

## Model-based estimation of contact forces in rotating machines

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**Abstract.** In this contribution, two model-based concepts for contact force estimation in rotating machines are introduced. The observer concepts base on the mathematical model of the affected structure. The performance of the observer techniques is shown in a simulation and in experimental results for a contact between a blade and the housing of a rotating machine.

### 1. Introduction

Model-based fault detection in rotating machines has been the focus of several scientific and industrial efforts during the last decade (1,3,4). The main task of fault detection in the area of rotating machines is to conclude the existence of a shaft crack or the existence of a contact between rotating and fixed parts. In both cases the disturbance can be interpreted as an external force acting on the system. In practice the measurement of contact forces is difficult. The forces can only be obtained via the resulting vibration effects (displacement, strain). In this contribution, model-based observer techniques are used to estimate contact forces between rotor and housing.

The paper introduces two different model-based Proportional-Integral-Observer (PIO) approaches and combines the two designs to solve the contact force estimation task including measurement noise. Both approaches use a minimal number of measurements (using one or two sensors). The observers are based on mathematical models of the elastic structure. An important advantage of the PIO approach is that no model of the affecting disturbance is necessary. This allows the application to problems, where no model is available or the known model is not useful because of its complexity. An application to estimate unknown impact forces acting on a fixed elastic beam structure via PIO was published in (1).

The classical PIO design as used in (1) works with high gains. External disturbances affected on a elastic structure can be estimated. But the conventional design is not particularly suitable for handling measurement noise, because the noise is amplified by the high gains. In (5) a PIO design is proposed which permits to attenuate measurement noise for state estimation. They reformulate the observer design in such a way that the measurement noise is not affected by the observer gain. In this paper, the classical PIO design as used in (1) is extended by the reformulation given in (5). The new combined PIO design achieves satisfying results while the measured signal is affected by measurement noise. Since measurement noise in practical applications cannot be completely suppressed, this is an important step to make the PIO more suitable for practical applications.

A simulation example, dealing with a rotating shaft and acting contact forces on a blade, is carried out to analyze the two different PIO approaches. Further experimental results of contact force estimation between a blade and a housing are presented (2).

### 2. Proportional-Integral-Observer (PIO)

External acting forces to an elastic structure influence the oscillation of the system. Via the known dynamics of the elastic system the unknown external inputs (forces) can be estimated by the PIO. The well known disturbance observer (DO) (3) needs a model of the affecting disturbances. The main advantage of the PIO is that no model for the affecting disturbance is necessary and a minimal number of measurements is required. This allows the application to problems, where no model is available or the

application of the model is not useful because of the complexity of the model. The problem of unknown external disturbances is described by

$$\dot{x} = Ax + Bu + Nn(x, u, t), \quad y = Cx, \quad (1)$$

with the state vector  $x$  of order  $n$ , the measurements vector  $y$  of order  $r_1$  and the input vector  $u$  of order  $m$ . The system matrix  $A$ , the input matrix  $B$  and the output matrix  $C$  are of the appropriate dimensions. The vector  $n(x, u, t)$  of order  $r_2$  describes the external unknown inputs in this case the external contact force. In (7) it is shown that for the application of the PIO the number of independent measurements  $r_1$  has to be equal or higher than the number of considered external inputs  $r_2$ . Another assumption for the application of the PIO is that the system is observable. The matrix  $N$  locates the disturbance to the system description and is assumed to be known. The structure of the classical PIO is given in Figure 1.

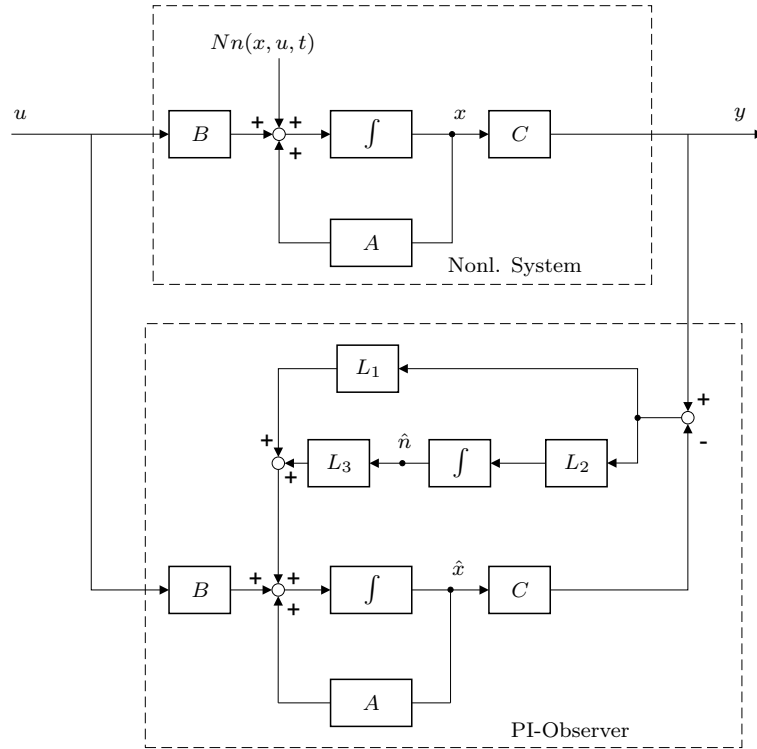


Figure 1: Structure of classical Proportional-Integral-Observer PIO

In contrast to the conventional Luenberger observer an integral of the estimation error is used in a second loop. The dynamics of the PIO is described by

$$\dot{\hat{x}} = A\hat{x} + L_3\hat{n} + Bu + L_1(y - \hat{y}), \quad \hat{y} = C\hat{x}, \quad (2)$$

$$\dot{\hat{n}} = L_2(y - \hat{y}). \quad (3)$$

The matrix  $L_3$  locates the disturbance and is a scaling of the matrix  $N$ . In (6) the proof is given that for high observer gains  $L_1$ ,  $L_2$  and  $L_3$ , scaled in an appointed ratio, the observer state vector  $\hat{x}$  reconstructs the system state vector  $x$  and the integral feedback  $\hat{n}$  represents the external disturbance  $n$ . The behaviour is

$$(x - \hat{x}) \rightarrow 0 \quad \text{and} \quad (n - \hat{n}) \rightarrow 0. \quad (4)$$

In (6) it is shown that systems with nonlinearities can be handled as linear systems with additional unknown external inputs which represent the nonlinearities.

With the estimation error  $e = \hat{x} - x$  the error dynamics have the expression

$$\begin{bmatrix} \dot{e} \\ \dot{\hat{n}} \end{bmatrix} = \begin{bmatrix} A - L_1 C & L_3 \\ -L_2 C & 0 \end{bmatrix} \begin{bmatrix} e \\ \hat{n} - n \end{bmatrix}. \quad (5)$$

The observer gains are chosen such that the error dynamics is stable (6). Now assuming that the measurement is affected by some measurement noise

$$y = Cx + d(t). \quad (6)$$

Assuming that the disturbance  $d(t)$  is a white noise and does not affect the rank observability of the system. In this case, the error dynamic is

$$\begin{bmatrix} \dot{e} \\ \dot{\hat{n}} \end{bmatrix} = \begin{bmatrix} A - L_1 C & L_3 \\ -L_2 C & 0 \end{bmatrix} \begin{bmatrix} e \\ \hat{n} - n \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} d. \quad (7)$$

Thus, it can be seen that the disturbance  $d(t)$  is multiplied by the high gains  $L_1$  and  $L_2$ . Hence, the classical design of PIO will in the case of relevant measurement noise not be able to realize satisfactory performance.

### 3. New design of the PIO for disturbance estimation

In (5), a new formulation to the PIO design for uncertain single-output linear systems is given. The new formulation is used to estimate states while measurement noise is attenuated. The integration part is

$$x_0(t) = \int_0^t y(\tau) d\tau, \quad y = Cx + d(t), \quad (8)$$

and

$$z = \begin{bmatrix} x_0 \\ x \end{bmatrix}. \quad (9)$$

A system without external disturbances, but affected by sensor noise, can be written as

$$\dot{z} = A_0 z + B_0 u + C_0^T d, \quad (10)$$

$$y_0 = C_0 z, \quad (11)$$

with  $C_0 = [1 \ 0 \ \dots \ 0]$ . The matrix pair  $(A_0, C_0)$  is assumed to be observable. Now, the system

$$\dot{\hat{z}} = A_0 \hat{z} + B_0 u + K_I (y_0 - C_0 \hat{z}), \quad (12)$$

is considered, where  $K_I$  is chosen so that the matrix  $(A_0 - K_I C_0)$  is stable. With the estimation error  $e = z - \hat{z}$ , the error dynamics becomes

$$\dot{e} = (A_0 - K_I C_0) e + C_0^T d. \quad (13)$$

From this it can be seen that the disturbance  $d(t)$  is not affected by the observer gain. In (5), a simulation example is carried out, to show the disturbance attenuating properties of this observer design. To attenuate the sensor noise in case of the estimation of external disturbances, the classical PIO for MIMO Systems as used in (1) will be reformulated by the design that is given in (5). The new combined observer has additional states for the unknown disturbances and an additional state to attenuate the sensor noise. The new observer design consists of two integrators. Now the estimation problem with assumed sensor noise is described by

$$\dot{\hat{z}} = A_0 \hat{z} + B_0 u + C_0^T d + N_0 n, \quad y_0 = C_0 \hat{z}, \quad (14)$$

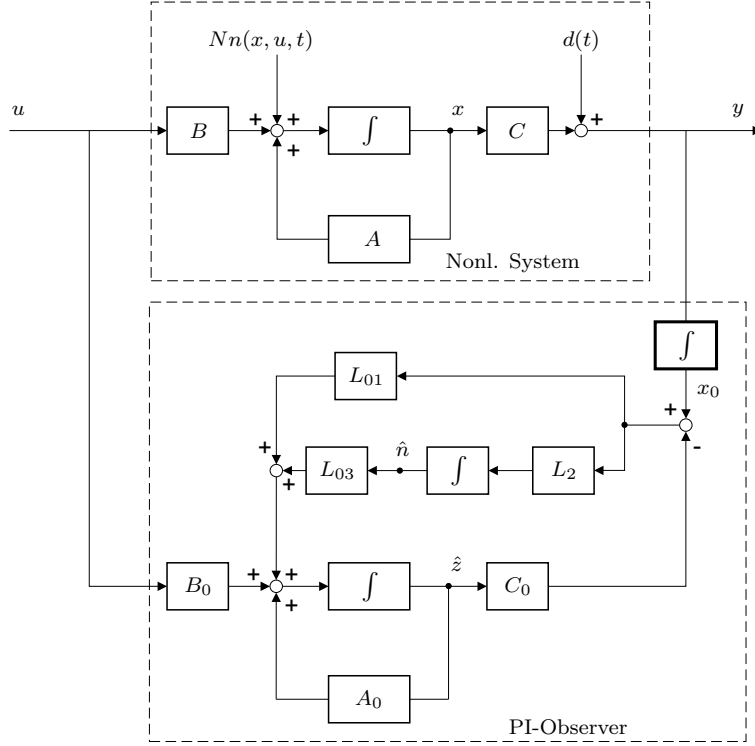


Figure 2: Structure of the new formulated PIO

where  $n$  is the external disturbance (to be estimated) and  $d$  represents the sensor noise. The matrices are given with

$$A_0 = \begin{bmatrix} 0 & C \\ 0_{n \times 1} & A \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad N_0 = \begin{bmatrix} 0 \\ N \end{bmatrix}. \quad (15)$$

The structure of the new PIO design, for the system given in Equation (14), is shown in Figure 2 and the dynamics is described by

$$\begin{bmatrix} \dot{\hat{z}} \\ \dot{\hat{n}} \end{bmatrix} = \begin{bmatrix} A_0 & L_{03} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{z} \\ \hat{n} \end{bmatrix} + \begin{bmatrix} B_0 \\ 0 \end{bmatrix} u + \begin{bmatrix} L_{01} \\ L_2 \end{bmatrix} (y_0 - \hat{y}_0), \quad (16)$$

where  $L_{03} = N_0$  and  $L_{01}$  are of the appropriate dimensions. Let the estimation error be  $e = \hat{z} - z$ , then the error dynamics will become

$$\begin{bmatrix} \dot{e} \\ \dot{\hat{n}} \end{bmatrix} = \begin{bmatrix} A_0 - L_{01}C_0 & L_{03} \\ -L_2C_0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \hat{n} - n \end{bmatrix} - \begin{bmatrix} C_0^T \\ 0 \end{bmatrix} d. \quad (17)$$

In Equation (17) it can be seen that the disturbance  $d$  is not amplified by the observer gains. The new considered system is assumed to be observable. Therefore, high gains  $L_{01}$ ,  $L_2$  and  $L_{03}$  can be chosen such that the stabilizing terms prevails over the perturbation term  $C_0^T d$ .

#### 4. Simulation results

As a simulation example, a contact between a rotor blade and a housing is simulated as shown in Figure 3(a). The contact force has an impact character and affects for a short time with a high amplitude the blade (beam). Figure 3(b) illustrates the structure of the model. The blade is modeled using 5 equal finite beam elements. Strain gages are used for measurement. In simulation, the one side fixed beam is

elastically deflected by an excitation force acting perpendicular to the beam axis. After an appointed displacement, an impact force acts on the end of the beam. The impact force is calculated using a nonlinear contact model. The acting force has the dynamic character of a high frequency impact force with a high amplitude. With the measurement of the strains (curvature in the nodes), the contact force can be estimated by the PIO. The length of the beam is 505 mm and the rectangular cross-section area is 25 mm  $\times$  5 mm. The beam material is steel.

In Figure 4(a) the real (simulated) and the estimated contact force are compared. In this case, the measurement is not affected by sensor noise. Figure 4(b) illustrates the simulated and estimated displacement of the contact point. The PIO is able to estimate the contact force as well as the displacement in the contact point. The results show that the observer technique works well and give similar results to those presented in (1).

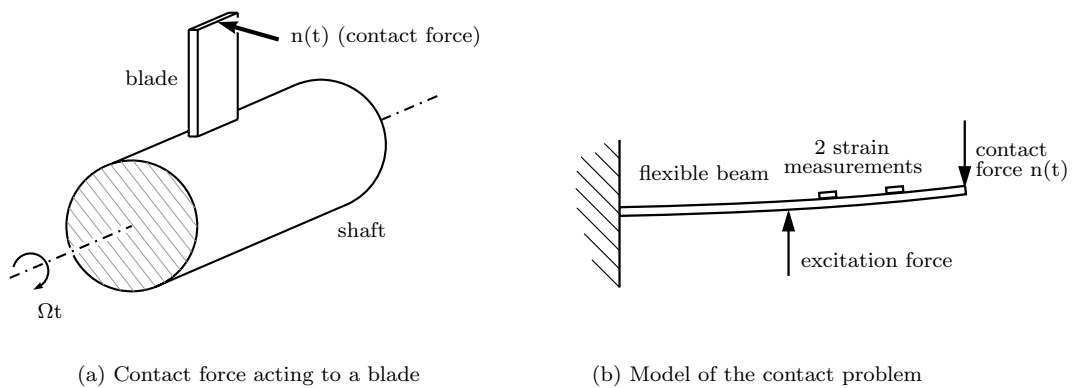


Figure 3: Simulation Example

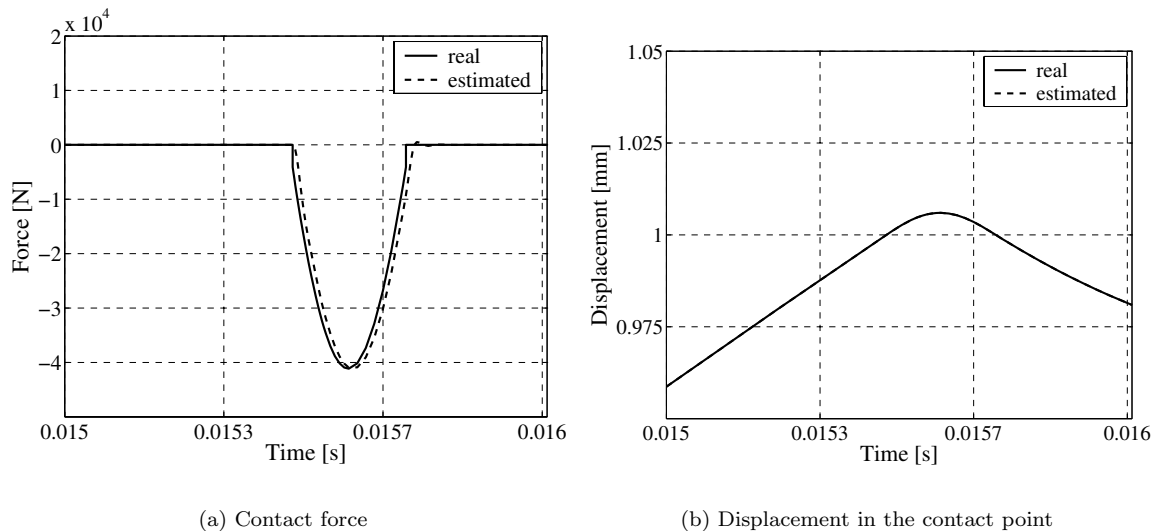


Figure 4: Contact without sensor noise (classical PIO)

For different frequencies of measurement noise, in Figures 5, 6 and 7 the real (simulated) and estimated contact forces are compared. The sensor noise is a white noise with an amplitude about 0.4 % of the amplitude of the maximum measurement signal. The results are shown for noise frequencies 1 kHz, 5 kHz

and 10 kHz. It can be seen that with the new formulated PIO the convergence of the estimated forces is smoother and the noise is smaller than with the classical PIO. The new formulated PIO achieves better results when measurement noise is present. Please note, that the contact forces to be estimated occur at  $t_1 = 0.0155$  s and  $t_2 = 0.0191$  s.

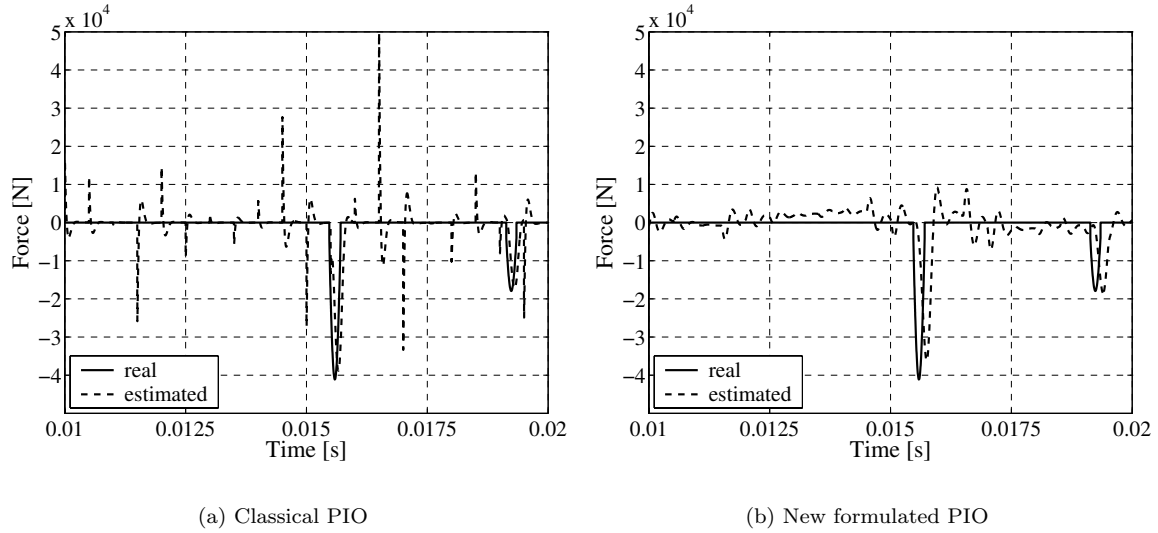


Figure 5: Estimated and real contact force (sensor noise 1 kHz)

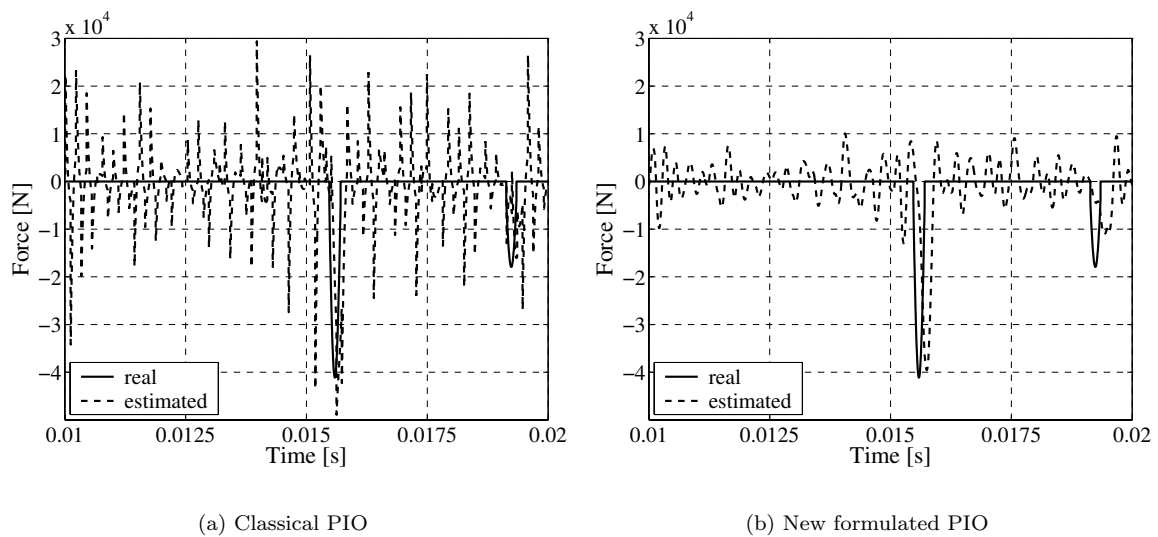


Figure 6: Estimated and real contact force (sensor noise 5 kHz)

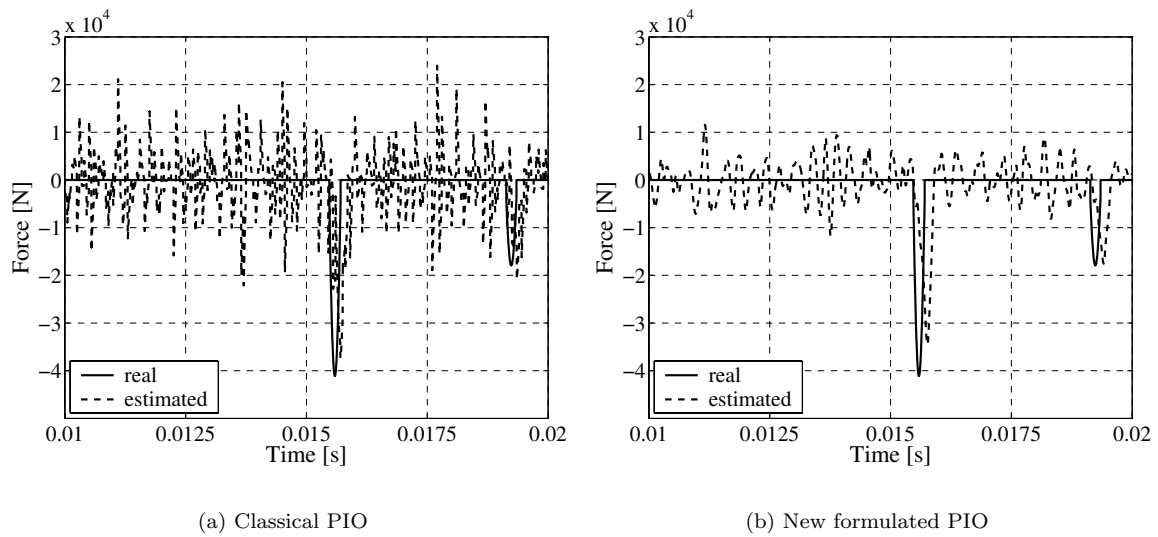


Figure 7: Estimated and real contact force (sensor noise 10 kHz)

## 5. Experimental results

A test-rig for experimental researches of friction effects between a rubbing turbine blade and the housing has been built up at the University of Essen (2). The experiment is illustrated in Figure 8(a), a draft scheme is shown in Figure 8(b). A small beam is clamped perpendicular to a shaft. The dimension of the shaft is chosen in the way that the rotor holds the rotating speed during the rubbing (contact) process. The blade rubs along the curved rubbing surface once per revolution. The contact force at the axial and vertical direction is measured by quartz force sensors, which are mounted between the rubbing surface and the supporting structure. Two strains on the blade are measured by strain gages and are used as input signals for the PIO.

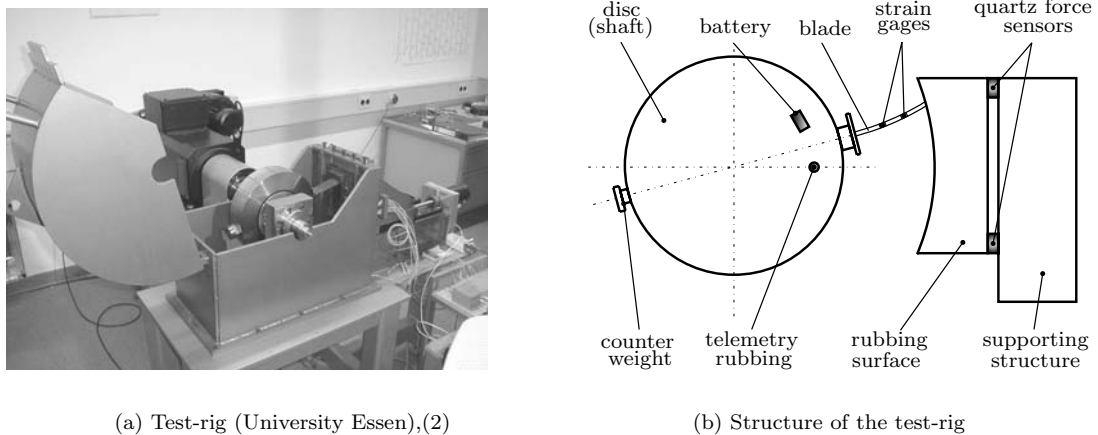


Figure 8: Experiment for contact force estimation for a rubbing blade

In Figure 9 the estimated and the measured force are shown (2). The force is estimated by the classical observer design. The result shows a good estimation of the characteristic of the contact force. The PIO is able to reconstruct these high frequency effects. The time delay is not caused by the observer itself. It

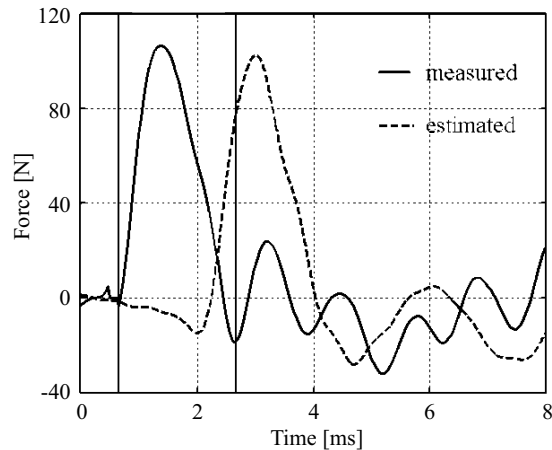


Figure 9: Measurement and estimated contact force (2)

is assumed that the effects are caused by the telemetry transmitter. The telemetry transmitter has also filtering properties in such a way that the measurement noise is suppressed by the signal transmission.

## 6. Conclusions

The paper introduces two model-based Proportional-Integral-Observer techniques for input estimation of unknown forces (or torques). This is used to estimate contact forces in rotating machinery. The proposed observer concepts can also be applied for other elastic structures. In this contribution a new formulation of the classical PIO is for the first time introduced. An additional integrator in the structure of the PIO leads to an additional state and permits to attenuate the measurement noise in such a way that the sensor noise is not amplified by the high observer gains. Simulation and experimental results show the satisfactory properties of the PIO. The next step of the work is to improve the attenuating properties of the new formulated PIO for sensor noise and to apply and analyze the PIO approach in real experimental environment. Also the attenuation of model uncertainties has to be analyzed.

## 7. References

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