Active Flutter Suppression of a Nonlinear Aeroelastic System Using PI-Observer

Fan Zhang and Dirk Söffker

Abstract In this paper a novel robust control is proposed for the purpose of active flutter suppression of a nonlinear 2-D wing-flap system in the incompressible flow field. The controller consists of an optimized robust stabilizer in the form of state feedback control and a Proportional-Integral Observer (PI-Observer). The optimized robust stabilizer is based on the former study about the time-domain robust stable criterion and obtained by a numerical optimization process. The PI-Observer is taken to estimate not only the system states but the bounds of the nonlinearities which are necessary for the constraints of the optimization process. The simulation results are given to show the performance of this control design approach in suppressing the flutter and the limit cycle oscillations.

1 Introduction

It is well-known that nonlinearities, no matter structural or aerodynamical, may exhibit a variety of responses that are typically associated with nonlinear regimes of response including Limit Cycle Oscillation, flutter, and even chaotic vibrations [1] in aeroelastic systems. And significant decays of the flutter speed may happen and cause unexpected or even fatal accidents. Therefore, it is necessary to take uncertainties and nonlinearities into account in aeroelastic problems.

In studies of flutter suppression of nonlinear systems, an aeroelastic model has been developed based on the research of the benchmark active control technology (BACT) wind-tunnel model designed at the NASA Langley Research Center [2–

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5]. For this kind of model a set of tests have been performed in a wind tunnel to examine the effect of nonlinear structure stiffness. And control systems have been designed using linear control theory, feedback linearizing technique, and adaptive control strategies [6–13]. The methods in these contributions, such as model reference adaptive control approaches [9], backstepping design methods [10, 11], robust control design with high gain observers [12] and so on, stand for the general approaches dealing with the effect of structural nonlinearities in aeroelastic problems.

Different from the methods above, this paper proposes a new active control strategy to suppress the instability caused by structural nonlinearities, thereby keeping the system robustly stable. The control strategy starts from the time-domain robust stability criterion for the system with structural uncertainties [14]. A useful conclusion is deduced from this criterion in this paper that, if such a controller exists that the largest singular value of the solution to one certain Lyapunov equation is minimized within the constraints given by the criterion, the system will not only keep stable but also gain the largest robustness against a uncertain disturbance. An optimization procedure is adopted to find such a robust controller.

The optimization procedure requires the information of unknown states and the bounds of perturbations, which can be provided by the PI-Observer. Advanced simulation results of the PI-Observer are given in [15, 16]. Actual experimental results of the used special disturbance observer, the PI-Observer, are given in [17, 18]. The online estimation of the bounds of the uncertain parameters which are understood as unknown external input to known systems for diagnosis and control, is already realized in several theoretical and experimental applications and publications [17, 19]. With the help of the PI-Observer, an optimized robust controller can be realized and the nonlinear system is stabilized in the way of the state feedback control.

This contribution is organized in the following way: Section 2 introduces the configuration of the nonlinear aeroelastic model; in Section 3 the new robustness measurement is developed here firstly,providing the theoretical basis of this paper; in Section 4 the design of the PI-Observer and how it estimates the nonlinear effect will be introduced briefly; in Section 5 the control strategy is reformulated in context of flutter suppression of the aeroelastic model; Section 6 will give the simulation results, where it can be seen that the proposed controller performs well against structural nonlinearities, with flutter being suppressed at different wind speed.

2 Problem Statement: Configuration of the Nonlinear Aeroelastic Model

The configuration of the nonlinear 2-D wing-flap system is shown in Figure 1. This model has been widely used in the aeroelastic studies [6–8]. The two degrees of freedom, the pitching and plunging movement, are respectively restrained by a pair of springs attached to the elastic axis(EA) of the airfoil. A single trailing-edge control surface is used to control the air flow, thereby providing more maneuverability

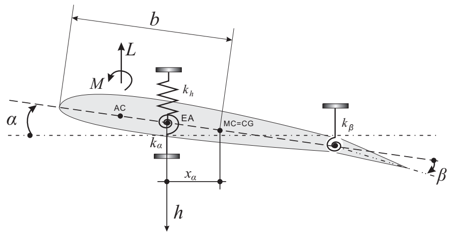

Fig. 1 2-D wing-flap aeroelastic model.

to suppress instability. This model is accurate for airfoils at low velocity and has been confirmed by wind tunnel experiments [6, 10].

The government equations of this model are given as

$$\begin{bmatrix} m_T & m_W x_\alpha b \\ m_W x_\alpha b & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix}, \tag{1}$$

where plunging and pitching displacement are denoted as h and α respectively. In Eq. (1) m_W is the mass of the wing, m_T is the total mass of the wing and its support structure, c_α and c_h are the pitch and plunge damping coefficients respectively, k_α and k_h are the pitch and plunge spring constants respectively. The variables M and L denote the aerodynamic lift and moment. In the case when the quasi-steady aerodynamics is considered, M and L should be of the form as

$$\begin{aligned} L &= \rho U^2 b c_{l_\alpha}\left[\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a\right) b \frac{\dot{\alpha}}{U}\right] + \rho U^2 b c_{l_\beta}\beta\ , \\ M &= \rho U^2 b^2 c_{m_\alpha}\left[\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a\right) b \frac{\dot{\alpha}}{U}\right] + \rho U^2 b^2 c_{m_\beta}\beta\ , \end{aligned} \tag{2}$$

where c_{l_α} and c_{m_α} are the lift and moment coefficients per angle of attack and c_{l_β} and c_{m_β} are lift and moment coefficients per angle of control surface deflection β.

The nonlinearity is supposed to exist in the pitching spring constant k_α and has the form of a polynomial of α,

$$k_\alpha = \sum_{i=0}^{4} k_{\alpha_i}\alpha^i = k_{\alpha_0} + k_\alpha^*(\alpha)\ , \tag{3}$$

where $k_\alpha^*(\alpha) = \sum_{i=1}^{4} k_{\alpha_i}\alpha^i$. The coefficients k_{α_i}, $i = 0, 1, \ldots, 4$ determined from experimental data given in [10] are

$$[k_{\alpha_i}] = [6.833 \quad 9.967 \quad 667.685 \quad 26.569 \quad -5084.931]^T\ . \tag{4}$$

Defining the state vector $x(t) = [\alpha(t),\ h(t),\ \dot{\alpha}(t),\ \dot{h}(t)]^T$, one can obtain a state variable representation of Eq. (1) in the form

$$\dot{x}(t) = A_n\, x(t) + k_\alpha^*\, N\, \alpha(t) + B\, \beta(t)\,,$$
$$y(t) = C\, x(t)\ , \tag{5}$$

where $C = [1\ 0\ 0\ 0]$, as the only measurable state is the pitch angle α. The explicit expressions of $A_n,\ N,\ B$ are given in [10].

3 Formulation of Robust Stability Control Problem

Assume a perturbed system can be described by the sum of a linear nominal system and uncertain perturbations as

$$\dot{x}(t) = (A + E)x(t)\ , \tag{6}$$

where A is a $n \times n$ real Hurwitz matrix denoting the nominal system, E is a $n \times n$ perturbation matrix and can be expressed as

$$E = \sum_{k=1}^{r} k_i E_i, \quad i = 1, 2, 3, \ldots, r\ , \tag{7}$$

where E_i is a constant matrix which shows how the uncertain parameter k_i perturbs the nominal matrix A.

Let P be the solution of the following Lyapunov equation:

$$A^T P + P\, A + 2\, I = 0\,. \tag{8}$$

Define P_i as

$$P_i = \frac{1}{2}(E_i^T\, P + P\, E_i)\,. \tag{9}$$

Following the results given in [14], the system (6) will be asymptotic stable if

$$\sum_{i=1}^{r} |k_i|\, \sigma_{\max}(P_i)\ < 1\,, \tag{10}$$

where symbol $\sigma_{\max}(\cdot)$ denotes the largest singular value. The proof of this result is given in [14].

Being more robust stable means that the system can keep stable with larger perturbation. Correspondingly, when the stable condition Eq. (10) is satisfied, if a certain controller is found to make each $\sigma_{\max}(P_i)$ in Eq. (10) minimized, the system (6) can bear the largest perturbation of $|k_i|$ and therefore will be robust stable against the perturbation $|k_i|$.

Therefore, the understanding of this robust control can be formulated as: the goal to make the system (6) most robust stable can be achieved by such a stabilizing controller that it makes each $\sigma_{\max}(P_i)$ minimized, under the constraints of Eqs. (8) and (10).

4 Estimation of System States and Unknown Effects via PI-Observer

Equation (10) shows that the bound of the uncertain perturbation $|k_i|$ is necessary for the constraints of the optimization process. For systems with structural nonlinearities, a PI-Observer can estimate the states and the unknown perturbations acting upon the nominal system [16], which is explained here briefly.

The system to be controlled is assumed as a nominal known system with additive unknown external inputs/unknown effects $n(t)$ and additive measurement noise $d(t)$

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Nn(t) \ , \\ y(t) &= Cx(t) + d(t) \ . \end{aligned} \tag{11}$$

Assume that the unknown input effect which includes model uncertainties and disturbances is caused by the uncertainty modeled in Eq. (11). So the uncertainty can be calculated if the estimation of $n(t)$ is available. In the sequel, the task is reduced to estimate the unknown effects $n(t)$.

A PI-Observer design [16] can be written by

$$\begin{aligned} \begin{bmatrix} \dot{\hat{z}} \\ \dot{\hat{n}} \end{bmatrix} &= \underbrace{\begin{bmatrix} A & N \\ 0 & 0 \end{bmatrix}}_{A_e} \begin{bmatrix} \hat{z} \\ \hat{n} \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} u + \underbrace{\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}}_{L} (y - \hat{y}), \\ \hat{y} &= \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} \hat{z} \\ \hat{n} \end{bmatrix} + \begin{bmatrix} d(t) \\ 0 \end{bmatrix}. \end{aligned} \tag{12}$$

The error dynamics becomes

$$\begin{bmatrix} \dot{e}(t) \\ \dot{f}_e(t) \end{bmatrix} = \begin{bmatrix} A - L_1 C & N \\ -L_2 C & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ f_e(t) \end{bmatrix} + \begin{bmatrix} L_1\, d(t) \\ L_2 \dot{(t)} - \dot{n}(t) \end{bmatrix} . \tag{13}$$

Assuming that the extended system is observable and the feedback matrices L_1 and L_2 can be calculated by solving the Riccati equation

$$A_e P + P A_e{}^T + Q - P C_e{}^T R^{-1} C_e P = 0 \ , \tag{14}$$

the observer feedback matrix L is denoted by

$$L \ = \ \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = P C^T R^{-1}. \tag{15}$$

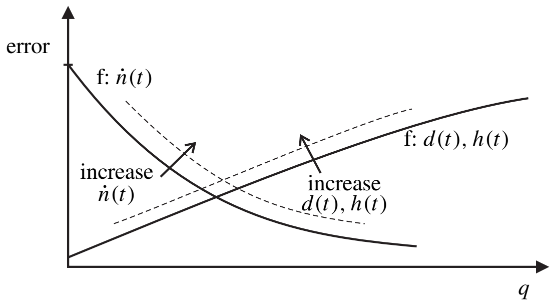

Fig. 2 Schematic behavior of the estimation error [18].

If the extended system is observable, the gains $\|L_2\|$ increases under some conditions by increasing the control design parameter, which is here the coefficient q of the weighting matrix for the Riccati solution. To achieve an approximative decoupling from the unknown inputs, here from the uncertainties Nn to the states x

$$\|[Is - (A_e - LC_e)]^{-1} N_e\|_\infty < \epsilon \,, \tag{16}$$

$\epsilon \to 0$ is required, so the weighting parameter has to be $q \to \infty$. In practical applications, the parameter should be $q \gg 1$, which yields from

$$q \gg 1 \quad \text{to} \quad \|L_2\| \gg \|L_1\|. \tag{17}$$

The important remark here is that the design parameter q can not be arbitrary increased. The estimation error depending on the LTR design parameter q is illustrated qualitatively in Figure 2. The curve $f\,:\,\dot{n}(t)$ in Figure 2 denotes the error caused by the derivative of the unknown inputs and the curve $f\,:d(t), h(t)$ denotes the error caused by the uncertainties. The optimal parameter q depends on the qualities of the model and the measurement and on the derivative of the unknown input.

As a result, in the best case, the PI-Observer can estimate the external input as well as the internal states.Additional background and details of the approach are given in [16, 18].

5 Robust Control of the Nonlinear Aeroelastic system

For the nonlinear aeroelastic system (5), suppose the state feedback control, $\beta(t) = -Kx(t)$, is implemented to realize the robust control, where K is the state feedback matrix.

Substitute $\beta(t) = -Kx(t)$ into Eq. (5), the close loop system can be expressed as

$$\dot{x}(t) = \tilde{A}\,x(t) + k_\alpha^*\,\tilde{N}\,x(t), \tag{18}$$

where $\tilde{A}$ and $\tilde{K}$ with proper dimensions are given as

$$\tilde{A} = (A_n - B\,K), \quad \text{and} \quad \tilde{N} = [\,N \mid 0\,]\,. \tag{19}$$

It can be seen that Eq. (18) has the same form as Eq.(6). The nonlinear term K_α^* can be treated as the uncertain part. The estimation of $|k_\alpha^*|$ can be obtained simultaneously by a PI-Observer. To suppress the flutter in system (5), regarding the robust control strategy in Section 3, the state feedback controller K should be found by solving the following optimization problem:

$$\begin{aligned} \min. \quad & \sigma_{\max}(P_i)\,, \\ s.t. \quad & P \in \left\{ P : \tilde{A}^T P + P\,\tilde{A} + 2I = 0 \right\}\,, \\ & |k_\alpha^*|\,\sigma_{\max}(P_i) < 1. \end{aligned} \tag{20}$$

Now the problem relies on the optimization process to deduce the matrix K. In this contribution, due to the fact that the system has only four states and one nonlinearity, the optimization can be taken numerically. The whole control loop consists of two relatively different parts: the PI-Observer estimates system states and the nonlinear perturbation online and returns these values to the optimizer, while the optimizer finds the optimal state feedback matrix K which is used to keep the system robust stable against the nonlinear perturbation.

6 Simulation Results

In this section, numerical results for the robust stability control of such a system are presented. The values of the model parameters are taken from [10] as

$$\begin{aligned} &\rho = 1.225\,\text{kg/m}^3 && b = 0.135\,\text{m}\,, && c_{l_\alpha} = 6.28\,, \\ &c_\alpha = 17.43\,\text{Ns/m}\,, && c_h = 27.43\,\text{Ns/m}\,, && c_{l_\beta} = 3.358\,, \\ &k_h = 2844.4\,\text{N/m}\,, && c_{m_\alpha} = (0.5 + a)c_{l_\alpha}\,, && c_{m_\beta} = -0.635\,, \\ &m_W = 2.0490\,\text{kg}\,, && x_\alpha = [0.0873 - (b + a\,b)]/b\ \text{m}\,, && \\ &m_T = 12.387\,\text{kg}\,, && \text{and } I_\alpha = m_W\,x_\alpha^2\,b^2 + 0.0517\,\text{kg/m}^2\,. && \end{aligned} \tag{21}$$

Suppose at $t = 0$ s the state feedback matrix K is given by the LQR method with respect to the nominal system matrix A_n. This makes sense because it provides an asymptotical stable system at $t = 0$ s, i.e., $\tilde{A}$ is a Hurwitz matrix. Take this K as the initial condition for the optimization process.

Following the robust stability control strategy introduced before, the simulation is performed with different value of a and U.The optimization process is performed by genetic-algorithm-based procedure because of its ability to find the global minimum with less sensitivity to the initial conditions and to solve problems with nondifferentiable objective functions. The initial conditions for the state variables of the sys-

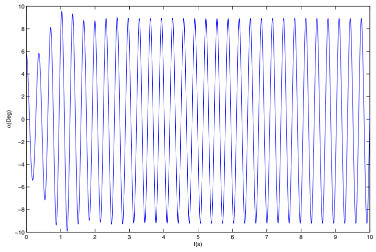

Fig. 3 System open-loop response of α.

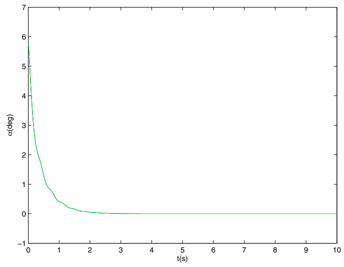

Fig. 4 Time history of pitching motion ($U = 16$ m/s, $a = -0.6847$).

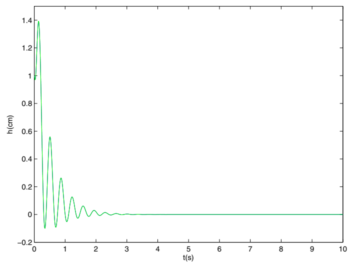

Fig. 5 Time history of plunging motion ($U = 16$ m/s, $a = -0.6847$).

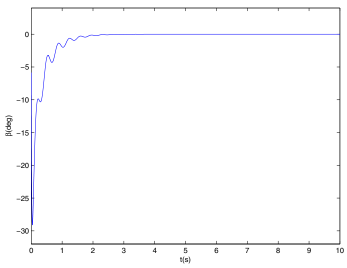

Fig. 6 Time history of control input ($U = 16$ m/s, $a = -0.6847$).

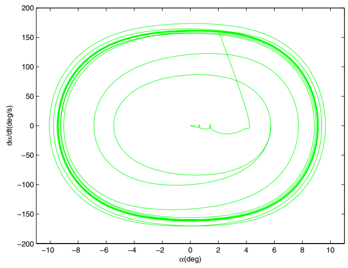

Fig. 7 LCO suppression of pitching motion, control implemented at $t = 5$ s ($U = 16$ m/s, $a = -0.6847$).

tem are selected as $\alpha(0) = 5.75$ (deg), $h(0) = 0.01$ m, $\dot{\alpha}(0) = 0$ (deg/s), and $\dot{h}(0) = 0$ m/s. The initial conditions for the estimated states of the observer are as the same as those of the system. The initial condition of the estimation of the nonlinearity is set to 0. The uncontrolled system is not asymptotic stable, which can be seen from the simulation of the open loop response shown in Figure 3.

Simulation of the close-loop system is performed with different wind speed U and structural parameter a (nondimensional distances from midchord to the elastic axis). Figures 4–6 shows the time histories of pitching, plunging, and control surface deflection with $U = 16$ m/s and $a = 0.6874$. It can be seen from the figures both the pitching motion and the plunging motion are quickly regulated to the original

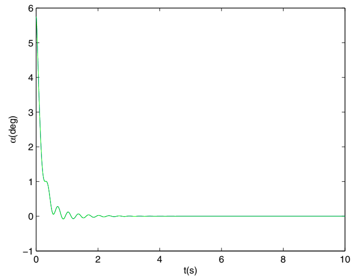

Fig. 8 Time history of pitching motion ($U = 20$ m/s, $a = -0.8$).

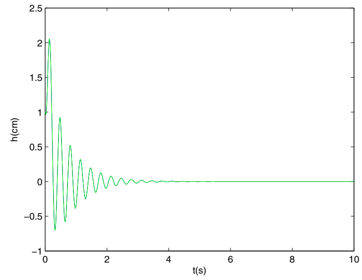

Fig. 9 Time history of plunginging motion ($U = 20$ m/s, $a = -0.8$).

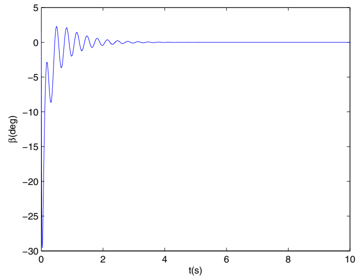

Fig. 10 Time history of control input ($U = 20$ m/s, $a = -0.8$).

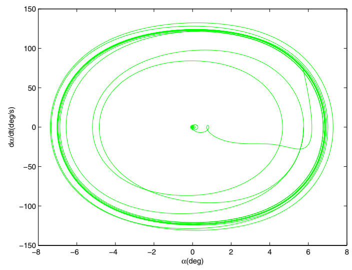

Fig. 11 LCO suppression of pitch motion, control implemented at $t = 5$ s ($U = 20$ m/s, $a = -0.8$).

within 2.5 seconds. When the wind speed $U = 20$ m/s, which is much higher than the flutter speed of the nominal system, the simulation results are given in Figures 8–10 and show that the system is also asymptotic stable with the presence of the robust active control, neglecting the system nonlinear effects.

Figures 7 and 11 show the LCO suppression with different U and a. The system is allowed to evolve open loop response for 5 seconds at first to observe the development of the LCO. At $t = 5$ s the active controller is turned on and the open-loop oscillation is immediately attenuated.

7 Conclusion

In this contribution a novel robust state feedback control strategy is proposed to stabilize an aeroelastic system with structural nonlinearities, illustrated by an example of flutter suppression in a 2-D wing-flap system with nonlinear stiffness in an incompressible flow field. A PI-Observer is used to estimate both the system states and the nonlinear perturbation. With the information provided by the PI-Observer, based on the new conclusion obtained from the robust stability criterion in time domain, an optimization procedure is utilized to find the optimal state feedback matrix for the purpose of flutter suppression. The simulation results are presented to illustrate the ability of this approach in suppressing the instability of the aeroelastic model against its nonlinear perturbation.

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