Universität Duisburg-Essen
Faculty of Engineering
Department of Computer Science and Applied Cognitive Science
Theoretical Computer Science Group
Prof. Dr. Barbara König

Exercise sheet 9

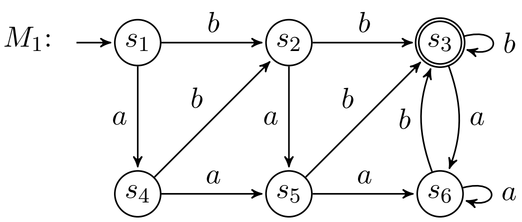
Automaten und Formale Sprachen

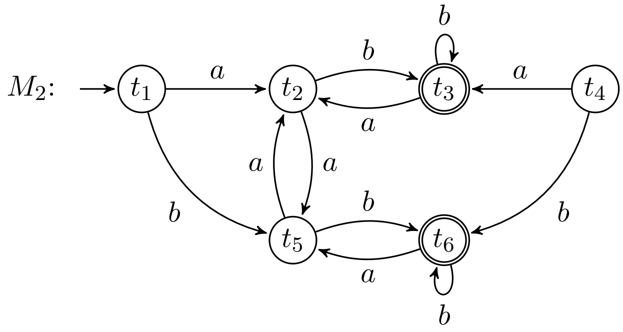
Sommersemester 2019, Teaching assistant: Dennis Nolte, Lara Stoltenow

Submission[1]: Monday, June 17, 2019, 10:00 Uhr

Exercise 28 *Equivalence of regular languages* (6 points)

Let the following deterministic finite automata M_1 and M_2 over the alphabet $\Sigma = \{a, b\}$ be given:





Check whether both deterministic finite automata are equivalent. Two finite automata are equivalent, if the following holds:

$$T(M_1) = T(M_2)\,.$$

First of all construct the minimal automata of M_1 and M_2 by means of the algorithm presented in the lecture (4 points) and argue with the aid of the minimal automata, why M_1 and M_2 are (not) equivalent (2 points).

Indicate all intermediate steps of the algorithm. Submissions without intermediate steps do *not* achieve points!

Note: Minimal automata for a language are unique up to the naming of states.

[1] *Options to submit your solutions:* Letterbox next to LF 259 (Campus Duisburg) or via Moodle `https://moodle.uni-due.de/course/view.php?id=15777`

Exercise 29 *Regular languages and Myhill-Nerode equivalence* (6 points)

Show by means of the Myhill-Nerode Theorem, whether the following languages over the alphabet $\Sigma = \{a, b\}$ are regular or not:

(a) $L_1 = \{w \in \Sigma^* \mid \#_a(w) = \#_b(w)\}$ (2 p)

(b) $L_2 = \{a^{2n} \mid n \in \mathbb{N}_0\}$ (2 p)

(c) $L_3 = \{a^n b^m \mid n, m \in \mathbb{N}_0 \land 1 \leq n \leq m\}$ (2 p)

Exercise 30 *Decidability* (8 points)

Prove that the following problems are decidable by giving an algorithm for each problem that solves it. Assume that each language is given by a deterministic finite automaton. Justify the correctness of your algorithms!

(a) Let L_1, L_2 be regular languages. Does the intersection of L_1 and L_2 contain infinitely many words? (2.5 p)

(b) Let L_1, L_2 be regular languages. Is the union of L_1 and L_2 equal to the set of all words? (2.5 p)

(c) Let L_1, L_2 be regular languages over the alphabet Σ. Is L_2 the complement of L_1? (3 p)

Note: Your algorithms can use the algorithms presented in the lecture.

(In total, there are **20** points in this exercise sheet.)