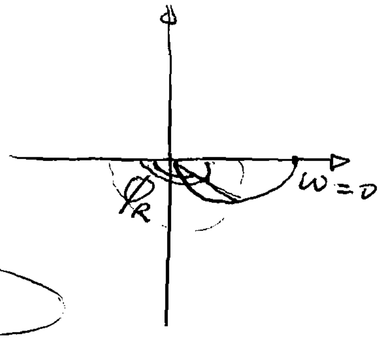
$$\Rightarrow \quad \frac{-7K_p}{1+\frac{29}{10}+1} > -1$$

$$\Rightarrow \quad K_p < \frac{7}{10}$$

d) Phase margin $\phi_R = 135°$

$$|\angle G_o(j\omega_s)| = 180° - 135° = 45°$$

From the figure on the right,



$$\angle G_o(j\omega_s) = -45°$$

$$\Rightarrow \mathrm{Re}(G_o(j\omega_s)) = -\mathrm{Im}(G_o(j\omega_s))$$

$$\Rightarrow 7K_p = \frac{K_p(1-10\omega_s^2)}{\omega_s}$$

$$10\omega_s^2 - 7\omega_s - 1 = 0$$

$$\Rightarrow \omega_s = \frac{7+\sqrt{89}}{20} \approx 0.8217$$

e) when $K_p = 0.1$, $K_d = 0.5$

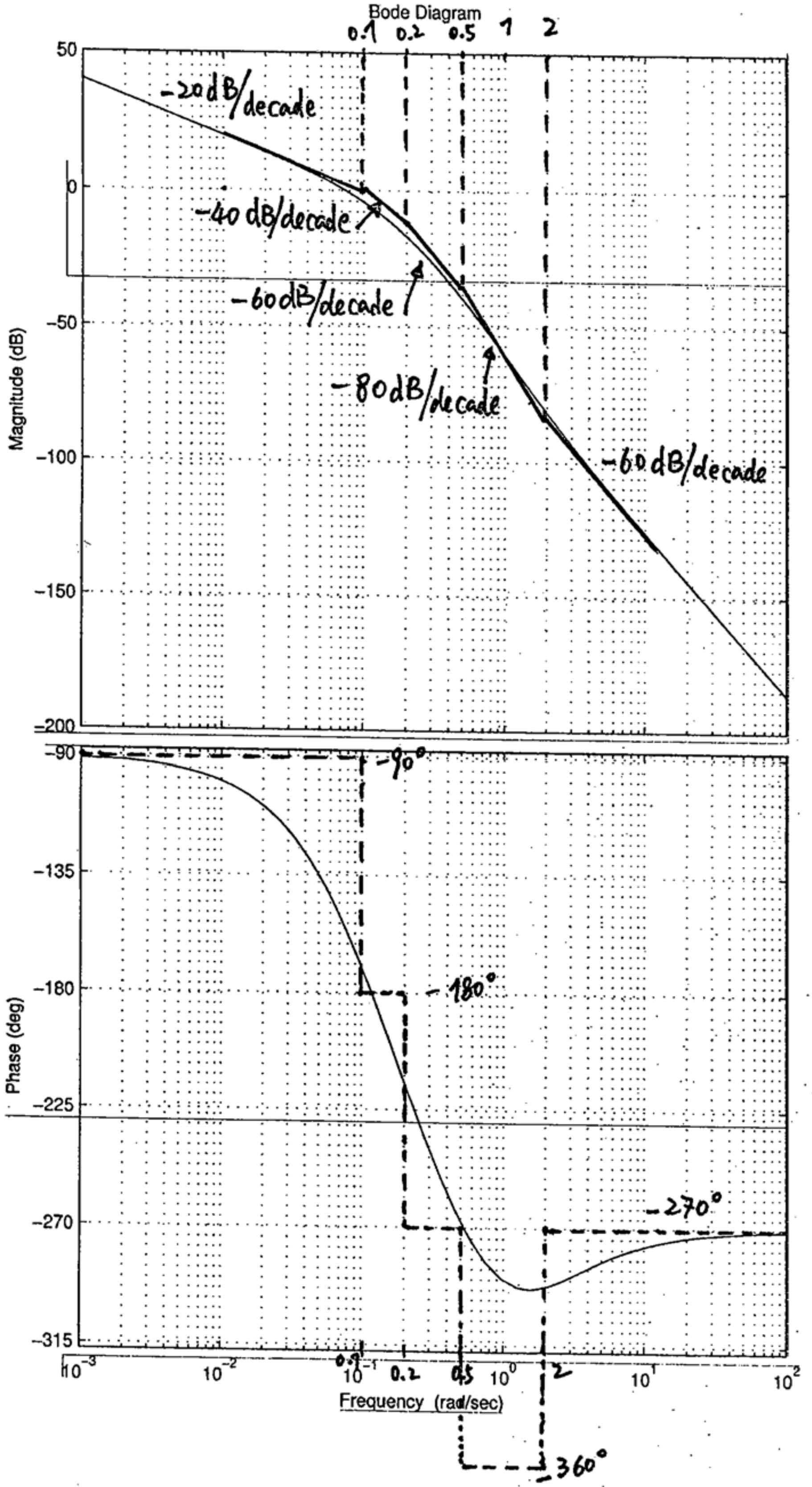
$$G_o(s) = \frac{0.1(0.5s+1)}{s(5s+1)(2s+1)(10s+1)}$$

poles : $s_1 = 0$, $s_2 = -\frac{1}{5}$, $s_3 = -\frac{1}{2}$, $s_4 = -\frac{1}{10}$

zero : $z_1 = -2$.

f.|

4-④


Bode Diagram
0.1 0.2 0.5 1 2
-20dB/decade
-40dB/decade
-60dB/decade
-80dB/decade
-60dB/decade
Magnitude (dB)
50
0
-50
-100
-150
-200
-90
-135
-180
-225
-270
-315
Phase (deg)
-90°
-180°
-270°
-360°
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
0.1 0.2 0.5 2
Frequency (rad/sec)

Problem 5

a) ODEs

$$G_1(s) = \frac{X_1(s)}{U(s)} : \dot{x}_1 = -a x_1 + u$$

$$G_2(s) = \frac{X_2(s)}{U_2(s)} : \ddot{x}_2 = -b\dot{x}_2 + u_2$$

$[U_2(s) = X(s) + U(s)]$

b)
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -a & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$$

Eigenvalues : $|A - \lambda I| \overset{!}{=} 0$

$$\Rightarrow \begin{vmatrix} -a-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -b-\lambda \end{vmatrix} \overset{!}{=} 0$$

$$\Rightarrow -(a+\lambda)(b+\lambda)\lambda \overset{!}{=} 0$$

$$\lambda_1 = 0, \quad \lambda_2 = -a, \quad \lambda_3 = -b$$

Transfer function : $G(s) = \dfrac{s + (a+1)}{s\left[s^2 + (a+b)s + ab\right]}$

$\Rightarrow$ no pole-zero-cancelation $\Rightarrow$ Poles $\hat{=}$ Eigenvalues

Problem 5

c) System order $n = 3$

d) characteristic equation $C(s)$:

$$C(s) = \underbrace{1}_{a_3} \cdot s^3 + \underbrace{2}_{a_2} \cdot s^2 - \underbrace{35}_{a_1} s$$

According to HURWITZ: $a_0 = 0 \Rightarrow$ instable

- - - - - - - - - - or - - - - - - - - - - - - -

$$C(s) = s\left(s^2 + 2s - 35\right)$$

Pole $s_1 = 0 \Rightarrow$ not asymptotical stable

e)

$$G_{u_2 \to x_2}(s) = \frac{X_2(s)}{U_2(s)}$$

$$= \frac{G_2(s)}{1 - G_2(s) \cdot G_3(s)}$$

$$= \frac{1 + \frac{1}{T_4} s}{\underbrace{\frac{1}{T_4}}_{a_3} s^3 + \underbrace{\left(1 - \frac{5}{T_4}\right)}_{a_2} s^2 + \underbrace{5}_{a_1} s + \underbrace{\frac{1}{6}}_{a_0}}$$

...

Problem 5

... e) Asymptotic stable if...

① all coefficients of characteristic equation exist and have the same sign:

$a_3: \frac{1}{T_4} > 0 \Rightarrow T_4 > 0$

$a_2: 1 - \frac{5}{T_4} > 0 \Rightarrow T_4 > 5$

$a_1: 5 > 0$ ✓

$a_0: \frac{1}{6} > 0$ ✓

and ② HURWITZ

$H_1 = |a_2| = 1 - \frac{5}{T_4} \overset{!}{>} 0 \Rightarrow T_4 > 5$ ✓

$$H_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = 5\left(1 - \frac{5}{T_4}\right) - \frac{1}{6} \cdot \frac{1}{T_4} \overset{!}{>} 0$$

$$\Rightarrow T_4 > \frac{151}{30}$$

$H_3 = a_0 \cdot |H_2| = \frac{1}{6} \cdot |H_2| > 0$, if $H_2 > 0$

$\Rightarrow H_3 > 0$ for $T_4 > \frac{151}{30}$

$\Rightarrow$ Asymptotical stable for $T_4 > \frac{151}{30}$.

6 a) - The plant is not stable. (poles with pos. real part)

$$- G_{01} = 15 \cdot K_{R1} \cdot \frac{s^2 + 2s + 2}{(1-s)(4-s)(5-4s+s^2)}$$

$$- G_{02} = 15 \cdot K_{R2} \cdot \frac{(\frac{1}{2}s+1)(s^2+2s+2)}{(1-s)(4-s)(5-4s+s^2)}$$

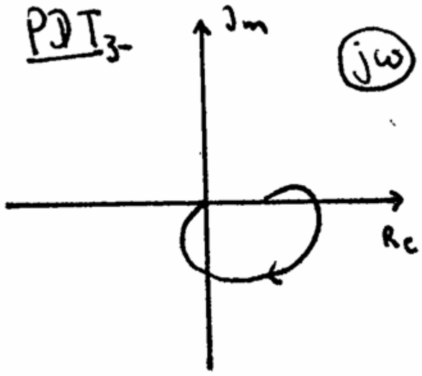
b) - No, both open-loop systems are not stable.

c) $k_S = 1{,}5$; $k_{R1} = 2$; $k_{R2} = 4$

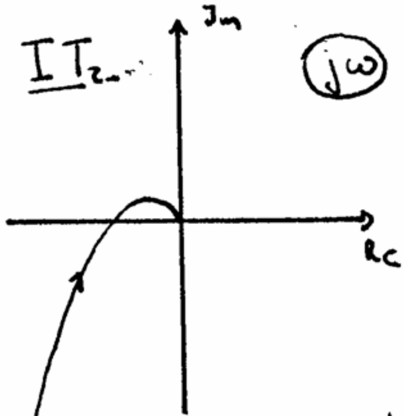
d) - Both open-loop systems have 4 poles with pos. real part.

- 1st open-loop system encl. the crit. point 2 times
  $\hat{u} \neq -n^+$ —> closed-loop system is not stable.
- 2nd open-loop system encl. the crit. point 4 times
  $\hat{u} = -n^+$ —> closed-loop system is stable.

e) PDT_3 -



IT_2 -



f) - If the crit. point is enclosed, closed-loop system is unstable

- The polar plot of the PDT_3 system never encl. the crit. point independently of the gain —> stable
- It depends on the gain, whether the polar plot of the IT_2 system encl. the crit. point (unstable) or not (stable)

6 g) plant: PDT_4 contr. 1: P contr. 2: PT_1

The plant is asymptotic stable
(all poles have neg. real parts)

h) $s_{1,2} = -1 \pm 2i$ $s_3 = -2$ $s_4 = -3$

$D_{1,2} = \frac{1}{\sqrt{5}}$ $\omega_{0_{1,2}} = \sqrt{5}$ $\omega_{03} = 2$ $\omega_{04} = 3$

$D_3 = D_4 = 1$

i) system 1: 4 sep. branches, 3 branches go to inf.

system 2: 5 sep. branches, 4 branches go to inf.

j) system 1: $\sigma_{w1} = -\frac{2}{3}$ $\varphi_{1_1} = 60°$, $\varphi_{1_2} = 180°$, $\varphi_{1_3} = 300°$

system 2: $\sigma_{w2} = -\frac{3}{2}$ $\varphi_{2_1} = 45°$, $\varphi_{2_2} = 135°$, $\varphi_{2_3} = 225°$

$\varphi_{2_4} = 315°$

k) see next page

l) $G_2 = \frac{k_{R3}}{(1+\frac{1}{2}s)(1+\frac{2}{5}s+\frac{1}{5}s^2)} = -1$ $\Rightarrow k_{R3} > -1$

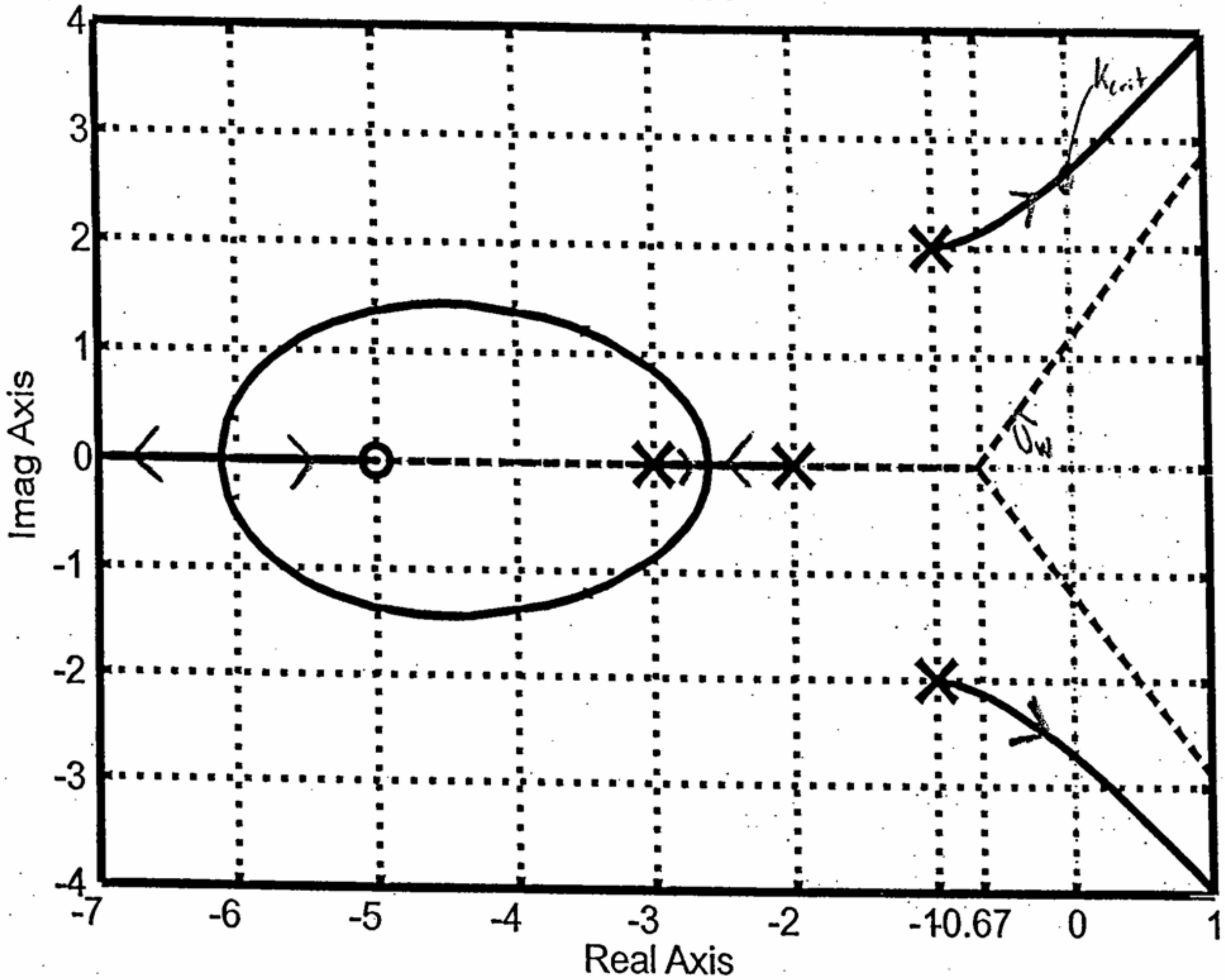
$$H = \begin{bmatrix} 4 & 10+10k_{R3} & 0 \\ 1 & 9 & 0 \\ 0 & 4 & 10+10k_{R3} \end{bmatrix}$$

$|H_1| > 0$

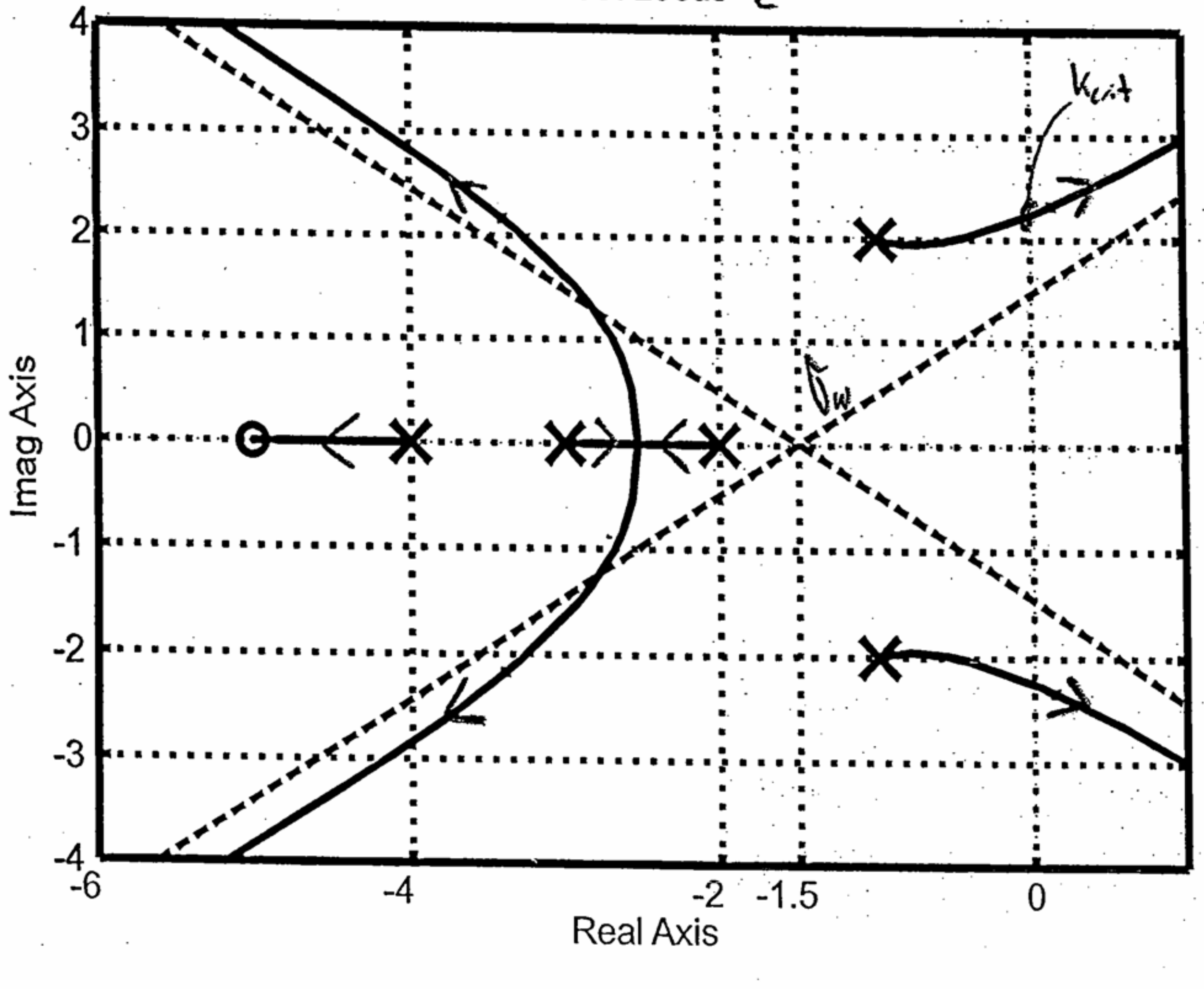
$|H_2| > 0$ with $k_{R3} < 2{,}6$

k)

Root Locus 1



Root Locus 2



6 m) Die Regelstrecke ist stabil und weist näherungsweise aperiodisches Übergangsverhalten auf.

n) $K_S = 4 \quad T_t = 3_s \quad T = 2_s$

P-Regler: $k_p = \frac{1}{6} \qquad G_p(s) = \frac{1}{6}$

PI-Regler: $k_p = \frac{3}{20} \quad T_I = 9{,}99 \qquad G_{PI}(s) = \frac{3}{20}\left(1 + \frac{1}{10s}\right)$

o) Die Regelstrecke ist stabil und kann zeitweise im grenzstabilen Bereich betrieben werden.

p) $K_{crit} = 3 \qquad T_{crit} = 4s$

PID-Regler : $k_p = 1{,}8 \quad T_I = 2s \quad T_D = 0{,}48s$

$$G_{PID}(s) = 1{,}8\left(1 + 0{,}48\,s + \frac{1}{2s}\right)$$