Universität Duisburg-Essen
Faculty of Engineering
Department of Computer Science and Applied Cognitive Science
Theoretical Computer Science Group
Prof. Dr. Barbara König

UNIVERSITÄT DUISBURG ESSEN

Open-Minded

Exercise sheet 6

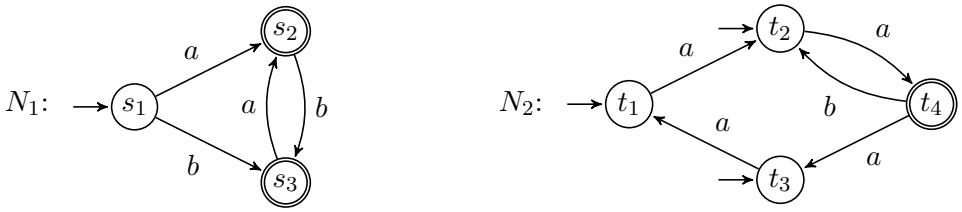
Automaten und Formale Sprachen

Sommersemester 2019, Teaching assistant: Dennis Nolte, Lara Stoltenow

Submission[1]: Monday, May 27, 2019, 10:00 Uhr

Exercise 18: Conversion of finite automata to regular expressions (6 points)

Let the following non-deterministic automata N_1 and N_2 be given:



Convert both non-deterministic automata N_1 and N_2 to regular expressions α and β. Use the procedure from the lecture! The regular expressions α and β have to describe the same languages that are accepted by the automata N_1 and N_2. Thus the following should hold:

$$L(N_1) = L(\alpha) \quad \text{and} \quad L(N_2) = L(\beta)\,.$$

Indicate all intermediate steps of your transformation.

Note: If you apply the rule S (deletion of loops) as late as possible, the resulting regular expressions will be smaller.

[1] *Options to submit your solutions:* Letterbox next to LF 259 (Campus Duisburg) or via Moodle `https://moodle.uni-due.de/course/view.php?id=15777`

Exercise 19: Conversion of regular expressions to finite automata (6 points)

Let $\Sigma = \{a, b\}$ and let the following regular expressions over Σ be given:

$$\alpha = a^*b(a \mid b)^* \quad \text{and} \quad \beta = (ab \mid b)^*(a \mid \varepsilon).$$

(a) Which languages $L(\alpha)$ and $L(\beta)$ are described by these regular expressions? Use words or set notation. (2 p)

(b) Convert both regular expressions α and β to non-deterministic automata M_α and M_β. Use the procedure from the lecture! The automata M_α and M_β must accept the same language as those described by the regular expressions α and β. Thus the following should hold:

$$L(\alpha) = T(M_\alpha) \quad \text{and} \quad L(\beta) = T(M_\beta)\,.$$

Indicate all intermediate steps of your transformation. Finite automata for a and b do not have to be given separately. (4 p)

Exercise 20: Questions about closure properties (8 points)

Decide whether the following statements hold for arbitrary languages $L_1, L_2 \subseteq \Sigma^*$. Give either a proof or a counterexample in each case. Answers *without* any justification achieve *no* points.

Note: For the construction of counterexamples you can use the fact that nonregular languages exist. Examples for such nonregular languages are $\{a^n b^n \mid n \geq 0\}$ or $\{w \in \{a, b\}^* \mid \#_a(w) = \#_b(w)\}$.

(a) If $L_1 \cup L_2$ is regular, then at least one of the languages L_1 or L_2 is regular. (1.5 p)

(b) If $L_1 \cup L_2$ is not regular, then at least one of the languages L_1 or L_2 is not regular. (1.5 p)

(c) If L_1 is regular and L_2 is not regular, then $L_1 \cup L_2$ is also not regular. (1.5 p)

(d) If L_2 is regular and $L_1 \subseteq L_2$ holds, then L_1 is also regular. (1 p)

(e) If L_1 is regular and $L_1 \subseteq L_2$, then L_2 is also regular. (1 p)

(f) If L_1 is regular, then the language $L_2 = \{xy \mid x \in L_1 \wedge y \notin L_1 \wedge y \in \Sigma^*\}$ is also regular. (1.5 p)

(In total, there are **20** points in this exercise sheet.)