Course Control Theory
WiSe 2019/20

Room: SG 135
Time: Fr 3.00–6.30pm (lecture and exercise)
Practical exercise: 2nd part of semester

Assistants: Mark Spiller, M.Sc.
WEB: http://www.uni-due.de/srs

Manuscript

Notes:
The collected material is prepared for the use only in connection with the lecture.
It is not allowed to use this material outside the lecture of Prof. Söffker.

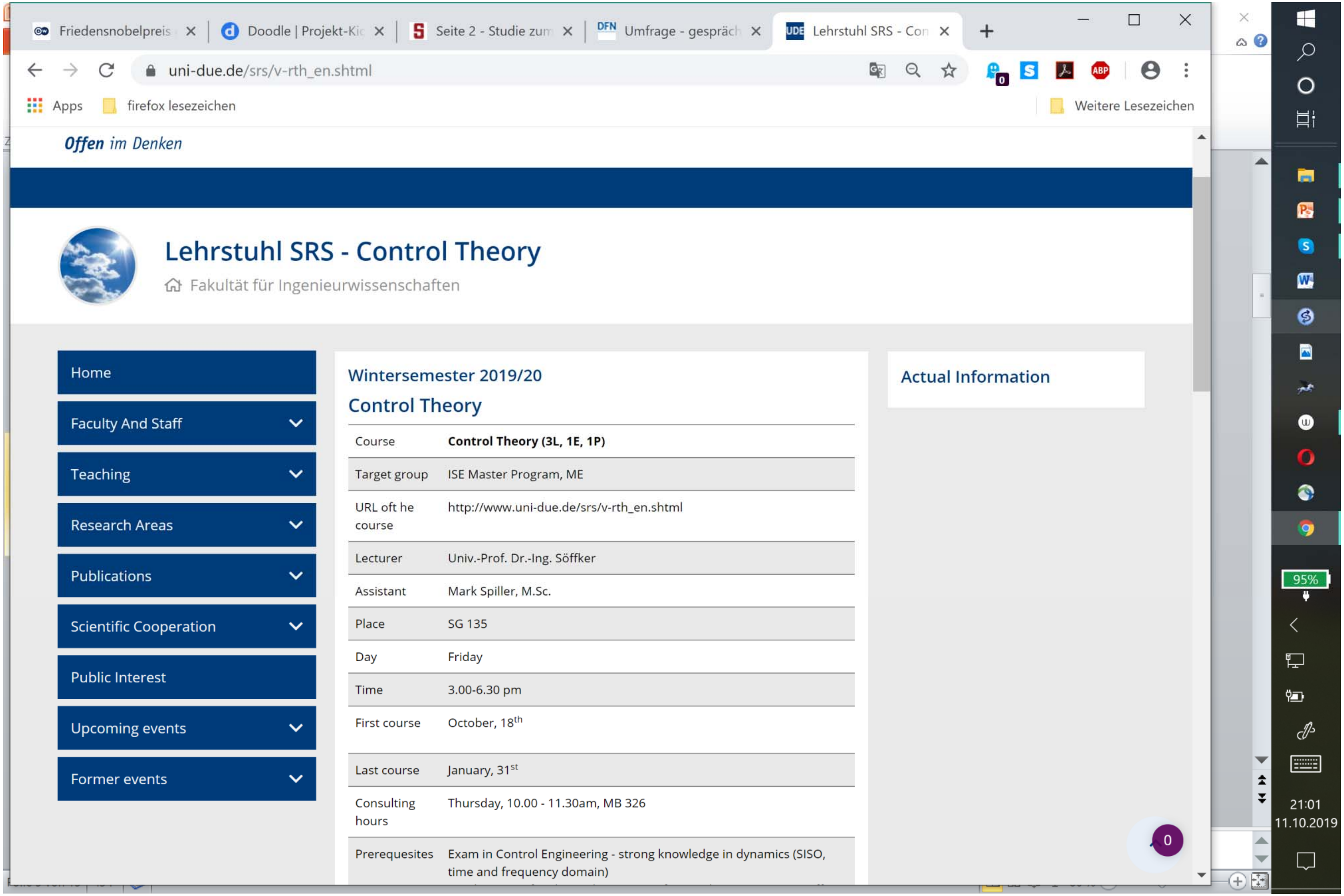
The reprinted figures are taken – if nothing different is mentioned - from the textbook of Prof. Lunze and are free for use in connection with this textbook-based course.

Add. remarks I

uni-due.de/srs/Lehrveranstaltungen.shtml

						...am 14.10.2019.
	Control Engineering (L, E)	Rothe/ Söffker/ Mitarbeiter/in	Monday	Oct 14th	Regular exam, written exam, closed book	NEW! Lecture material first lecture 'for free': print it out and take it with you for the first lecture. [PDF]
		MB 144	08.00am – 11.00am	Jan 20th		
PRACTICAL EXERCISE	Practical Exercise System Dynamics und Control Engineering (P)	Söffker/ Doctoral candidates	daily	Dec 2nd	**Central Attestation on Dec 2nd during lecture time of SD**	Practical Exercise: will be arranged 1st week of the semester **(enrollment necessary)** Lists of authorized participants for the resits for 14th October
		MB 325 (ms), MB 327 (dr), MB 028 (hs)	between 8.00am and 5.00pm	Jan 2020		
MASTER	Control Theory (L, E)	Söffker/ Doctoral candidates	Friday	Oct 18th	Regular exam, written exam, closed book	
		SG 135	3pm - 6.30pm	Jan 31st		
PRACTICAL EXERCISE	Preparatory Practical Exercise Control Engineering (P)	Söffker/ Doctoral candidates	daily	Nov 4th	--	Practical Exercise: will be arranged 1st week of the semester **(enrollment necessary)**
		MB 025 (hs)	between 8.00am and 5.00pm	Jan 2020		
PRACTICAL	Practical Exercise Control Theory (P)	Söffker/ Doctoral candidates	daily	Dec 20th	**Central Attestation on Dec 20th during lecture time**	Practical Exercise: will be arranged 1st week of the semester **(enrollment necessary)**
		MB 323 (hrt)	between	Jan		

Add. remarks Ic



Wintersemester 2019/20

Control Theory

Course	**Control Theory (3L, 1E, 1P)**
Target group	ISE Master Program, ME
URL oft he course	http://www.uni-due.de/srs/v-rth_en.shtml
Lecturer	Univ.-Prof. Dr.-Ing. Söffker
Assistant	Mark Spiller, M.Sc.
Place	SG 135
Day	Friday
Time	3.00-6.30 pm
First course	October, 18th
Last course	January, 31st
Consulting hours	Thursday, 10.00 - 11.30am, MB 326
Prerequesites	Exam in Control Engineering - strong knowledge in dynamics (SISO, time and frequency domain)

Add. remarks Id

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Lecturer	Univ.-Prof. Dr.-Ing. Dirk Söffker
Assistant	Mark Spiller, M.Sc.
Place	SG 135
Day	Friday
Time	3.00-6.30 pm
First course	October, 18th
Last course	January, 31st
Consulting hours	Thursday, 10.00 - 11.30 am, MB 326
Prerequisites	Exam in Control Engineering - strong knowledge in dynamics (SISO, time and frequency domain) For those, not familiar with contents of Control Engineering, we offer a Preparatory Practical Exercise. Please see the separate announcements.
Literature	Textbooks (> Library): Ogata, K.: Modern Control Engineering 3. Edition, Prentice H. Lunze, J.: Regelungstechnik 2, Springer Ludyk, G.: Theoretische Regelungstechnik Vol 1/2, Springer Franklin, G.: Feedback Control of Dynamic Systems, 4th ed.
Content	(see table below)
Practical Exercise	Mandatory, individually graded. See separate announcement. **(Attention: Attestation for practical exercises will take place on December, 20th during lecture time. Please check the relevant announcements).**
Exam	Written exam, closed book, in the examination period.

Content:

Unit	Topic:	Chapter (Ogata):	Chapter (Lunze) :
1	State space	11.1 – 11.5	1-2.6
2	Controllability and observability	11.6 f.	3
3	Pole placement	12.1-12.4	6
4	State observers	12.5-12.6	8.1-8.2
5	Design of servo systems / Robust control	12.7 f.	4-5 7.1-7.5
6	Liapunov stability	13.1-13.3	
7	Model reference control	13.4-13.5	
8	Quadratic optimal problems	13.6	7.1-7.5
9	Advanced approaches		
10	Discrete systems / discrete design		11-14

Course	**Practical Exercise Control Theory (1P)** comprising three experiments: • Control of the Inverted Pendulum (ip) • Observer-based Control for a Torsional Oscillator (brt) • Disturbance Estimation in Rotating Machines (de)
Attendance mandatory:	Participants of the course Control Theory (ISE Master Program, ME)
URL of the course	http://www.uni-due.de/srs/v-cth-an1-Praktikum.shtml
Examiners	Ph.D. students/scientific co-workers
Coordination	Dr.-Ing. Sandra Rothe, praktikum-srs@uni-due.de
Attestation date	**December, 20th during lecture time** (Dates and seat numbers will be announced on our homepage a few days before the attestation).
First lab dates	January, 7th
Last lab dates	January 31st
Place (labs)	MB 323 (brt), MB 325 (de, ip)
Lab days	Daily
Time	Dates between 8.00 am - 05.00 pm
Consulting hours	Thursday, 10.00 am – 11.30 am, MB 326
Scripts	Scripts for each experiment are located on the SRS homepage. Those have to be worked through until the attestation date as they are the basis for the attestation.
Attestation	You have to succeed one central attestation for all experiments in order to participate at the labs. **The attestation is only offered at the a.m. date. There is no (!) possibility to change the attestation date or to repeat the attestation in the same term.** Resit of this attestation is in the first semester week of the following term. Participation at the labs without a successfully passed attestation is not possible.
Registration	**Mandatory registration at the examination office in 5th and 6th week of the semester (same procedure as for examinations).** ONLY registered participants are allowed to take part in the attestation. A deregistration is only possible via email to praktikum-srs@uni-due.de latest 1 week (full 7 days) before the attestation date. Nonappearance leads to the grading fail for all three experiments. A deregistration after participation at the attestation is not possible.

<table>
<tr><td>Realization of labs</td><td>The experiments are held in English language. The participants are grouped in teams of 6 students and assigned to fixed lab dates. A central date exchange service by the chair will not be provided, but a change-of-dates-forum is arranged in moodle (Control Theory (CTh) – Practical Exercise (WiSe 19/20)). The participants are allowed to switch their dates with another accepted student on their own risk. If the switching party does not participate, the original advised student loses the right to participate. The doctoral candidate conducting the lab has to be informed at the beginning of the experiment about a date’s switch. All participants will be checked if their participation is accepted. Not accepted students are not allowed to take part.</td></tr>
<tr><td>Grading / fail</td><td>Your performance will be graded:
<table>
<tr><th>Criteria</th><th>Grade</th></tr>
<tr><td>- Attestation passed and
- Active participation at the lab</td><td>1,0</td></tr>
<tr><td>- Attestation passed but
- No active participation at the lab</td><td>3,0</td></tr>
<tr><td>- Attestation failed, or
- Nonappearance/delay</td><td>5,0
(failed)</td></tr>
</table>

Grading with 5,0 (failed), all experiments and the attestation have to be repeated. Grades will be reported to the examination office like other examination results.
The experiments have to be completed within one semester (including the repetition period of the directly following semester). Grades are 1,0 or 3,0, or the experiments have to be repeated completely.

The pass of the practical exercise is connected with:
1) Attestation: Each participant has to succeed one central written attestation for all experiments in order to participate at the labs.
2) Verification of identity: Participation at the attestation is only possible if your identity can be verified. For verification of your identity you have to show your Student-ID, or your passport, or your Aufenthaltstitel at the attestation date and in the beginning of the labs. If the ID cannot be accepted or is not correct, the student loses the right to participate.
3) Presence: The exercise starts exactly at the announced time. Participants who are not present until 5 minutes after start of the exercise will be graded as being “not present”, regardless of reasons. Nonappearance leads to the grading fail for all three experiments.
4) Active participation at the practical experiment.</td></tr>
<tr><td>Additional information</td><td>It is recommended to conduct the labs in the proposed order as failed attempts lead to worse grades or failed trials.</td></tr>
</table>

Veranstaltungsablauf WiSe 2019/20
Winter term time table
(Leitung/Head: Univ.-Prof. Dr.-Ing. Dirk Söffker) (V0, August 2019)

Lehrstuhl Steuerung, Regelung und Systemdynamik
Chair of Dynamics and Control

Verantaltung Course	**Kalenderwoche** Calendar week	42	43	44	45	46	47	48	49	50	51	2	3	4	5	**Prüfung** Exam
Systemdynamik	v-sd															**Schriftlich** Written
Control Engineering	v-ce															**Schriftlich** Written
Control Theory	v-cth															**Schriftlich** Written
Functional Safety	v-fs															**Schriftlich** Written
Notlauf und Diagnose (Söffker/Wolters)	v-ndts															**Schriftlich** Written
Prozessautomatisierungs-technik (Jelali)	v-pat															**Schriftlich** Written
Advanced Control Lab 1*	p-ac1															**Antestat+Bericht** Attestation+Report
Praktikum/Practical Exercise SD/CE*	p-rt															**Antestat** Attestation
Praktikum/Practical Exercise CTh/RTh*	p-cth/rth															**Antestat** Attestation
Vorbereitungspraktikum/ Preparatory Practical Exercise CE	p-pce															-
Mechatroniklabor/ MachineLab/ Teamprojekt/ Praxisprojekt	l-me/ma/te/pr															**Abschluss-präsentation** Final presentation

Legende:

Vorlesung,Übung/Lecture,Exercise	
Veranstaltung, geblockt/Blocked course	
Praktika/Practical Exercises	
Labor/Labs	
Prüfung,Antestat/Exam,Attestation	

* **Bitte beachten Sie den gesonderten Veranstaltungsablauf für die Praktika Regelungstechnik und Systemdynamik, Regelungstheorie sowie Advanced Control Lab.**
Please consider the separate time table for the practical exercises Control Engineering and System Dynamics, Control Theory as well as Advanced Control Lab.

Add. remarks IIb

Praktikum Regelungstheorie (RTh)
Practical Exercises Control Theory (CTh)
(Leitung/Head: Univ.-Prof. Dr.-Ing. Dirk Söffker) (V0, August 2019)

Lehrstuhl Steuerung, Regelung und Systemdynamik
Chair of Dynamics and Control

Wintersemester 2019-20 / Winter term 2019-20

Semesterwoche/-week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Anmeldung/Registration					HISinOne										
Antestat/Attestation	Wiederh. RTh									CTh					
Versuche/Labs		Wiederh. RTh		Preparatory Lab CE (hs)*							CTh: ip, de, brt Language: English				
Vorlesung/Lecture	Lecture CTh														

Sommersemester 2020 / Summer term 2020

Semesterwoche/-week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Anmeldung/Registration					HISinOne										
Antestat/Attestation	Resits CTh							RTh							
Versuche/Labs		Resits CTh							RTh: ip, de, brt Sprache: Deutsch						
Vorlesung/Lecture	Vorlesung RTh														

Wichtig! In der 5. und 6. Semesterwoche muss die **Anmeldung im Prüfungsamt** zum Praktikum erfolgen. **Ohne gültige Anmeldung ist keine Teilnahme am Praktikum möglich!** Eine Abmeldung vom Praktikum ist nur bis spätestens 1 Woche vor dem Antestattermin per Mail an praktikum-srs@uni-due.de möglich.

Important! In the 5th and 6th week of the term you have to **register at the Examination Office** for the practical exercise. **Without valid registration a participation in the labs is not possible!** You may unsubscribe from the labs date at least 1 week before the attestation date only by mail to praktikum-srs@uni-due.de.

* Registration for the Preparatory Practical Exercise is possible by mail to praktikum-srs@uni-due.de until October 25th, 2019.

Let's talk about the problems (today: challenges)
- What is so difficult about control technology?
- Why seems the subject particularly difficult for some students?
- Why is it annoying me (Söffker,) if students need more time for their studies due to delayed exams in System Dynamics and Control than necessary?
- Why do some students tell interesting stories and then do not know how simple and schematic lectures, exercises, and practicals are organized, or suffer from perceived disadvantages?

Short answers:
- Who is not being present, can not listen.
  > Personal and mental presence in lecture and practice are strongly recommended
- Control engineering and system dynamics contents are different and seems to be for many students an (affordable) challenge.
- I am sometimes disappointed because we know the problems, give many hints and offers, and these is not perceived by some people, eg. Due to non presence, not reading documents and not preparing in a suitable manner, or even appointments (practicals) are simply ignored etc.

What is the optimal strategy for fast study and good results?
(if necessary: wake up), being present and listening, read the material and recommend books, learn continuously, take all/most of the SRS offers and prepare properly for the exams (attestation for practical plus exam > take time (10-14 days if you are continuously working, if not: 3-4 weeks).

Detailed answers related to challenges with respect to System Dynamics and Control
What is the problem and how to overcome this?

Content

- Strong formal, method-oriented nature of the content
- Strong mathematization
- Constructive teaching content
  > exponentially rising speed of course

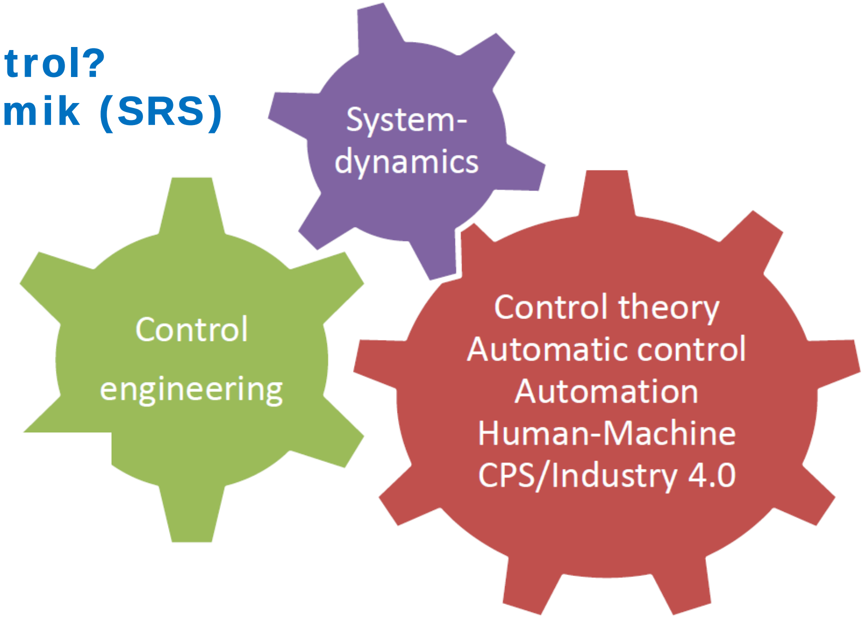
Form

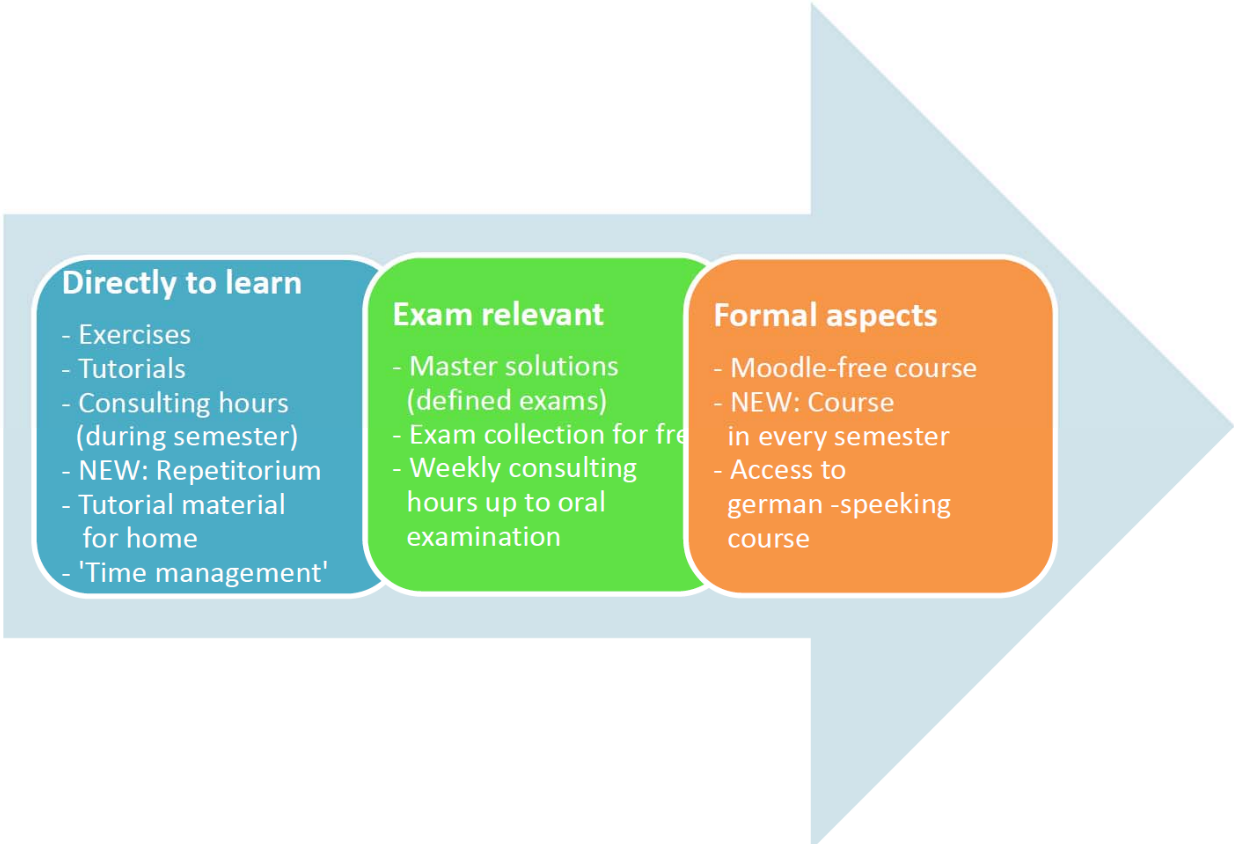
- Mathematization leads to a very strong level of abstraction
- > Transferability of the content vs. practical understanding
- Mathematization requires mathematical understanding
- > Mathematical understanding necessary

Waht to do?

- **WATCH OUT!** This subject is different.
- **Invest time**, take advantage of ALL offers of the chair.
- >> Understand exams and content. >> We are happy to help.
- **Take no semi-knowledge (from anywhere).**

What are the offers of the Chair Dynamics and Control?
(in german: Steuerung, Regelung and Systemdynamik (SRS)

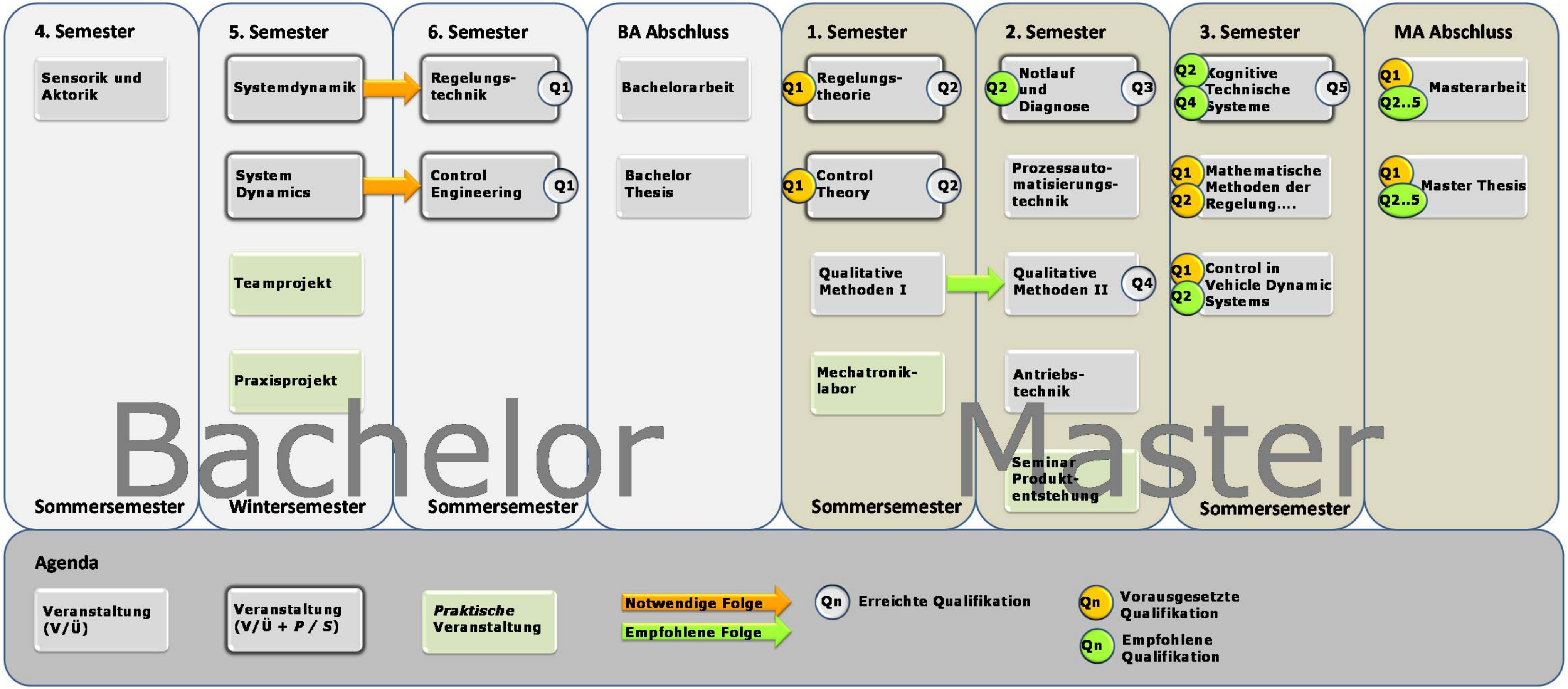




... for your success outcome.

Add. remarks III

Lehrangebot Lehrstuhl SRS (empfohlene Veranstaltungsfolge, Stand 03.11)



Methods (Control, Diagnosis, Automated Systems)
- Observers, Control, Fault Detection, Fault Diagnosis
- Cognition (Integrated Learning, Planning, Decision Making, ...)
- Filtering, Classifikation, Fusion (signals, information, ...)

> Robustness
> Modeling
> Reliability, Safety

Methods/ Mechatronics/ System Dynamics
- Modeling using automata
- Adaptive and reliable classification, able to learn
- Observers
- Nonlinear dynamics
- Power management of hybrid powertrains/ wind energy systems

Structural Health Monitoring
- Acoustic Emission
  > Sensors
  > Filters
  > Fault diagnosis
  > Prognosis
- Fault detection and fault diagnosis methods
- Wear and aging
  > Data-driven modeling
- Safe- und lifetime-oriented operation

Cognitive Technical Systems
- Knowledge-based and individualized assistence of HMS (example: driver assistance)
- Situation recognition of complex situations
- Situated automata, able to learn
- Safe actions and interactions

LU-1 Introduction to MIMO systems

What is the content of lecture units 1 and 2?

>> You should learn about the prerequisites of the course, especially about some terms and methods.
You should learn that time is required to pass the exam and in case of missing required knowledge to learn from books.

>> You should get sensible for the MIMO view of the dynamics of systems.
> This is illustrated with static and dynamic examples.

>> The differences between SISO and MIMO are declared.
> Representation, Methods

>> The main differences between SISO and MIMO are developed by changing the viewpoint from SISO to MIMO
> State space representation, Frequency domain, Couplings of elements, Time behavior and integration

LU-1 Introduction to MIMO systems

1.1 Characterisctics of MIMO systems (MIMO: Multi Input - Multi Output) $\neq$ SISO (single . . .)

Inputs and Outputs: Number of in/outputs > 1; $(\geq)$

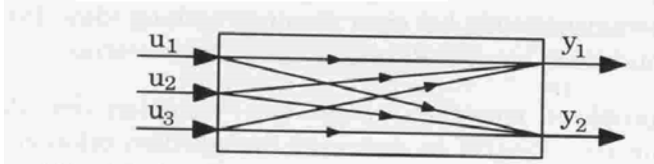


Abb. 1.1: Mehrgrößensystem

Complex dynamics: Several inputs are connected to several outputs

Coupled systems: Small / individual subsystems are effecting each other

Result: Expression / modeling is necessary to analyse the couplings and their relations to inputs and outputs
$\rightarrow$ Internal states

Practical examples I:

- Heat exchanger

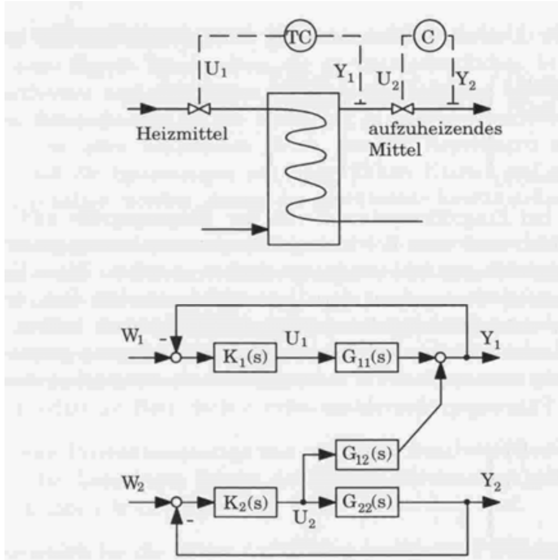
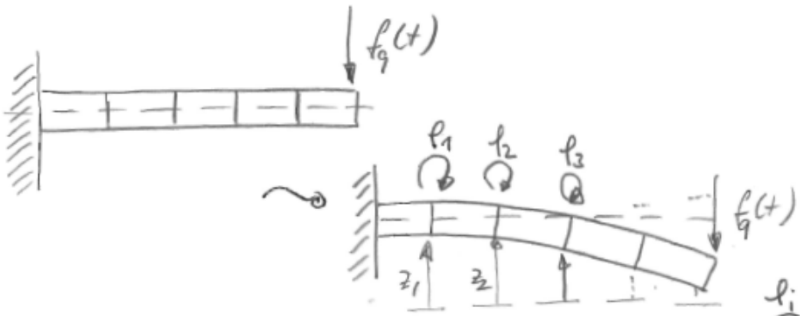


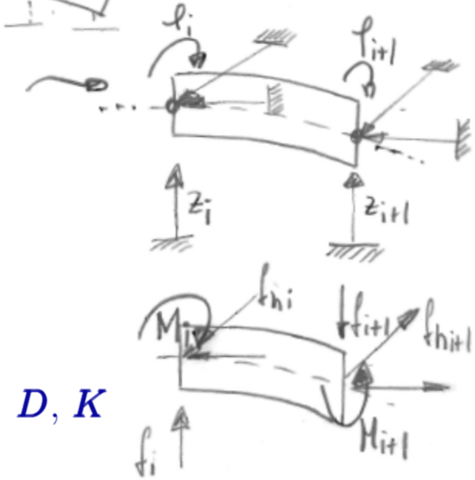
Abb. 1.2: Mehrgrößenregelung eines Wärmetauschers

LU-1 Introduction to MIMO systems

Practical examples II:

- Elastic beam (robot beam)





$$\boldsymbol{z} = \begin{bmatrix} z_1 \\ z_1' \sim \varphi_1 \\ z_2 \\ z_2' \sim \varphi_2 \\ \vdots \\ z_5 \\ z_5' \sim \varphi_5 \end{bmatrix}$$

Element matrices

- Stiffness
- Mass
- Damping

$\Rightarrow$ Overall system matrices $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{K}$

$$\boldsymbol{M}\ddot{\boldsymbol{z}} + \boldsymbol{D}\dot{\boldsymbol{z}} + \boldsymbol{K}\boldsymbol{z} = \boldsymbol{f}(t)\,, \;|\cdot \boldsymbol{M}^{-1}$$

$$\boldsymbol{z},\, \dot{\boldsymbol{z}},\, \ddot{\boldsymbol{z}}\,(t{=}0) = \boldsymbol{z}_0,\, \dot{\boldsymbol{z}}_0, \ddot{\boldsymbol{z}}_0,$$

$$\boldsymbol{z}\,(t) = \left[z_1,\, z_1',\, z_2,\, z_2',\, \ldots,\, z_5,\, z_5'\right]^T$$

State space description:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}$$

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{z} \\ \dot{\boldsymbol{z}} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\boldsymbol{z}} \\ \ddot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{z} \\ \dot{\boldsymbol{z}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{f} \end{bmatrix}$$

1.2 Representation of variables

- *m*-dim. input vector ***u***(*t*)

$$\boldsymbol{u}(t) = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$$

- *r1*-dim (*r*-dim). output vector ***y***(*t*)

$$\boldsymbol{y}(t) = \begin{bmatrix} y_1 \\ \vdots \\ y_{r1} \end{bmatrix}$$

- *r2*-dim. vector of reference ***w***(*t*)

$$\boldsymbol{w}(t) = \begin{bmatrix} w_1 \\ \vdots \\ w_{r2} \end{bmatrix}$$

- *p*-dim. disturbance vector ***d***(*t*)

$$\boldsymbol{d}(t) = \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}$$

> written form

> spoken form

> A special case: the SISO system

1.3 Analysis and control goals in the context of MIMO systems

- SISO control design goals : stability, reference control, disturbance compensation, dynamic requirements plus
- Analysis of the dynamics → Goal: Use of couplings > decoupling

- Analysis of the inner couplings → What acts how?

(- Guaranty of the functionality of the control loop) → Robustness against failures of actuators and sensors)

1.4 Transformation/Use of the SISO methods to analysis and synthesis of MIMO-systems possible?

- Description (state space (1:), transfer function $G(s)$, weighting function $g(t)$)

- Description tools to describe the dynamic behavior (poles , zeros , Hurwitz)

- Signal models (internal model principle) (→ works for single transfer pathes)

- Root locus

- Frequency domain-based control approaches (→ but: more complex relations)

1.5 New ideas and methods:

- Analysis of internal couplings

- Design of control structures using internal couplings

$$(1:)\ \dot{x} = Ax + Bu$$
$$y = Cx + Du$$

LU-2 Description and behavior of linear MIMO systems

2.1 Description techniques for SISO systems

- Differential equations

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 \dot{y} + a_0 y = b_q u^{(q)} + \ldots + b_0 u$$

- Transfer functions

$$g(s) = \frac{b_q s^{(q)} + b_{(q-1)} s^{(q-1)} + \ldots + b_1 s + b_0}{a_n s^{(n)} + a_{(n-1)} s^{(n-1)} + \ldots + a_1 s + a_0} = \frac{y(s)}{u(s)}$$

- State space description

$$\underbrace{\dot{x} = Ax + bu,}_{\text{State equation}} \qquad \underbrace{y = cx + du}_{\text{Output equation}}$$

2.2 Description techniques for MIMO systems

- Set of linear differential equations
  > Vector differential equation

$$\sum_{j=1}^{n} a_{ij} \frac{\mathrm{d}^j y_i}{\mathrm{d}t^j} = \sum_{k=1}^{m} \sum_{l=1}^{q} b_{kl} \frac{\mathrm{d}^l u_{ik}}{\mathrm{d}t^l}, \quad (i = 1, \ldots, r)$$

$$\frac{\mathrm{d}^j y_i}{\mathrm{d}t^j}(t{=}0) = y_{0,ij} \quad (i = 0, \ldots, n-1)$$

Resp. equations describing inner states

- Canonical regular form (diagonal canonical form)
  > (effect of inputs, effect of outputs, modal measurements)

$$\dot{x} = Ax + Bu + Ed$$
$$y = Cx + Du + Fd$$

x	:	State vector
u	:	Input vector
d	:	Disturbance vector
y	:	Output vector
A	:	System matrix
B	:	Input matrix
C	:	Output matrix
D	:	Transmittance matrix
E, F	:	Disturbance input matrices

  > Calculating modes (eigenvalue equation: eigenvalues, eigenvector equation: eigenvectors)

$$\texttt{det}\left(\underbrace{\lambda}_{\lambda_i} I - A\right) \overset{!}{=} 0$$
$$Av_i = \lambda_i v_i$$
$$V = \left[v_1 \; v_2 \; \dots \; v_n\right] \quad \texttt{(Modalmatrix)}$$

  > Decomposing the system (calculating the modes)

$$\Rightarrow AV = V\texttt{diag}\left[\lambda_i\right]$$
$$V^{-1}AV = \texttt{diag}\left[\lambda_i\right]$$

$$\tilde{B} = V^{-1}B, \quad \tilde{C} = CV$$

2.3 Description of the MIMO-system in frequency domain

$$\begin{aligned}
\dot{x} &= Ax + Bu \\
sx(s) &= Ax(s) + Bu(s) \\
(sI - A)\,x(s) &= Bu(s) \qquad \Big| \cdot (sI - A)^{-1} \\
x(s) &= (sI - A)^{-1}\,Bu(s)
\end{aligned}$$

$$\begin{aligned}
y(s) &= Cx(s) + Du(s) \\
y(s) &= \left[C\,(sI - A)^{-1}\,B + D\right] u(s)
\end{aligned}$$

- Transfer function matrix (be careful > MIMO-case)

$$G(s) = \frac{y(s)}{u(s)} = \underbrace{\left[C\,(s\,I - A)^{-1}\,B + D\right]}_{\texttt{Transfer function matrix}}$$

- Frequency domain description of a MIMO systems
  > elementwise analog to the SISO case

$$\begin{bmatrix} y_1(s) \\ \vdots \\ y_i(s) \\ \vdots \\ y_r(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & \dots & \dots & \dots & g_{1m}(s) \\ \vdots & \ddots & \vdots & & \vdots \\ \vdots & \dots & g_{ij}(s) & \dots & \vdots \\ \vdots & & \vdots & \ddots & \vdots \\ g_{r1}(s) & \dots & \dots & \dots & g_{rm}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ \vdots \\ u_j(s) \\ \vdots \\ u_m(s) \end{bmatrix}$$

$$g_{ij}(s) = k_{ij} \frac{\prod_{i=1}^{q_{ij}} (s - s_{0i})}{\prod_{e=1}^{n_{ij}} (s - s_i)}$$

- Connections between the frequency domain and the state space

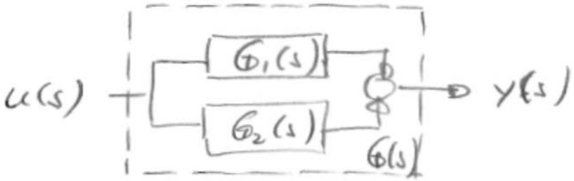
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$\rightarrow G = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

> Transfer behavior in state space: Rosenbrock system matrix

$$\begin{bmatrix} (sI - A) & -B \\ C & D \end{bmatrix} \begin{bmatrix} x(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} x_0 \\ y(s) \end{bmatrix} \qquad \rightarrow P(s) \begin{bmatrix} x(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} x_0 \\ y(s) \end{bmatrix}$$

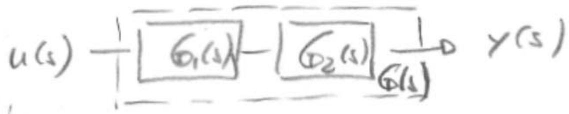
2.4 Parallel and series network of MIMO transfer elements



$$G(s) = G_1(s) + G_2(s)$$

Feedback:

$$G(s) = \underbrace{G_1(s)}_{\text{Plant}} \cdot \left[I + G_2(s)\, G_1(s)\right]^{-1}$$



$$G(s) = G_2(s) \cdot G_1(s)$$

2.5 (Time-)behavior of MIMO systems (of the state-space description)

- Equations of motion

$$x(t) = \phi(t) \cdot x_0 + \underbrace{\int_0^t \phi(t-\tau)\, Bu(\tau)\, \mathrm{d}\tau}_{\text{Convolution integral}}, \quad \phi(t) = \mathrm{e}^{At}, \; x_0 = x(t=0)$$

- Output equations of motion

$$y(t) = Cx(t) = C\phi(t)\, x_0 + C \int_0^t \phi(t-\tau)\, Bu(\tau)\, \mathrm{d}\tau$$

- Instationary (transient) and stationary (# constant) behavior

$$x(t) = \underbrace{x_{\text{trans}}}_{\text{Transient response}} + \underbrace{x_{\text{stat}}}_{\text{Stationary behavior}}$$

What you should learm from LU 1-2:

>> The fact that MIMO systems link several inputs and outputs with one another leads to complex dynamics, i.e. based on the dynamic couplings. From this it results that the known fast controller design strategy (SISO: PID-approaches etc.), which are used in SISO approaches and which realizes a simple connection between the controller input and system output, can not be used with MIMO systems.

>> A system analysis (stability, modal characteristics, observability, controllability) always precedes the controller design (synthesis) in the case of MIMO systems.

>> The state space representation permits a systematic representation of all linear MIMO systems. The analysis methods are independent from the number of the states as well as from the dynamics.

>> The transfer function matrix shows the dynamic effects between the several inputs and outputs.

>> The structural properties are shown in the time domain using the state space representation, in the frequency domain using the Rosenbrock matrix.

>> The state space representation is the linear and explicit special case of the general representation for the time integration of dynamic systems (in explicit form).