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Exercise sheet 2

Automaten und Formale Sprachen
Sommersemester 2019, Teaching assistant: Dennis Nolte, Lara Stoltenow

Submission[1]: Monday, April 29, 2019, 10:00 Uhr

Exercise 4: Grammars and their languages (6 points)

(a) Let $\Sigma = \{a, b\}$. In the following several grammars G_i are given. For every grammar G_i give the language generated by that grammar. You may use set notation (preferably) or your own words.

(i) $G_1 = (\{S\}, \Sigma, P, S)$, where P is defined as: $S \to \varepsilon \mid aSb$ (1 p)

(ii) $G_2 = (\{S, A, B\}, \Sigma, P, S)$, where P is defined as: (1.5 p)

$$S \to AB \qquad A \to aA \mid \varepsilon \qquad B \to bB \mid \varepsilon \qquad ab \to ba \qquad ba \to ab$$

(b) In the following we give different languages L_i. For every L_i give a grammar that generates exactly that language.

(i) $L_1 = \{w \in \{0, 1\}^* \mid w \text{ is a binary representation of an odd number}\}$ (1 p)

(ii) $L_2 = \emptyset$, where the alphabet is $\{a, b\}$. (1 p)

(iii) $L_3 = \{w_1 w_2 \mid w_1 \in \{a, b\}^*, w_2 \in \{b, c\}^*\}$ (1.5 p)

[1] *Options to submit your solutions:* Letterbox next to LF 259 (Campus Duisburg) or via Moodle `https://moodle.uni-due.de/course/view.php?id=15777`

Exercise 5: Grammar and Chomsky hierarchy (7 points)

(a) Let $\Sigma = \{a, b\}$. Classify the following grammars as accurately as possible with respect to the Chomsky hierarchy. Specify the language ($L(G_i) = \{\dots\}$) which is generated by the grammar. Additionally, specify its Chomsky type.

(i) Let $G_1 = (\{S\}, \Sigma, P, S)$, where P is defined as follows: (2 p)

$$S \to aSb \mid aS \mid \varepsilon$$

(ii) Let $G_2 = (\{S, A\}, \Sigma, P, S)$, where P is defined as follows: (2 p)

$$S \to aA \mid a \mid bS$$
$$A \to aS \mid bA \mid b$$

(b) Let $\Sigma = \{a, b\}$. Give a grammar of maximal Chomsky type for each of the following languages, where type-3 is the largest and type-0 the smallest Chomsky type:

(i) $L_3 = \{w \in \Sigma^* \mid w \text{ contains at least one } b\}$ (1.5 p)

(ii) $L_4 = \{a^n b^k \mid n, k \in \mathbb{N}_0 \wedge n < k\}$ (1.5 p)

Exercise 6: Word problem (7 points)

Check by means of the algorithm for the word problem, which was presented in the lecture, whether the following words are contained in the language of the given grammars:

(a) $G_1 = (\{S, X\}, \{a, b\}, P, S)$, where P is defined as follows:

$$\begin{aligned} S &\to aX \\ X &\to aXb \mid bXa \mid ab \mid ba \\ aX &\to Xa \end{aligned}$$

Decide whether the word $baaba$ is part of the language $L(G_1)$ or not. (3 p)

(b) Let $G_2 = (\{S, A, B\}, \{a, b\}, P, S)$, where P is defined as follows:

$$S \to SAB \mid aB \qquad aA \to aa \qquad aB \to ab$$
$$bB \to bb \qquad BA \to AB$$

Decide whether the word $aaaabb$ is part of the language $L(G_2)$ or not. (3 p)

(c) Give a – preferably small – type-0 grammar and a word generated by the grammar, such that the algorithm for the word problem does not detect that the word is generated by the grammar. (1 p)