Universität Duisburg-Essen
Faculty of Engineering
Department of Computer Science and Applied Cognitive Science
Theoretical Computer Science Group
Prof. Dr. Barbara König

UNIVERSITÄT
DUISBURG
ESSEN

Open-Minded

Exercise sheet 1

Automaten und Formale Sprachen

Sommersemester 2019, Teaching assistant: D. Nolte, L. Stoltenow

Submission[1]: Tuesday, April 23, 2019, 08:30[2]

Exercise 1: Short questions   (6 points)

For each of the following statements, indicate whether it is true or not. Justify your answers! Answers without justification receive *no points*.

(a) $\{1, 2, 3\} = \{2, 3, 1\}$ (1 p)

(b) $12 \in \{1, 2, 3\}$ (1 p)

(c) $\{3\} \in \{1, 2, 3\}$ (1 p)

(d) $\{3\} \in \big\{\{1\}, \{2\}, \{3\}\big\}$ (1 p)

(e) $\{3\} \subseteq \{1, 2, 3\}$ (1 p)

(f) $\{3, 4\} \subseteq \{1, 2, 3\}$ (1 p)

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[1] *Options to submit your solutions:* Letterbox next to LF 259 (Campus Duisburg) or via Moodle
`https://moodle.uni-due.de/course/view.php?id=15777`
[2] Only this time, since Monday is a public holiday

Exercise 2: Sets   (8 points)

(a) Determine the powerset $\mathcal{P}(M_i)$ of the following sets:

    (i) $M_1 = \{\{1\}, \{1, 2\}\}$ (1 p)

    (ii) $M_2 = \{a, (a, b), b\}$ (1 p)

    (iii) $M_3 = \{1\} \times \{1, 2\}$ (1 p)

    (iv) $M_4 = \{\emptyset\}$ (1 p)

(b) Give the following sets by means of the set notation presented in the lecture:

    (i) The set of all even, natural numbers.   (1 p)

    (ii) The set of all pairs (2-tuples) of natural numbers, where the first element is equal to the square of the second element.   (1 p)

    (iii) The set of all pairs (2-tuples), where both elements are subsets of the natural numbers and the first element is a subset of the second element.

        (1 p)

    (iv) The set of all functions which map natural numbers to natural numbers and where the number 5 is mapped to itself.   (1 p)

Exercise 3: Adventure (Level 1) (6 points)

You are an adventurer on a treasure hunt and you are given the map below, over the alphabet Σ: Dragon (d), Arch (b), Door (t), Sword (w), Treasure (a), River (f) and Key (l). Your mission is as follows: You start at the start state and you have to reach the end state, where the following conditions must be fulfilled.

(S) You must find two treasures. If you reach the same treasure field several times, you gain a new treasure every time.

(T) You can only go through doors, if you have found a key before. Every key opens every door.

(D) If you meet a dragon, you must jump into a river directly afterwards, otherwise the dragon will set you on fire. This is not needed anymore once you have found a sword, because you can kill the dragon early enough in this case. If you reach the same dragon field several times, there is a new dragon every time.
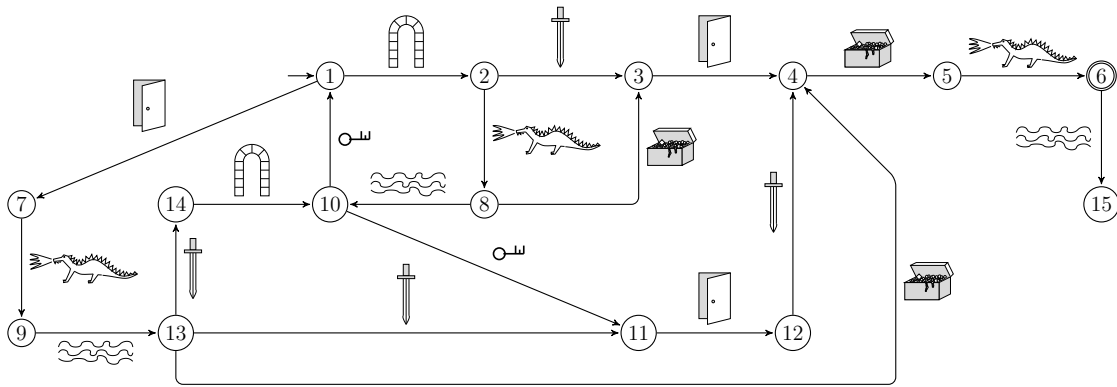
(a) Explain why the following words are not a solution for the given adventure (i.e. why do these words not fulfill the conditions (S), (T), (D) respectively). (1.5 p)

 ($tdfaad$)

 ($bdfltwad$)

($bdfltdfaadf$)

(b) Find all shortest solutions for the given adventure. A solution is a shortest one, if there is no other solution with a fewer number of symbols. *Note:* There are two shortest solutions. (1 p)

(c) Does the given adventure have finitely or infinitely many solutions? Justify your answer! (2 p)

(d) Give your own adventure (i.e. draw a new map – do *not* just give another solution for the map on the sheet) that has at least one solution. (1.5 p)

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