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Exercise sheet 7

Automaten und Formale Sprachen

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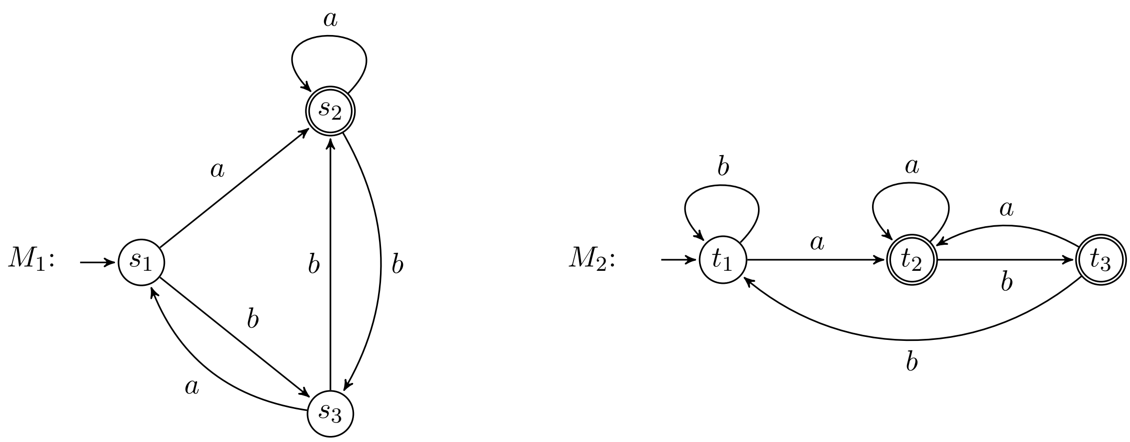
Submission[1]: Monday, June 3, 2019, 10:00 Uhr

Exercise 21 *Closure properties of regular languages* (7 points)

Let $\Sigma = \{a, b\}$ and let the following deterministic finite automata $M_1 = (Z_1, \Sigma, \delta_1, s_1, E_1)$ and $M_2 = (Z_2, \Sigma, \delta_2, t_1, E_2)$ be given:



(a) (i) Construct by means of the cross product construction given in the lecture a deterministic finite automaton M_S, which accepts the language

$$T(M_S) = T(M_1) \cap T(M_2)$$

(3 p)

Note: You only have to specify reachable states.

(ii) Give a deterministic finite automaton M_V, which accepts the language

$$T(M_V) = T(M_1) \cup T(M_2)$$

(2 p)

Note: You can construct this automaton using an adapted version of the cross product construction.

(b) Describe for two arbitrary deterministic finite automata $M' = (Z', \Sigma, \delta', s', E')$ and $M'' = (Z'', \Sigma, \delta'', s'', E'')$ how to construct a deterministic finite automaton M that accepts the language

$$T(M) = T(M') \cup T(M'')$$

Briefly describe why your procedure is correct! (2 p)

[1] *Options to submit your solutions:* Letterbox next to LF 259 (Campus Duisburg) or via Moodle `https://moodle.uni-due.de/course/view.php?id=15777`

Exercise 22 *Pumping Lemma for beginners* (6 points)

Show by means of the Pumping Lemma for regular languages, that the following languages are not regular:

(a) $L_1 = \{ab^k c^m \mid k, m \in \mathbb{N}_0 \wedge k > m\}$ (3 p)

(b) $L_2 = \{a^k b^{(k^2)} \mid k \in \mathbb{N}_0\}$ (3 p)

Your proof should have the following form:
Let n be an arbitrary natural number. We choose the word $x =$ ☐. Then $x \in L$ and $|x| \geq n$ holds. We can decompose x in the following ways, such that $|uv| \leq n$, $|v| \geq 1$:

1) $u =$ ☐, $v =$ ☐, $w =$ ☐, where ☐
2) $u =$ ☐, $v =$ ☐, $w =$ ☐, where ☐
3) $u =$ ☐, $v =$ ☐, $w =$ ☐, where ☐

For every decomposition there is an index i such that $uv^iw \notin L$. For the decompositions mentioned above, we choose the indices as follows:

1) $i =$ ☐, such that $uv^iw =$ ☐ $\notin L$, because ☐
2) $i =$ ☐, such that $uv^iw =$ ☐ $\notin L$, because ☐
3) $i =$ ☐, such that $uv^iw =$ ☐ $\notin L$, because ☐

According to the Pumping Lemma L is therefore not regular.

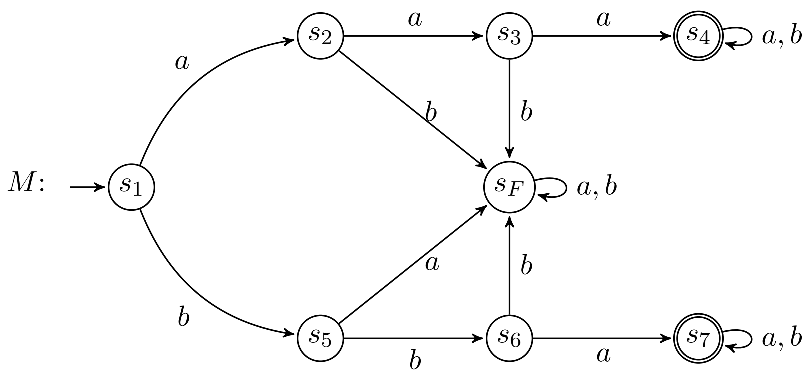
Note: The number of different decompositions depends on the chosen word and the way of describing the decompositions.

Exercise 23 *Language of a state* (7 points)

You already know that the language of a deterministic finite automaton (DFA) is the set of all words w such that beginning at the start state and reading the word w, the DFA ends up in a final state. We now define the *language of a state* $Z(s_i)$ as the set of words w such that beginning at a given state s_i and reading the word w, the DFA ends up in a final state. Formally:[2]

$$Z(s_i) = \left\{ w \in \Sigma^* \,\middle|\, \hat{\delta}(s_i, w) \in E \right\}$$

Now let the following deterministic automaton M be given:



As an example, starting at s_2, a final state can be reached by reading aa, therefore $aa \in Z(s_2)$. On the other hand, $b \notin Z(s_2)$, $ab \notin Z(s_2)$, $a \notin Z(s_2)$, since reading all these words results in a non-final state being reached.

(a) For each state of M, specify the language of this state. You can use set notation ($Z(s_i) = \{\dots\}$) or regular expressions ($Z(s_i) = L(\dots)$). (4 p)

(b) Recall the definition of Myhill-Nerode equivalence:

$$x \equiv_L y \iff \text{for all } z \in \Sigma^* \text{ it holds that } (xz \in L \Leftrightarrow yz \in L)$$

We have $\hat{\delta}(s_1, aa) = s_3$ and $\hat{\delta}(s_1, bb) = s_6$. Also, it is easy to see that $Z(s_3) = Z(s_6)$ (if you did not solve subtask (a), then you may simply assume that this is true). Based on this, what is the relationship between the words aa and bb? Give a short justification for your answer. (2 p)

(c) Specify the Myhill-Nerode equivalence class of the word aa ($[aa]_{\equiv_{T(M)}} = \{\dots\}$). (1 p)

(In total, there are **20** points in this exercise sheet.)

[2] $\hat{\delta}(s_i, w) = s_k$ means that if a DFA is in state s_i and reads w, then it is in s_k afterwards, see also slide 90.