An Approach to Affect the Probability of Failure by Changed Operation Modes

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ABSTRACT: The SRCE-concept (Söffker 2000) provides a framework and strategy to calculate the actual reliability of a system under operation. The maximum possible liable utilization may be controlled by changed operation mode. In consideration of a given reliability the amount of utilization can be extended. This concept may be used in terms of optimization of maintenance strategies, in terms of a limp home mode for safety critical applications or even to support the operator to organize the further utilization of the systems under operation. The important step in this concept is the calculation of a reliability characteristic depending on the actual applied load and the load history of the system (Wolters & Söffker 2004). In this contribution the formulation of the interrelations between the desired amount of utilization under a given probability of failure and the applied usage (in terms of load) is given.

1 INTRODUCTION

The utilization of the maximum life cycle of a technical system or component is getting more and more important. Because of economical or safety reasons, operators have to use technical components up to the maximum possible life. This demand can only be achieved, if knowledge about the actual state of the components concerning failure and the knowledge about how to change the operating mode to expand life time are given. There are several reasons to expand the life of a technical system or component which will probably fail if the actual operation mode continues. For example to achieve a date at which maintenance measures are feasible (due to the accessibility of the system and organizational or economical reasons respectively) or for inaccessible systems like space probes, satellites or autonomous submarines to complete a defined mission.

2 THE CONCEPT

The Safety and Reliability Control Engineering Concept (SRCE), first introduced by Söffker & Rakowsky (1997) describes a method to achieve information about the actual probability of failure depending on the individual actual and past utilization expressed in loads. Furthermore it describes an approach how to expand the maximum amount of utilization (in terms of mileage, cycles, times etc.) taking into account that the probability of failure will not exceed an unacceptable value.

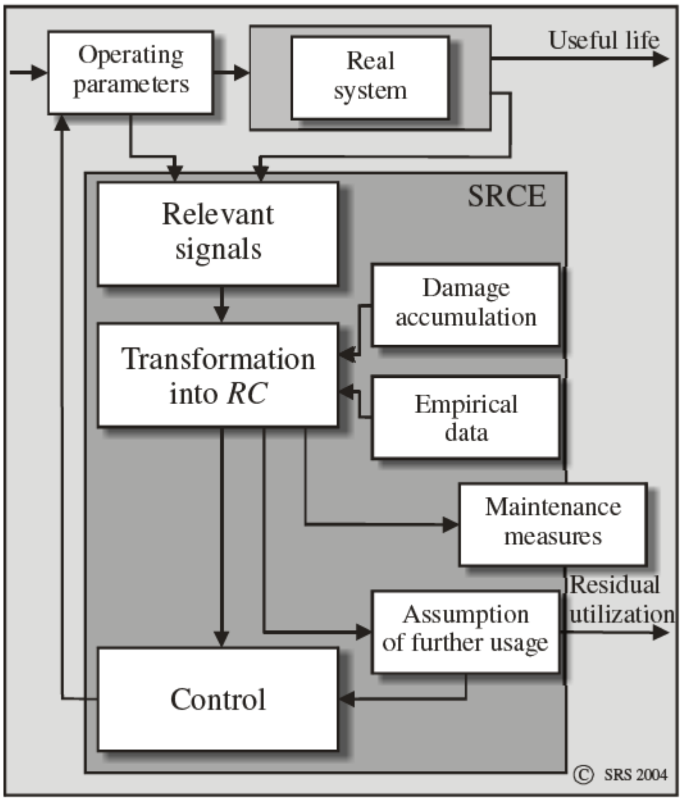

Figure 1. The structure of the Safety and Reliability Control Engineering Concept.

The concept is illustrated in Figure 1. It uses special information from a real, individual system which runs under defined operational conditions and has an unknown individual useful life. This Information are load or stress representing signals and can be obtained either by signal based measurements or by model based measurements (Söffker 1995).

Depending on the type of stress (mechanical, electrical or environmental) and depending on the type of damage, caused by the stress like fatigue, corrosion, wear etc. a special damage accumulation law (DAL) which is assumed to be valid for the damaging effect has to be chosen. Examples for this DAL are the Palmgren-Miner Rule (inclusively its modifications), the Marco-Starkey Theory, the Stanley theory and some more (Hwang & Han 1985).

Also a database with information about the relation between applied stresses and the expected useful life with its statistical spread is needed.

With this three modules,

- the actual stress,
- the valid DAL and
- the database,

reliability characteristics (*RC*) like the probability of failure and the failure rate can be calculated. In contrast to classical *RC*, this new *RC* and its development depends on the actual and past usage of the system instead of time only. For this reason, this *RC* is a quantity of the further possibility of an individual, used component's utilization. The use of this *RC* can be as an indicator for condition-based maintenance or supervision. In this case the *RC* can be used to initialize preventive maintenance actions or it may be used to support shut-down decisions. It can also be used to forecast the residual utilization of the system. In this case assumptions about the further usage of the system have to be given. The assumption of the further usage defines the maximum utilization (specified by the unacceptable probability of failure) while the residual utilization can be calculated as the difference between maximum and actual amount of utilization.

Beside this information the *RC* can be used to influence the maximum utilization of the system by changing the operating parameters. The changing of the operating parameters has of course to be limited to the range in which a minimum degree of functionality (comparatively to a limp home mode) is guaranteed. The intervention in the operating mode may be necessary because a safety critical mission (or another not to be aborted mission) has to be complete with unexpected appearing overstresses. The further usage of the system with these stress-level may lead to an untimely stop of the mission, if no intervention is performed.

The influences of the loads and stresses on the *RC* are known from the part of the transformation from the relevant signals to the *RC* (Figure 1). The inverse of this relation can now be used to determine the wanted operating parameters. With these parameters the system has to be run to achieve a given amount of utilization without exceeding a defined probability of failure.

The database representing the relation between the stresses and the maximum utilization can be expressed as a two- or multi-dimensional (depending on the amount of combined stresses) characteristic curve of different test series (Haibach 2000, Simoni 1999). In these curves the maximum utilization is inversely proportional to the applied stress. Because of inhomogeneities in the material, dimensional tolerances and scattering in the manufacturing process, the maximum utilization is scattered which has to be taken into account.

Figure 2 illustrates such a relation. These curves, here called S-U-R diagrams (or S-N-R-diagrams; normally the letter N stands for number of cycles, while U is for any kind of utilization) have to fulfill some assumption mentioned in Wolters & Söffker (2004) to be applicable in the SRCE-Concept.

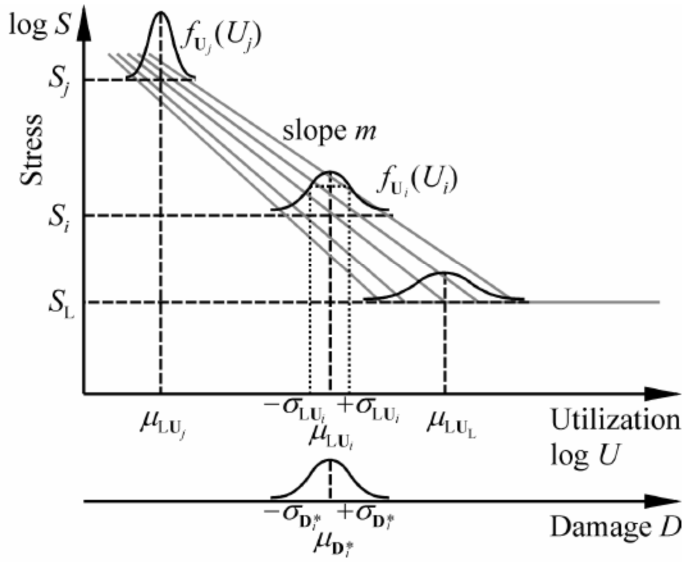

Figure 2. The S-U-R Life-Model.

The ordinate of a S-U-R diagram represents the stress S while the abscissa represents the maximum utilization U of the system. Such diagrams contain linear, straight and decreasing lines, when plotted in log-log coordinates. Each line represents a defined probability of the scattered maximum utilization U_i for a certain stress S_i. Hence U_i is a random variable (RV) $\boldsymbol{U}_i$ with a probability density function (PDF) $f_{Ui}(U_i)$. As an assumption $\boldsymbol{U}_i$ follows a log-normal distribution LN(μ_{LUi}, σ_{LUi}) with parameters μ_{LUi} and σ_{LUi} depending on the applied stress S_i (Min, Xiaofei & Qing-Xiong 1995). The stresses S_i are assumpted to be deterministic. There exists a limit S_L (endurance limit), underneath which applied stresses result in infinite utilization.

Different known additive and multiplicative DAL can be generally represented by

$$D_k = \sum_{i=1}^{k} d_i = \sum_{i=1}^{k} c\,\alpha(u_i, S_i, \ldots)^q \beta(D_{k-1}) \ , \tag{1}$$

where D_k is the cumulated damage up to usage k, d_i is the damage fraction of usage i, c is a constant, $\alpha_i(u_i, S_i, \ldots)$ is the damage. It depends on at least the number of usages u_i with the stress value S_i. The parameter q describes non-linearity. For additive DAL the function $\beta\ (D_{k-1}) = 1$ while for multiplicative DAL $\beta\ (D_{k-1}) = D_{k-1}$ (according to Desmond 1985). By using a DAL, an amount for the maximum or critical damage D^*_i assigning the failure of the component is specified. The value depends on the material properties, the applied stress and the operational conditions. A second abscissa is added to Figure 2, representing the damage. Since the system state *Failure* is defined as $D_i \geq D^*_i$, as well as $u_i \geq U_i$ (with u_i as the actual utilization at Stress S_i), the critical damage $\boldsymbol{D}^*_i$ follows a normal-distribution $N(\mu_{\mathbf{D}^*i}, \sigma_{\mathbf{D}^*i})$ with $\mu_{\mathbf{D}^*i} = 1$ (per definition) and $\sigma_{\mathbf{D}^*i} = \sigma_{\mathbf{U}i} \cdot \mu_{\mathbf{U}i}^{-1}$. The PDF $f_{\mathbf{D}i}(D^*_i)$ has got the same shape as $f_{\mathbf{U}i}(U_i)$ and can be plotted at the same position, shown in Figure 2.

The damage fraction d_i is, possibly among other parameters, calculated as a function of $\boldsymbol{U}_i$ (or indirect as a function of S_i). That results in a RV $\boldsymbol{d}_i$ following a log-normal distribution $LN(\mu_{\mathbf{Ld}i}, \sigma_{\mathbf{Ld}i})$. Following the central limit theorem, the damage D_i is also a RV following a normal-distribution as $N(\mu_{\mathbf{D}i}, \sigma_{\mathbf{D}i})$. By scaling the second abscissa with $\mu_{\mathbf{D}^*i} = 1$ it is possible to plot the PDF of $\boldsymbol{D}_i$ as illustrated in Figure 3.

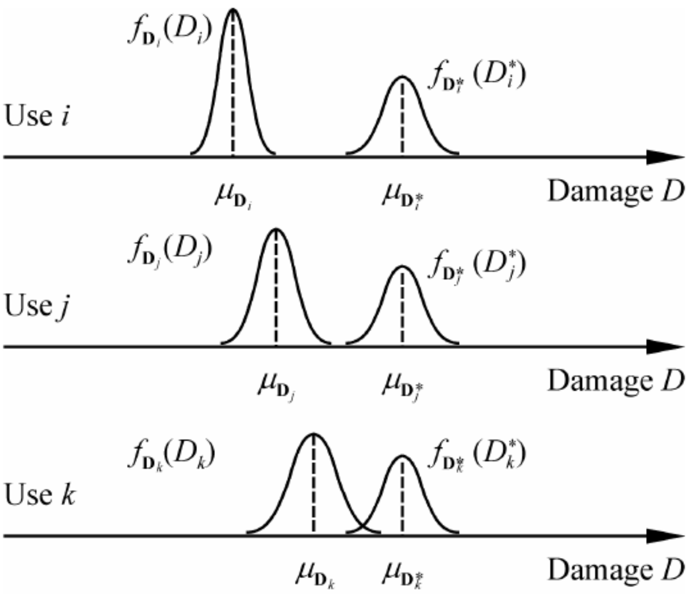

Figure 3. PDF of damage $\boldsymbol{D}_i$ and critical damage $\boldsymbol{D}^*_i$ at different point of uses with $i < j < k$.

For uses with stresses above S_L the PDF of the accumulated damage starts to overlap or even exceed the PDF of the critical damage. This area of overlapping, the interference area, indicates the probability

$$\Pr\{\boldsymbol{D}_i^* \le \boldsymbol{D}_i\} = F_{\mathbf{D}_i^*}(\boldsymbol{D}_i) = \int_{-\infty}^{\infty} F_{\mathbf{D}_i^*}(\delta) f_{\mathbf{D}_i}(\delta)\, d\delta \tag{2}$$

that the accumulated damage may be equal or greater than the critical damage. Here $F_{\mathbf{D}^*i}(\cdot)$ is the cumulative density function. This probability of failure depends only on

– the accumulated damage $\boldsymbol{D}_i$ representing the load history and
– the actual critical damage $\boldsymbol{D}^*_i$ which is associated with the actual load.

As a result, equation (2) represents a probability of failure depending on the load and load history of the system.

Equation (2) can be solved (Kapur & Lamberson 1997) for the above mentioned assumption that $\boldsymbol{U}_i$ follows a log-normal distribution

$$F_{\mathbf{D}_i^*}(\boldsymbol{D}_i) = \Phi\left(\frac{\mu_{\mathbf{D}_i} - 1}{\sqrt{\sigma^2_{\mathbf{D}_i^*} + \sigma^2_{\mathbf{D}_i}}} \right) = \Phi(z) \ , \tag{3}$$

where $\Phi(\cdot)$ is the standardized normal cumulative function, which is tabulated.
The standard normal variable z has to be a very large negative number in order to minimize the probability of failure. This can only be achieved by a large negative numerator and a small positive denominator.

The numerator can be rewritten as

$$\mu_{\mathbf{D}_i} - 1 \Leftrightarrow \sum_{i=1}^{k} \left[\left(1 + \frac{\sigma^2_{\mathbf{U}_i}}{\mu^2_{\mathbf{U}_i}} \right) \frac{1}{\mu_{\mathbf{U}_i}} \right] - 1 \tag{4}$$

and the denominator as

$$\sqrt{\sigma^2_{\mathbf{D}_i^*} + \sigma^2_{\mathbf{D}_i}} \Leftrightarrow \sqrt{\sum_{i=1}^{k} \left[\frac{\mu^2_{\mathbf{U}_i} \cdot \sigma_{\mathbf{U}_i} + \sigma^2_{\mathbf{U}_i}}{\mu^4_{\mathbf{U}_i}} \right]} . \tag{5}$$

The numerator and denominator depend only on the expectation $\mu_{\mathbf{U}i}$ and the deviation $\sigma_{\mathbf{U}i}$ of the maximum utilization $\boldsymbol{U}_i$ which can be calculated as

$$\mu_{\mathbf{U}_i} = \exp^{\left[\mu_{\mathrm{LU}_i} + \frac{1}{2} \sigma^2_{\mathrm{LU}_i} \right]} \tag{6}$$

and

$$\sigma_{\mathbf{U}_i} = \sqrt{\exp^{\left[2\mu_{\mathrm{LU}_i} + \sigma_{\mathrm{LU}_i} \right]} \left(\exp^{\sigma^2_{\mathrm{LU}_i}} - 1 \right)} \tag{7}$$

from the given $\mu_{\mathrm{LU}i}$ and $\sigma_{\mathrm{LU}i}$ derived from the S-U-R diagram. To get a large negative numerator, the following conditions have to be fulfilled

$$\mu_{\mathbf{U}_i} >> 1 \wedge \mu_{\mathbf{U}_i} > \sigma_{\mathbf{U}_i} \tag{8}$$

while for a very small positive denominator the conditions

$$\mu_{\mathbf{U}_i} >> 1 \wedge \sigma_{\mathbf{U}_i} \approx 0^+ \tag{9}$$

has to be fulfilled.

From Figure 2 it is obvious, that a decrease of stress results in an increasing expectation $\mu_{\mathrm{U}i}$ which meets the first condition of the conjunctions (8) and (9). Usually the expectation is greater than the standard deviation so that the second part of (8) is also fulfilled. With decreasing stress the standard deviation $\sigma_{\mathrm{U}i}$ increases, hence the second condition of (9) is difficult to keep. Nevertheless a small positive denominator will be obtained since the second condition of the conjunction (8) is fulfilled.

With the also often used assumption, that $\boldsymbol{U}_i$ follows a Weibull-distribution like Weibull(n, λ) equation (2) can be written as

$$F_{\mathbf{D}_i^*}(\boldsymbol{D}_i) = 1 - \Phi\left(-\frac{\mu_{\mathbf{D}_i}}{\sigma_{\mathbf{D}_i}}\right) - \frac{1}{\sqrt{2\pi}}\left(\frac{1}{n\cdot\sigma_{\mathbf{D}_i}}\right) \times \int_0^\infty \exp^{\left[-\delta^\lambda - \frac{1}{2}\left\{\left(\frac{1}{n\cdot\sigma_{\mathbf{D}_i}}\right)\delta - \frac{\mu_{\mathbf{D}_i}}{\sigma_{\mathbf{D}_i}}\right\}^2\right]} d\delta \qquad (10)$$

The computation of this equation is more complex than equation (3). Tables for the results of numerical integration methods for various parameters are e.g. given in (Kapur & Lamberson 1977).

The following questions may generally be of interesting to the operator of a system:

1 How far is the actual consumption of the possible utilization?
2 Up to which level is it possible to increase the load (to get more performance) without a probabilistic failure of the system?

The first question can simply be answered by solving equation (3) or (10). The result is the probability

$$\Pr\{U_{\mathrm{A}} \geq \boldsymbol{U}_i\} = \int_0^{U_{\mathrm{A}}} \frac{1}{\upsilon\cdot\sigma_{\mathrm{LU}_i}} \exp^{\left[-\frac{1}{2}\left(\frac{\ln\upsilon - \mu_{\mathrm{LU}_i}}{\sigma_{\mathrm{LU}_i}}\right)^2\right]} d\upsilon \qquad (11)$$

that the actual amount of utilization U_{A} is equal or greater than the maximum utilization $\boldsymbol{U}_i$. For the constant load case the parameters $\mu_{\mathrm{LU}i}$ and $\sigma_{\mathrm{LU}i}$ will not change whereas for the changing load case, the parameters changes according to Figure 2. A decreasing load results in an increasing $\mu_{\mathrm{LU}i}$ and $\sigma_{\mathrm{LU}i}$, which means, the residual system life is greater than before. The residual life increases with decreasing loads and vice versa.

For the second question two effects have to be considered. The first is, that an increasing load results in a PDF of the maximum damage $\boldsymbol{D}^*_i$ which is closer to the actual damage $\boldsymbol{D}_i$ and changes its shape to a narrower PDF (analogue to the first case). The second is that the growth of $\boldsymbol{D}_i$ will be faster with increasing loads. For a given probability of failure the possible load can be calculated by solving equation (2). The problem is that this equation contains three parameters depending on the load so all three have to be adjusted.

3 ILLUSTRATION

A short illustration to explain the potential of the proposed approach is given in Figure 4.

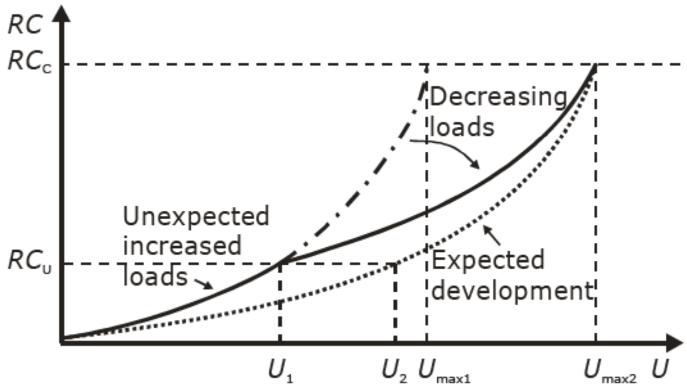

Figure 4. Illustration of the proposed approach.

Here the RC is plotted over the accumulated utilization U. The RC represents characteristics like the failure rate or the probability of failure, which should be as small as possible. The value RC_{U} describes a value at which an expected amount of utilization has to be achieved. It may be appropriate to have more than one RC_{U} like $RC_{\mathrm{U1}} < RC_{\mathrm{U2}} < \ldots < RC_{\mathrm{C}}$. The higher value RC_{C} represents a critical value of RC at which a shutdown of the system has to be arranged.

The dotted line represents the progression of the RC as expected for the system. The solid line represents the actual RC of the system depending on its stress history and its actual stress. At a certain $RC = RC_{\mathrm{T}}$ the actual accumulated amount of utilization U_1 is compared with the expected amount U_2. If $U_1 < U_2$ with a certain level of tolerance, the operation parameters have to be changed to decrease the applied loads. Without any change and the assumption of a further development of the stress related to the past load development, the critical $RC = RC_{\mathrm{C}}$ will be achieved with an amount of utilization U_{max1}. This is indicated by the dash-dot line. The result would be that the expected operational end, defined by U_{max2}, will not be achieved.

If the effect of limited operating parameters and its resulting stresses to the RC is known, then the change results in the development indicated by the solid line from U_1 to U_{max2}, which has a smaller gradient than the dash-dot line. Hence the level of the maximum performance and load is limited by the value of the maximum RC expressed by the solid line.

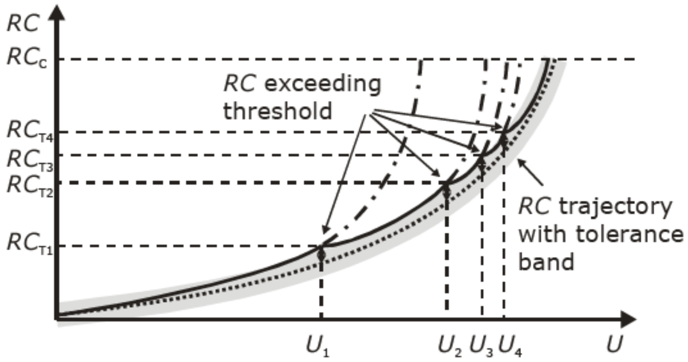

Figure 5. Control of the *RC*.

It is also possible to control the *RC*. In Figure 5 RC_{T1},..., RC_{T4} indicate values of *RC* which exceed a given threshold. At these points of utilization the load has to be changed in order to prevent an unacceptable growth of the *RC*, indicated by the dash-dot lines. The threshold may be given by a trajectory of the *RC* with a defined tolerance.

The potential of this approach is the possibility to extend the maximum amount of utilization of the individual system and to control its probabilistic failure behavior by changing the operating parameters or mode.

4 CONCLUSIONS

In this contribution, the mathematical relations belonging to the SRCE concept are given in a more detailed way. This concept makes it possible to control a quantitative characteristic, which is a prediction of the future reliability performance of an individual system with its own loads and load history. The changing of this characteristic by the use of empirical correlations like the mentioned S-U-R-diagrams yields to the PDF of the damage $\boldsymbol{D}_i$, which represents the accumulated load history (the stress) and the PDF of the critical damage $\boldsymbol{D}^{*}_{i}$, as the limit of utilization of the system (the strength). The formulated stress-strength/interference model can be solved for special assumption of PDF. How the parameters have to be changed is given by some conditions.

The main task for future research will be to obtain more corresponding data of the S-U-R curves by experimenting on a test-rig. In these experiments the damage is defined as the effect of fretting which may appear in wear plates. The operational parameters in this case are different surface pressures, the dimension of the abrasive material inside the lubrication and maybe other parameters like the feed speed etc. Furthermore the approach of controlling the system's life by changing operating parameters by considering a given probability of failure has to be detailed.

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