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Detection of Cracks in Turborotors—A New Observer Based Method

The clear relation between shaft cracks in turborotors and vibration effects measured in bearings can be established by model-based methods very well. Here a new concept has been presented, based on the theory of disturbance rejection control, extended for nonlinear systems and applied on a turborotor. Simulations have been done, showing the theoretical success of this method, especially for reconstructing disturbance forces as inner forces caused by the crack. Calculating the relative crack compliance as the ratio of additional compliance caused by the crack and undamaged compliance a clear statement about the opening and closing, and therefore for the existence of the crack, and about the crack depth is possible. Theoretically it has been shown that it is possible to detect a crack with very small stiffness changes which corresponds to a crack depth of 5 percent of the radius of the rotor.

1 Introduction

Propagating fatigue cracks can have profound effects on the reliability of rotating machinery. An early crack warning can considerably extend the durability of these very expensive machines, increasing their reliability at the same time. A detailed study of the vibrational behavior of cracked rotating shafts, therefore, is an important problem for engineers working in the area of the dynamics of machines.

Methods which are normally used for monitoring the machine and also for crack detection can be divided into classical and modern methods.

The classical methods consist of measurements taken of oil temperature in bearings or control the vibration peaks with regard to maximum allowed values. Also coastdown-measurements are done (Zimmer and Bently, 1985). For all these ways the experience of the machine operator is very important, because none of the classical methods provides an obvious statement about the crack.

The modern methods for failure detection are called Vibration Monitoring Systems (VMS). For these FFT- and Cepstrum Analysis are done, also statistical methods and/or pattern recognition are used (Peter, 1985; Ericsson, 1985). These methods have a great potential, because it is possible to use them without dismounting any part of the machine or even stopping the machine.

Through simulations and experiments, correlations between the crack and the caused phenomena, i.e., vibrations of the rotor or of the bearings which can be measured, are to be found. It is very difficult to conclude the existence of a shaft crack, because there is no clear relation between the crack and the caused phenomena. In this way the main problem is establishing a clear and unambiguous relation between the crack and the caused phenomena.

There are a lot of crack models, but the typical effect of the crack is already described by the simplest one. This typical behavior is the breathing of the crack, and is modeled by the "Hinge-Mechanism" of Gasch (1976). For the detection method presented in this paper "real crack signals" can be used, whereby for simulation purposes all crack models can be considered.

The incorporation of the stiffness change resulting from a crack into the equations of motion was dealt with in several papers. Gasch (1976) and Henry and Okah-Avae (1976) considered the nonlinear mechanism of a breathing crack with different elasticities for open and closed crack, described in body-fixed rotating coordinates. Mayes and Davies (1980) correlated some experimental results with their theoretical background and suggested a method for calculation of the changing stiffness due to a crack. Grabowski and Mahrenholtz (1982) used modal formulations to investigate the vibrational behavior of realistic cracked rotor systems, developing a crack mechanism and using it in their dynamic rotor model. Bently (1981, 1982) and Muszynska (1982) investigated the dynamics of cracked systems by the development of both demonstration rigs and practicable crack detection systems based on their own theoretical work. In Muszynska (1982), both gaping and breathing cracks were considered and modeled by local changes in stiffness. Currently, the vibrations of cracked rotors and the detection of cracks are active fields of research, e.g., Schmalhorst (1989), Wauer (1990), and Papadopoulos and Dimarogonas (1983, 1987). In this way the system behavior will be described and monitored well.

Here another principal way of crack detection is suggested. Based on the theory of disturbance rejection control, developed for linear control systems, an extension for reconstructing nonlinear signals as external disturbance forces is used (Müller,

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1990). In this way the crack is interpreted as an external disturbance. According to the theory of estimating unknown disturbances of a control system, simple measurements of displacements and/or velocities of the vibration system are used to reconstruct these additional time signals by state observers to obtain estimates of the nonlinear effects. The state observer is based on the known linear part of the vibration system and a linear fictitious model, which approximates the crack. The principle idea with a first application of this observer based crack detection method is published by Söffker and Bajkowski (1991).

2 Method

Usual crack detection methods are based on signal analysis. As information only vibration signals are used. Further information about the mechanical system and about the fault we are looking for are not used. The method suggested in this paper is an observer-based method, so further information together with the measurable signals are used, e.g., the mechanical model of the rotor and the characteristics of the typical behavior of the crack.

The method of disturbance rejection control allows the reconstruction of disturbances, which cannot be measured (Johnson, 1972; Müller and Lückel, 1977). As a typical application the reconstruction of friction torques in robot joints was done by Ackermann (1989). Only with the compensation of friction effects it was possible to realize a highly accurate position control for elastic robots with rotatory joints (Müller et al., 1990). Other investigations were made to determine the Coulomb friction curve (Müller, 1990; Stelter, 1990). For these applications the mechanical system was of a low order. With an extension of the method of disturbance rejection control it is possible to reconstruct specific nonlinear crack forces.

Assume that the cracked mechanical system is described by

$$\mathbf{M}\ddot{\mathbf{z}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{z}} + \mathbf{K}\mathbf{z} = \mathbf{f}(t) + \mathbf{N}_n\mathbf{h}(\mathbf{z}(t),t) \tag{1}$$

with

- $\mathbf{z}, \dot{\mathbf{z}}, \ddot{\mathbf{z}}$ = displacement vector and its time derivatives of order n
- $\mathbf{M}$ = mass matrix
- $\mathbf{D}, \mathbf{G}$ = matrices of damping and gyroscopic effects
- $\mathbf{K}$ = stiffness matrix
- $\mathbf{f}(t)$ = vector of unbalances
- $\mathbf{N}_n$ = input matrix of nonlinearities
- $\mathbf{h}(\mathbf{z}(t),t)$ = vector of disturbances caused by the crack

The vector $\mathbf{h}(\mathbf{z}(t),t)$ contains the specific forces caused by the crack. To consider the crack influences in the equations of motion (1) a crack model is needed in the way that it describes the change in stiffness and/or damping coefficients e.g., by a crack element stiffness or damping matrix. Usual crack models (Gasch, 1976; Schmalhorst, 1989) use the change in stiffness coefficients, like Eqs. (2)–(4),

$$\mathbf{h}(\mathbf{z}(t),t) = [0_1 \ldots 0_i \;\; \mathbf{h}_{chcr}(\bar{\mathbf{z}}(t),t) \;\; 0_{i+1+e_n} \ldots 0_n]^T \tag{2}$$

with

$$\mathbf{h}_{chcr}(\bar{\mathbf{z}}(t),t) = \mathbf{K}_e[z_i \ldots z_{i+e_n}]^T, \tag{3}$$

$$\mathbf{K}_e = \begin{bmatrix} a_{11} & \cdots & a_{1e_n} \\ \vdots & \ddots & \vdots \\ a_{e_n1} & \cdots & a_{e_ne_n} \end{bmatrix} \quad \text{with } a_{ij} = a_{ji}, \tag{4}$$

where $\mathbf{K}_e$ (4) represents a general additional crack stiffness matrix of order e_n. Applying state space notation Eq. (1) is described by

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}(t) + \mathbf{N}\mathbf{n}(\mathbf{x}(t)), \\ \mathbf{y} &= \mathbf{C}\mathbf{x}. \end{aligned} \tag{5}$$

Here $\mathbf{x}$ denotes the $2n$-dimensional state vector (consisting of displacement and velocity variables), $\mathbf{A}$ is the $2n \times 2n$ system matrix and $\mathbf{b}$ represents the $2n$-dimensional vector of control inputs and/or excitation functions. The $2n \times f$ matrix $\mathbf{N}$ is the input matrix of the nonlinearities into the linear dynamical system. The vector $\mathbf{n}(\mathbf{x}(t),t)$ characterizes the f-dimensional vector of nonlinear functions. The m-dimensional vector $\mathbf{y}$ represents the measurements via the $m \times 2n$-dimensional matrix of measurements $\mathbf{C}$. It is assumed that the system parameters ($\mathbf{A}$, $\mathbf{N}$, $\mathbf{C}$) as well as the input and output time signals ($\mathbf{b}$, $\mathbf{y}$) are known. The task is to reconstruct the unknown nonlinearities (here the external disturbance forces of the crack)

$$\mathbf{n}(\mathbf{x}(t),t) \approx \hat{\mathbf{n}}(\hat{\mathbf{x}}(t)) \tag{6}$$

by applying state observers.

For consideration of external disturbances, the state space vector will be extended by a fictitious disturbance vector $\mathbf{v}(t)$,

$$\begin{aligned} \mathbf{n}(\mathbf{x}(t)) &\approx \mathbf{H}\mathbf{v}(t), \\ \dot{\mathbf{v}}(t) &= \mathbf{F}\mathbf{v}(t) + \mathbf{G}\mathbf{b}(t), \\ \dim \mathbf{v} &= s, \end{aligned} \tag{7}$$

to describe approximately the time behavior of the nonlinearities. The model matrices $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ must be chosen in accordance with the technical background about the system. Here $\mathbf{NH}$ couples the fictitious model (7) to the whole system. Because of the periodic changes in element-stiffness matrix $\mathbf{K}_e$ (4) the time signals of the nonlinearities can be considered as harmonic signals with corresponding frequencies. So the approximation of the disturbance forces can be modelled by oscillators ($r_{nl} = 2$). The matrices are of $\mathbf{N}\,[2n, f]$, $\mathbf{H}[f, (r_{nl} \cdot f)]$, $\mathbf{F}[r_{nl} \cdot f, r_{nl} \cdot f]$ order.

In this way the external forces caused by the crack are reconstructed by the estimates of the disturbance vector $\mathbf{v}(t)$. An approximation seems to be a disadvantage, but on the other hand, this flexibility and robustness "against" different crack models for simulations or real crack behavior in practice characterize this new crack detection method.

3 State Observers for Reconstruction of Nonlinear Effects

Applying (7), the extended system is obtained with the new system matrix $\mathbf{A}_e$,

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{v}}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A} & \mathbf{NH} \\ \mathbf{0} & \mathbf{F} \end{bmatrix}}_{\mathbf{A}_e} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{I} \\ \mathbf{G} \end{bmatrix} \mathbf{b}(t),$$

$$\mathbf{y}(t) = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \end{bmatrix}. \tag{8}$$

This extended system (8) with the new system matrix $\mathbf{A}_e$ could be observed by the extended state space observer if the system is completely observable (Luenberger, 1971). This requires a suitable choice of matrices $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ and measurements. The observability of the undamaged turborotor with bearings as a linear mechanical system normally is given, because of non-existent decoupled sub-systems. To guarantee the complete observability of the extended system (8), the following condition must be fulfilled,

$$\text{rank} \begin{bmatrix} \lambda\mathbf{I}_{2n} - \mathbf{A} & -\mathbf{NH} \\ \mathbf{0} & \lambda\mathbf{I}_s - \mathbf{F} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} = 2n + s, \tag{9}$$

with $m \geq s$, $(s \geq f)$, this means that there must be equal or more measurements than nonlinearities. The number of approximated nonlinearities mainly depends on the degrees of freedom of the crack model used in Eq. (4).

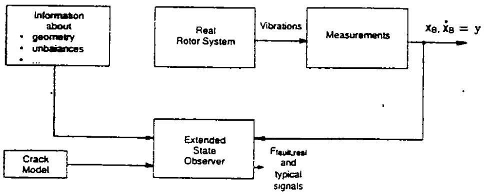

Fig. 1 General concept of crack detection by state observers

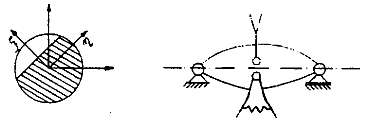

Fig. 2 Crack model of Gasch (1976)

The observer essentially consists of a simulated model with a correction feedback of the estimation error between real and simulated measurements,

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}}(t) \\ \dot{\hat{\mathbf{v}}}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A}-\mathbf{L}_x\mathbf{C} & \mathbf{NH} \\ -\mathbf{L}_v\mathbf{C} & \mathbf{F} \end{bmatrix}}_{\mathbf{A}_o} \begin{bmatrix} \hat{\mathbf{x}}(t) \\ \hat{\mathbf{v}}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{I} \\ \mathbf{G} \end{bmatrix}\mathbf{b} + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_v \end{bmatrix}\mathbf{y}. \tag{10}$$

The dynamical behavior of the observer is expressed by the system matrix $\mathbf{A}_o$. Using an identity observer different methods can be used like pole-placement or linear-quadratic optimal observer etc. The asymptotic stability of the observer can be guaranteed by a suitable design of the gain-matrices $\mathbf{L}_x$, $\mathbf{L}_v$. For a successful estimation the observer has to be asymptotically stable and usually the eigenvalues should be on the left side of those of the observed system $\mathbf{A}$. Furthermore the design should consider that only approximations instead of the nonlinearities are used, cf. Section 4.

The general concept of crack detection by state observers is shown in Fig. 1. With information about the mechanical system the extended observer can be built. With measurements taken only in bearings the observer reconstructs external forces caused by the crack. These forces depend on the crack depth and can be used for further calculations.

The approximation of the nonlinearities by linear fictitious models in Eqs. (7) has two different aspects:

- Instead of using nonlinear observers linear theory can be used.
- The conditions for using this approximation for reconstructing the signals of the nonlinear effects by state observer methods are weak, and they are often fulfilled for mechanical systems with band structure.

4 Application on a Simple Rotor Using the Crack Model of Gasch

In the literature (Wauer, 1990) a lot of crack models are mentioned, which are very sensitive, especially the FEM-crack model of Schmalhorst (1989). But the typical behavior of the crack, the "breathing" of the crack under weight influence is already described by the very simple model of Gasch (1976).

The crack model of Gasch, Fig. 2, is described in the rotating coordinate system by

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} h+h_a & 0 \\ 0 & h \end{bmatrix} \begin{bmatrix} F_\xi \\ F_\eta \end{bmatrix}. \tag{11}$$

The compliance h in the crack direction ξ will be increased with an additional compliance h_a in case of an open crack,

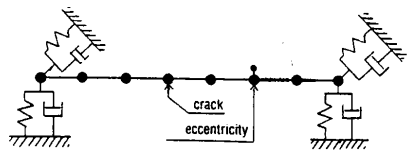

Fig. 3 Simple rotor

which depends on the crack depth. The relative crack compliance h_r as the ratio

$$h_r = \frac{h_a}{h} \tag{12}$$

is established by experimental investigations of Mayes and Davies (1984) for different crack depths. For very small cracks the values are approximated. The opening-condition of the crack can be formulated via the bending at the crack position or approximately via the displacements near the crack

$$\xi_i > \frac{\xi_{i-e_n} + \xi_{i+e_n}}{2}, \tag{13}$$

where $e_n = 2$.

Using the transformation matrix

$$\mathbf{T} = \begin{bmatrix} \cos(\Omega t+\beta) & \sin(\Omega t+\beta) \\ -\sin(\Omega t+\beta) & \cos(\Omega t+\beta) \end{bmatrix} \tag{14}$$

the element-stiffness-matrix $\mathbf{K}_e(\bar{\mathbf{z}}(t),t)$ for a discretized model like an MBS-formulation in the inertial coordinate system looks like

$$\mathbf{K}_e = \frac{-h_r}{h(1+h_r)} \begin{bmatrix} \sin^2(\Omega t+\beta) & \sin(\Omega t+\beta)\cos(\Omega t+\beta) \\ \sin(\Omega t+\beta)\cos(\Omega t+\beta) & \cos^2(\Omega t+\beta) \end{bmatrix} \tag{15}$$

where $\mathbf{K}_e$ depends on the opening condition (13) for the crack and on time, so the system in the inertial coordinates becomes a nonlinear and parametrically excited one.

The rotor used for theoretical investigations and simulations is described by the following assumptions, cf. Fig. 3:

Rotor as a lumped-mass-model, 7 beam elements, length, $l = 600$ mm; radius, $r = 140$ mm; frequency, $\Omega = 100\pi$ rad/s; eccentricity, $e_m = 0.02$ mm; stiffness of bearings, $k_s = 750$ kN/mm; damping as $\mathbf{D} = \alpha_{\text{mod}}\,\mathbf{M} + \beta_{\text{mod}}\,\mathbf{K}$, $\alpha_{\text{mod}} = 0$, $\beta_{\text{mod}} = 0.00001$; number degrees of freedom, $n = 16$; number of nonlinearities, $f = 2$; number of measurements, $m = 8$, measurements only in bearings as displacements and their velocities.

The system matrices are given in the Appendix.

According to the presented method the choice of the fictitious model (7) is be effected, because the time signals of the nonlinearities can be considered as harmonic signals with frequencies $\omega_{1,2}$. Therefore the fictitious model makes sense:

$$\ddot{v}_1 + \omega_1^2 v_1 = 0, \tag{16}$$

$$\ddot{v}_2 + \omega_2^2 v_2 = 0, \tag{17}$$

i.e.,

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\alpha, \tag{18}$$

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2 & 0 & 0 & 0 \\ 0 & -\omega_2^2 & 0 & 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}, \tag{19}$$

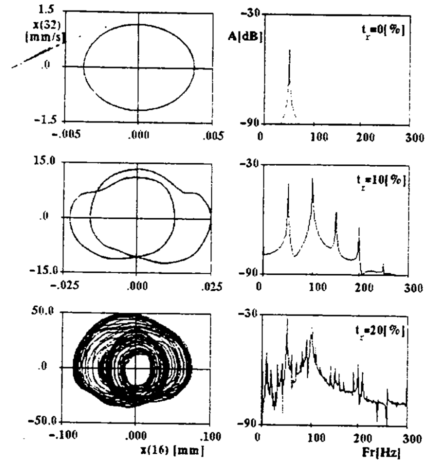

Fig. 4 Phase planes and FFT-analysis for different relative crack compliances h_r (right bearing, horizontal direction)

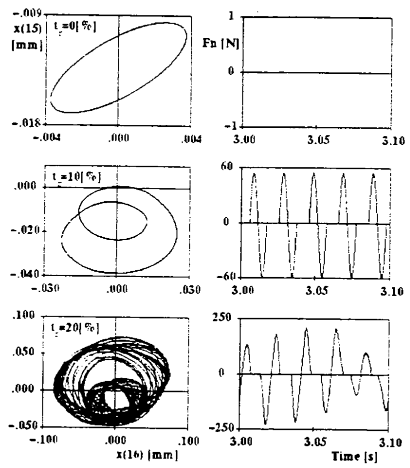

Fig. 5 Orbits and crack forces for different relative crack compliances h_r (orbit: right bearing, crack forces: right bearing, horizontal direction)

$$h_1(z_{i+1}(t),t) = n_1(x_{2i+1}(t),t) \approx v_1(t), \tag{20}$$

$$h_2(z_{i+2}(t),t) = n_2(x_{2i+2}(t),t) \approx v_2(t), \tag{21}$$

where α denotes the related $(2i,2i)$-element of the matrix $\mathbf{K}$.

The reconstruction of the characteristic relative crack compliance h_r, cf. Eq. (12), can be done by simple calculations using the matrix $\mathbf{T}$ (14), the estimated disturbance forces $\hat{v}_{1,2}$ (20, 21), displacements at crack position $\hat{x}_{(2i+1,2i+1)}$ (10) and phase information β. Because of the system order a Riccati observer design was used, which fulfills the following requirement:

$$\mathbf{A}_e\mathbf{P} + \mathbf{P}\mathbf{A}_e^T - \mathbf{P}\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C}\mathbf{P} + \mathbf{Q} = 0. \tag{22}$$

Since using approximations instead of the real nonlinearities the weighting matrices $\mathbf{R}$ and $\mathbf{Q}$ must be chosen specifically. For this the weighting $\mathbf{Q}$ of the measurements is split up into three blocks, like

$$\mathbf{Q} = \begin{bmatrix} q_1\mathbf{I}_n & 0 & 0 \\ 0 & q_2\mathbf{I}_n & 0 \\ 0 & 0 & q_3\mathbf{I}_{r_{nl}\cdot f} \end{bmatrix}, \tag{23}$$

with $q_1 = 10$, $q_2 = 1$ and $q_3 = (10^8 \div 10^{10})$. The weighting matrix $\mathbf{R}$ in Eq. (22),

$$\mathbf{R} = r\mathbf{I}_m \tag{24}$$

is used with $r = 0.01$ to 0.0001.

5 Simulations and Results

All calculations are done in FORTRAN double precision on a Control Data 4230 Workstation under BSD Unix V.3.2. For integration a Runge Kutta-Gill algorithm of fourth order was used, modified for consideration the switching condition for the crack.

5.1 Behavior of the Cracked Rotor. The simulations of the cracked rotor show an interesting behavior. Dependent on the relative crack depth

$$t_r = \frac{t}{r}, \tag{25}$$

with

t: crack depth, and
r: radius of the rotor at crack position,

respectively, the relative crack compliance h_r (12), the time behavior of the system changes in a special way.

For characterizing the behavior of the cracked rotor for different relative crack compliance h_r the phase plane plots and FFT-analysis of the right bearing in horizontal direction are shown in Fig. 4, the orbit of the right bearing and the disturbance forces F_n caused by the crack are shown in Fig. 5.

The linear behavior in undamaged case changes for greater h_r, i.e., for greater crack depths, to semiperiodic behavior. Considerations in vertical direction would show the same result.

Analogously to the investigations of Fritzen (1990), who found chaos in a cracked Laval rotor, the possibility of chaos here may also exist. Dependent on the system parameters, i.e., crack or damping coefficients, we can obtain different types of motions; periodic, almost-periodic, sub- or ultraharmonic, and either chaotic one. To classify the system behavior Lyapunov exponents can be calculated. They provide a qualitative and quantitative characterization of dynamical behavior and they are related to the exponentially fast divergence or convergence of nearby orbits in phase space. There are thirty three Lyapunov exponents $(2n + 1)$, and their sum should be negative since the system (1) is dissipative. One exponent corresponds to the direction parallel to the trajectory and its corresponding Lyapunov exponent is zero. The remaining exponents are negative or zero in the periodic states, whereas in the chaotic state at least one Lyapunov exponent is positive (Lichtenberg and Lieberman, 1983). As an indicator for chaos only the greatest Lyapunov exponent σ was calculated as a function of h_r from a single time series (Wolf et al., 1985). The results are shown in Fig. 6. The regions of chaotic behavior $(\sigma > 0)$ are interrupted by intervals of periodic behavior $(\sigma \leq 0)$.

For the illustration of chaotic vibrations only Poincaré maps and phase planes of the damaged rotor are presented. Figure 7 shows Poincaré maps and the projection of the trajectory (c) onto $[Lx(16), Lx(32)]$-phase plane for several values of h_r, in which (a) corresponds to point "1," (b) corresponds to point "2" and (c) corresponds to point "3" in Fig. 6, respectively. In this space periodic motion (c) appears as one dimensional closed orbit and its Poincaré section is zero-dimensional (two points), while chaotic orbits form complicated sets containing an infinite number of points ((a) and (b))

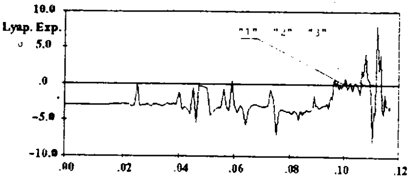

Fig. 6 Greatest Lyapunov exponent σ of the simulated cracked rotor as a function of h_r

(strange attractors and their dimension is a fraction greater than two).

5.2 Crack Detection by State Observers. Crack detection by state observers means a procedure with two steps:

- Estimation of crack forces $\hat{\mathbf{v}}(t)$ via the extended state space observer.
- Recalculating the coefficients of $\mathbf{K}_e(\hat{\mathbf{z}}(t),t)$ (4) or e.g., $\mathbf{h}_{chcr}(\hat{\mathbf{z}}(t))$ (3) for each time step and representing them in a favorable manner, e.g., dividing trough the nominal values of the undamaged case. Therefore phase information is useful.

Figures 8(a) and 9(a) show the crack forces F_n in horizontal direction for different crack depths. The observer estimates the signal very well, only in a few points the dynamic of the observer can not follow the real simulated signal. The external signal only exists if the crack opens, the maximal values depends on the crack depth. Using this estimation and the estimation about the displacements the normally unknown ratio $h_r = h_a/h$ in (12) can be recalculated, Figs. 8(b) and 9(b). As a function of time this ratio describes the variable compliance or stiffness depending on the phase angle in the rotating coordinate system. Hence it will be a clear indicator for cracks. Here the opening and closing of the crack is shown very clear and unambiguous. For both crack depths of 20 and 5 percent it is very clear to see: opening and closing of the crack in Gasch crack model. In contrast to this, calculations of undamaged rotor results in a ratio about 0.001.

5.3 Additional Remarks

- In this way it is not important that the crack model used for simulations possibly could not describe such minimal stiffness changes correctly. The proposed method is independent from crack models. In spite of using an approximation this scheme can be used to reconstruct minimal stiffness changes, independent from specific characteristics.
- The assumptions for using the new model-based method consist of the knowledge of the mechanical system parameters and the vector of unbalances. The mechanical parameters are normally known and are used for example for vibration calculations. The parameters of the vector of unbalances is normally identified by balancing as the rest vector of unbalances.
- For this crack detection method the crack location is supposed as to be known and is determined in the matrix **NH** of the observer scheme. To locate a real crack a series of observers has to be arranged. This means that for some possible and possibly for all possible locations different observers should be built. This task can be done by a parallel or a sequential working scheme. With regard to the slow growth of the crack and the possibly early indication of a sequential off-line application seems to be sufficient.
- It was assumed that the system behavior can be observed by measurements in the bearings. But it is well-known that the observability depends on the type of support. For example, if the rotor is modelled as a clamped-clamped or clamped-pinned beam, no motion along the rotor can be observed when measurements are taken at the supporting end. Therefore, we have to be aware of the dif-

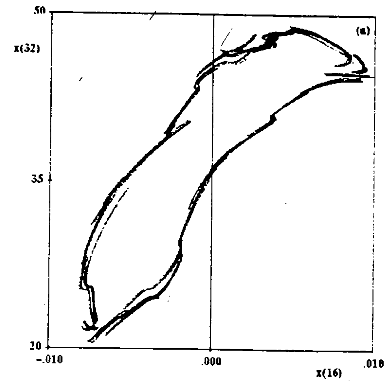

Fig. 7(a)

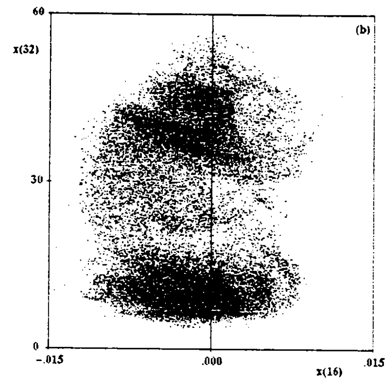

Fig. 7(b)

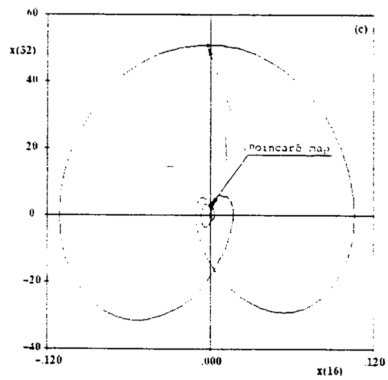

Fig. 7(c)

Fig. 7 Poincaré sections and phase planes for several values of h_r (right bearing, horizontal direction) (a): $h_r = 0.100$, (b): $h_r = 0.107$, (c): $h_r = 0.111$ ($t_n = 2\pi n/\omega$, $\omega = 100\pi$, $n = 50000$)

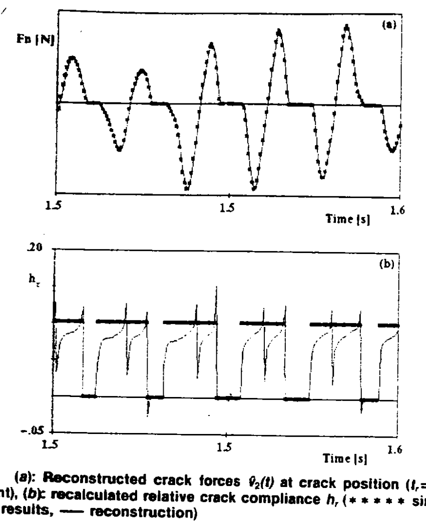

Fig. 8 (*a*): **Reconstructed crack forces $\theta_2(t)$ at crack position (t_r = 20 percent), (*b*): recalculated relative crack compliance h_r (* * * * * simulation results, —— reconstruction)**

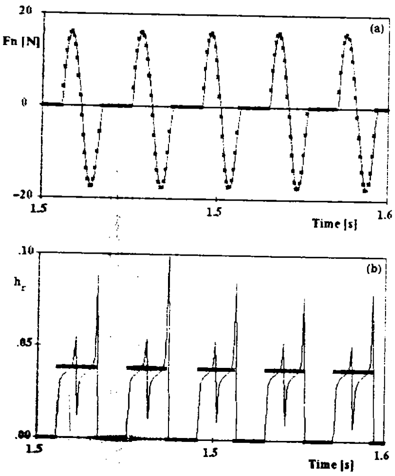

Fig. 9 (*a*): **Reconstructed crack forces $\theta_2(t)$ at crack position (t_r = 5 percent), (*b*): recalculated relative crack compliance h_r (* * * * * simulation results, —— reconstruction)**

ficulties caused by those limitations. But here dynamic bearings are considered assuring the observability of all modes which has the advantage of measurements at accessible locations only.

6 Results and Conclusions

The clear relation between shaft cracks in turborotors and caused phenomena in vibrations measured in bearings can be established by model-based methods very well. Here a new concept has been presented, based on the theory of disturbance rejection control, extended for nonlinear systems and applied to a turborotor. Simulations have been done, showing the theoretical success of this method, especially for reconstructing disturbance forces as inner forces caused by the crack. Calculating the relative crack compliance as the ratio of additional compliance caused by the crack and undamaged compliance a clear statement about the opening and closing, and therefore for the existence of the crack, and about the crack depth is possible. Theoretically it has been shown that it is possible to detect a crack with very small stiffness changes which correspond to a crack depth of 5 percent of the radius of the rotor. To what extent this success can be transferred into practice has to be analyzed by further investigations and experiments.

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APPENDIX

The matrices used in Eqs. (1) and (5) are as follows,

M: Mass matrix of order (16×16),

$$\mathbf{M}=\text{diag}\left[\frac{m}{2}\ \frac{m}{2}\ \underbrace{m \ldots\ldots m}_{12\ \text{times}}\ \frac{m}{2}\ \frac{m}{2}\right], \quad (26)$$

where $m=\pi r^2 l\rho$, $\rho=7860$ kg/m^3,

K: Stiffness matrix of order (16×16),

$$\mathbf{K}=\begin{bmatrix} k_1 & 0 & k_3 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & k_1 & 0 & k_3 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ k_3 & 0 & k_2 & 0 & k_3 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & k_3 & 0 & k_2 & 0 & k_3 & \cdots & \cdots & \cdots & 0 \\ \vdots \\ \vdots \\ 0 & \cdots & \cdots & \cdots & k_3 & 0 & k_2 & 0 & k_3 & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & k_3 & 0 & k_2 & 0 & k_3 \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & k_3 & 0 & k_1 & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & 0 & k_3 & 0 & k_1 \end{bmatrix} \quad (27)$$

where $k=12EJ/l^3$, $E=2.1\cdot 10^5$ N/mm^2, $k_s=7.5\cdot 10^5$ N/mm, $k_1=k+k_s$, $k_2=2k$, $k_3=-k$,

D: Damping matrix of order (16×16),

$$\mathbf{D}=\beta_{\text{mod}}\cdot\mathbf{K},\quad \beta_{\text{mod}}=0.00001, \quad (28)$$

f: Vector of unbalances of order (16×1),

$$\begin{aligned} &f_{2i}=0,\quad i=1,2,\ldots,8,\ \text{except} \\ &f_1=f_{15}=-mg/2, \\ &f_3=f_5=f_7=f_9=f_{13}=f_{15}=-mg, \\ &f_{11}=-mg+e_m\Omega^2 m_{ex}\sin(\Omega t+\beta), \\ &f_{12}=e_m\Omega^2 m_{ex}\sin(\Omega t+\beta-90\ \text{deg}), \end{aligned} \quad (29)$$

where $m_{ex}=7\ m$ – the mass of eccentricity, $\beta=0$ deg,

$\mathbf{N}_n$: Matrix of nonlinearities of order (16×2),

$$n_{n71}=n_{n82}=1, \quad (30)$$

the remaining elements are zero,

h: Vector of the crack forces of order (2×1),
in the case of the "closed" crack, i.e., if $\xi\le 0$ in Eq. (13), then $\mathbf{h}\equiv\mathbf{0}$.
in the case of the "open" crack, i.e., if $\xi>0$ in Eq. (13), or in the inertial coordinate frame

$$z_7\cos(\Omega t+\beta)+z_8\sin(\Omega t+\beta) > \frac{[(z_5+z_9)\cos(\Omega t+\beta)+(z_6+z_{10})\sin(\Omega t+\beta)]}{2}, \quad (31)$$

than the elements of the (2×1) vector **h** are as follows:

$$\begin{aligned} h_1&=[-z_7\cos^2(\Omega t+\beta)-z_8\sin(\Omega t+\beta)\cos(\Omega t+\beta)]\frac{k_2h_r}{1+h_r}, \\ h_2&=[-z_8\sin^2(\Omega t+\beta)-z_7\sin(\Omega t+\beta)\cos(\Omega t+\beta)]\frac{k_2h_r}{1+h_r}, \end{aligned} \quad (32)$$

all other elements are zero,

A: The system matrix of order (32×32),

$$\mathbf{A}=\begin{bmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D}\end{bmatrix}, \quad (33)$$

b: Vector of excitation of order (32×1),

$$\mathbf{b}=\begin{bmatrix}\mathbf{0} \\ \mathbf{M}^{-1}\mathbf{f}\end{bmatrix}, \quad (34)$$

N: Matrix of nonlinearities of order (32×2),

$$\mathbf{N}=\begin{bmatrix}\mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{N}_n\end{bmatrix}, \quad (35)$$

n: Vector of nonlinear functions of order (2×1),
in the case of the "closed" crack $\mathbf{n}\equiv\mathbf{0}$,
in the case of the "open" crack,

$$\begin{aligned} n_1&=[-x_7\cos^2(\Omega t+\beta)-x_8\sin(\Omega t+\beta)\cos(\Omega t+\beta)]\frac{k_2h_r}{1+h_r}, \\ n_2&=[-x_8\sin^2(\Omega t+\beta)-x_7\sin(\Omega t+\beta)\cos(\Omega t+\beta)]\frac{k_2h_r}{1+h_r}, \end{aligned} \quad (36)$$

C: Measurement matrix of order (8×32),

$$c_{ij}=0,\quad i=1,2,\ldots,8,\quad j=1,2,\ldots,32,\ \text{except} \quad (37)$$

$$c_{11}=c_{22}=c_{3,15}=c_{4,16}=c_{5,17}=c_{6,18}=c_{7,31}=c_{8,32}=1.$$