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Open-Minded

Output Analysis for Simulation Model

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Rechnergestützte Netzanalysen
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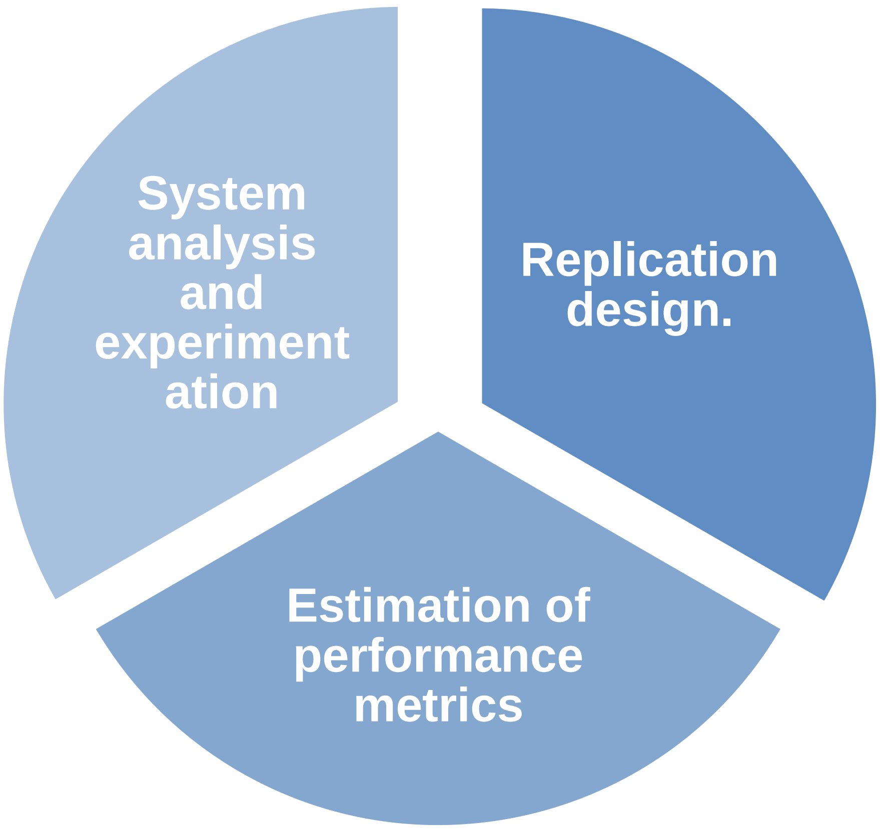
Introduction

- Output analysis is the modeling stage concerned with designing replications, computing statistics from them and presenting them in textual or graphical format.

- Output analysis focuses on the analysis of simulation results (output statistics).

- It provides the main value-added of the simulation enterprise by trying to understand system behavior and generate predictions for it.

Output Analysis



The main issues addressed by output analysis follow

Output Analysis

- **Replication design.** A good design of simulation replications allows the analyst to obtain the most statistical information from simulation runs for the least computational cost. In particular, we seek to minimize the number of replications and their length, and still obtain reliable statistics.
- **Estimation of performance metrics**. Replication statistics provide the data for computing point estimates and confidence intervals for system parameters of interest. Critical estimation issues are the size of the sample to be collected and the independence of observations used to compute statistics, particularly confidence intervals.
- **System analysis and experimentation.** Statistical estimates are used in turn to understand system behavior and generate performance predictions under various scenarios, such as different input parameters (parametric analysis), scenarios of operation, and so on. Experimentation with alternative system designs can elucidate their relative merits and highlight design trade-offs.

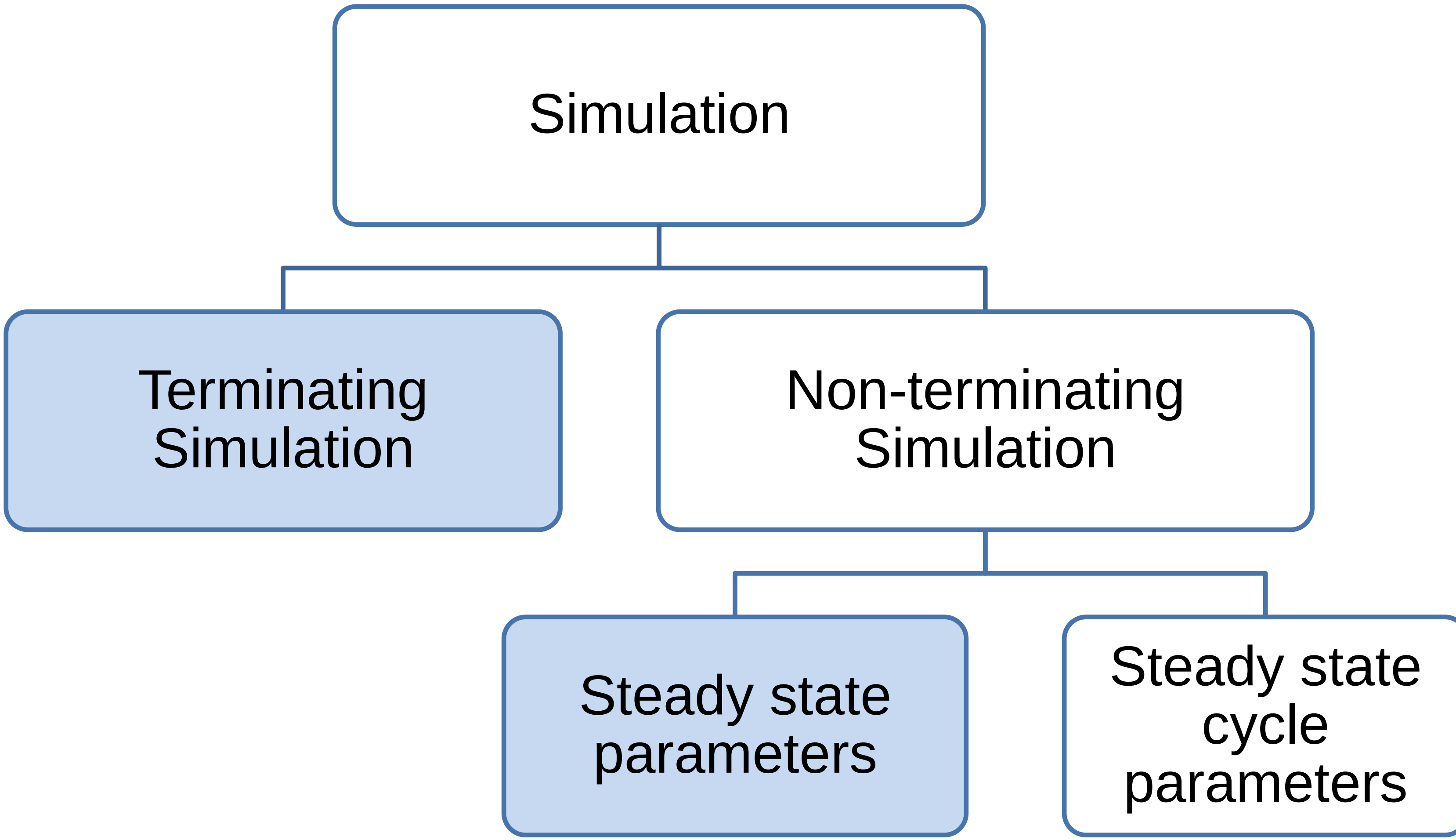
Output

- Bank with 5 tellers, one FIFO queue, open 9am-5pm
- Inter-arrivals: expo (mean = 1 min.), service times: expo (mean =4 min.)
- Measures from 10 runs (replications):

Replication	Number served	Finish time (hours)	Average delay in queue (minutes)	Average queue length	Proportion of customers delayed < 5 minutes
1	484	8.12	1.53	1.52	0.917
2	475	8.14	1.66	1.62	0.916
3	484	8.19	1.24	1.23	0.952
4	483	8.03	2.34	2.34	0.822
5	455	8.03	2.00	1.89	0.840
6	461	8.32	1.69	1.56	0.866
7	451	8.09	2.69	2.50	0.783
8	486	8.19	2.86	2.83	0.782
9	502	8.15	1.70	1.74	0.873
10	475	8.24	2.60	2.50	0.779

Inference:
There's variation across runs

Types of Simulation with regard to Output Analysis



Terminating Simulations

- **Terminating Simulation** is one that runs for some duration of time T_E, where E is a specified event (or set of events) which stops the simulation, Such simulation starts at time 0 under specified initial conditions and stop at the stopping time $T_{E.}$

- The different runs use independent random numbers and same initialization rule.

- The event E often occurs at a point when the system is cleaned out.

- Initial condition for a terminating simulation generally affect the desired measure of performance, these conditions should berepresentative of those actual system.

Examples

- A retail / commercial establishment e.g. Bank, has working hours 9 to 5, the object is tomeasure the quality of customer service in this specified 8 hours. Here the inital condition is number of customers present at time E(t)=0 (which is to be specified).

- An aerospace manufacturer recieves a contract to produce 100 airplanes, which must be delieverd within 18 months. The would like to simulate various manufacturing configuration to see which can meet the delivery at least cost.

- A company that sells a single product would like to deicde how many items to have in inventory during 120 month planning horizon. Given some inital inventory level, the object is determine how much order each month so as to minimize the expected averaging cost per month of inventory system.

Point Estimation

- **Point estimation** : Suppose the replication collects a sequence of n variates, (X1, . . . , Xn) yielding a corresponding sample of observations, (x1, . . . , xn). The estimator for the mean value parameter is the sample mean:

$$\bar{X} = \frac{1}{n}\sum_{j=1}^{n} X_j.$$

- Suppose we are interested in a continuous-time stochastic process (X_t : 0 < t< T) over some time interval [0, T], yielding a corresponding sample of observations, (x_t : 0<t<T). The estimator for the mean value parameter in this case is the point estimator

$$\bar{X} = \frac{1}{T}\int_0^T X_t \, dt$$

Statistical Analysis for *Terminating Simulations*

$$\bar{Y}(n) = \frac{\sum_{i=1}^{n} Y_i}{n}$$

unbiased estimator of μ

$$S^2(n) = \frac{\sum_{i=1}^{n} [Y_i - \bar{Y}(n)]^2}{n-1}$$

Sample Variance

$$\bar{Y}(n) \pm t_{n-1,1-\alpha/2} \frac{S(n)}{\sqrt{n}}$$

Confidence interval for μ

Terminating Simulations Example

Replication	Number served	Finish time (hours)	Average delay in queue (minutes)	Average queue length	Proportion of customers delayed < 5 minutes
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- Calculate the point estimate for average delay.
- Calculate expected average delay with an approximate 90% confidence interval.

T-table

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725

Non terminating Simulation
Steady-state parameters

Welch Method

Batch Mean Method

Replication/ Deletion Method

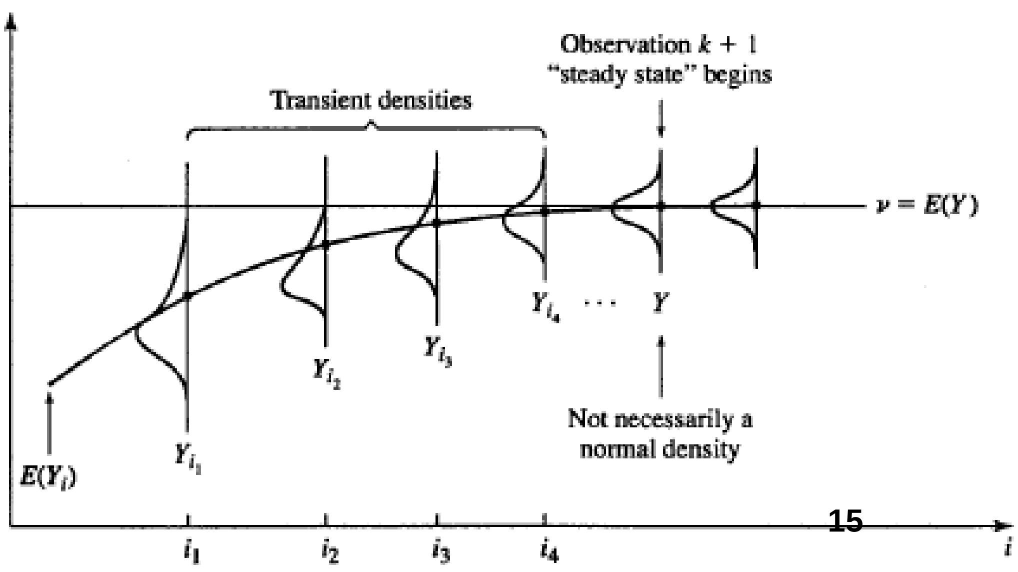
Non-Terminating Simulation

- **Non-Terminating Simulation** is a system that runs continuously, or at least a very long period of time, It starts at simulation time 0 under initial conditions defined by the analyst and runs for some analyst-defined period of time T_E, A steady-state simulation is a simulation whose objective is to study long-run behavior of a non-terminating system,
- A manufacturing system working 5 days a week and 16 hours, It is desired to estimate long-run production levels and production efficiencies for a relatively long period of 10 shifts.

- It should ebmentioned that real system do not have steady state distribution, since the characteristics of the system change over time. While in a model , it may steady state as the characteristics are assumed to be unchanged over time.

Transient and Steady-state Behaviour

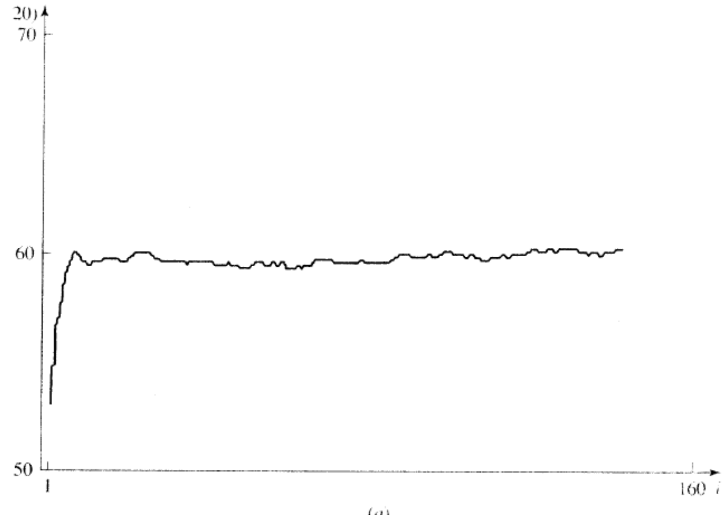
- **Transient State** : Its the output process for the initial condition *I*, at discrete time i.
- It shows the density of random variable Y vary from one replication to another.
- **Steady state:** It shows the distribution of the random variable from a a particualr point will be approximately same as each other.
- It doesnot depend on initial conditions I.



Initialiasation Bias

- Initialization bias: it's condition to choose initial values for the state variables that are not representative of the steady state distribution.

- Divide the simulation into two phases, warm-up phase and steady state phase. Data collection doesn't start until the simulation passes the warm-up phase.



Initialiasation Bias

- **Example:** let's say that you're modeling a factory making washing machines. When your simulation starts, the simplest initial state is for the factory to have no work-in-progress - that is, the factory has no washing machine parts in any part of the process. As the simulation runs, you introduce parts, which progress through the simulation until completed washing machines are shipped.
- Let's say that you count the number of washing machines shipped by the simulation. You can then estimate the mean number of washing machines shipped per hour as follows:
- mean hourly throughput = (number of washing machines shipped) / (simulation time in hours)
- But, we're going to see an initialization bias, because it takes time for the simulation to complete the first washing machine; we might not ship any washing machines for some time.

The Problem of Initial transient

- To over come the inital transient problem, warming up the model or inital data deletion is used.
- The idea is to delete some number of obeservation from the beginning of the run.
- The remaining observation are used to estimate.
- This represneted by $\bar{Y}(m,l)$ where m is n replication of lenght m and l is the warm up lenght.

Statistical Analysis for *Steady-state parameters (welch method)*

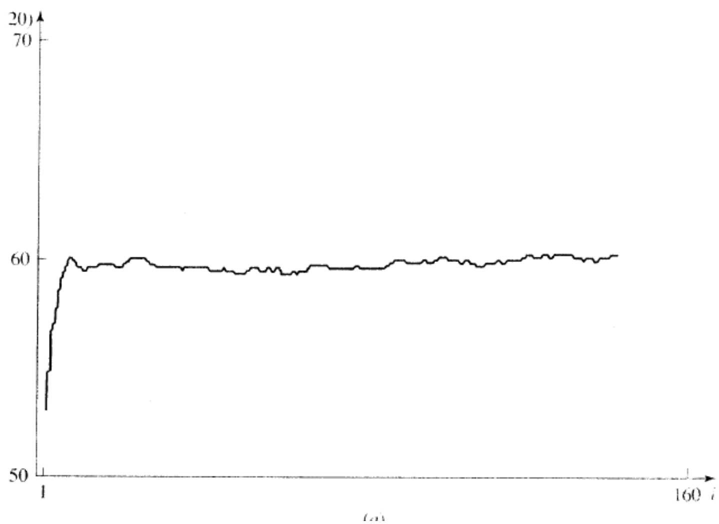
- Make n replications of the model (n≥5), each of length m, where m is large. Let be the ith observation from the jth replication (j = 1, 2, …, n; i =1, 2, …, m).
- for i =1, 2, …, m, let $\bar{Y}_i = \sum_{j=1}^{n} Y_{ji} \Big/ n$
- To smooth out the high frequency oscillations in $\bar{Y}_1, \bar{Y}_2$ define the moving average $\bar{Y}i(w)$ as follows (*w* is the window and is a positive integer such that w ≤ m/4

$$\bar{Y}_i(w) = \begin{cases} \dfrac{\sum_{s=-w}^{w} \bar{Y}_{i+s}}{2w+1} & \text{if } i = w+1, \ldots, m-w \\ \dfrac{\sum_{s=-(i-1)}^{i-1} \bar{Y}_{i+s}}{2i-1} & \text{if } i = 1, \ldots, m \end{cases}$$

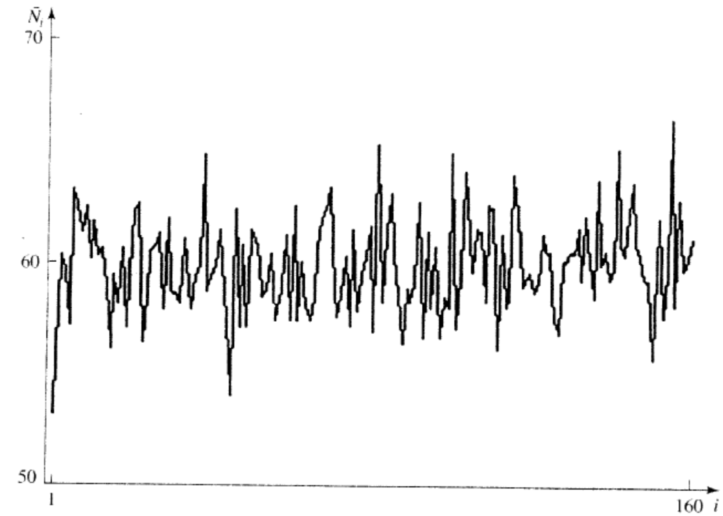
- Plot Yi(w) for i = 1, 2, …, m - w and choose I to be that value of i beyond which $Y_1(w)$, $Y_2(w)$, … appears to have converged

Welch’s method

Moving Average for Removing the Initial Bias



With initial bias



Averaged process for hourly throughputs in a small factory

Welch Method

- Recommendations on choosing n(replications), m, and w
- Initially, make n = 5 or 10 replications with m as large as practical
- m should be much larger than l and large enough to allow infrequent events (e.g.breakdowns) to occur a reasonable number of times
- Plot Yi(w) for several values of the window w and choose the smallest value of w for which the corresponding plot is “reasonably smooth”
- Use the plot to determine *l*
- If no value of w in step 3 of Welch’s procedure is satisfactory, make 5 or 10 additional runs of length m. Repeat step 2 using all available replications

Replication/Deletion Approach

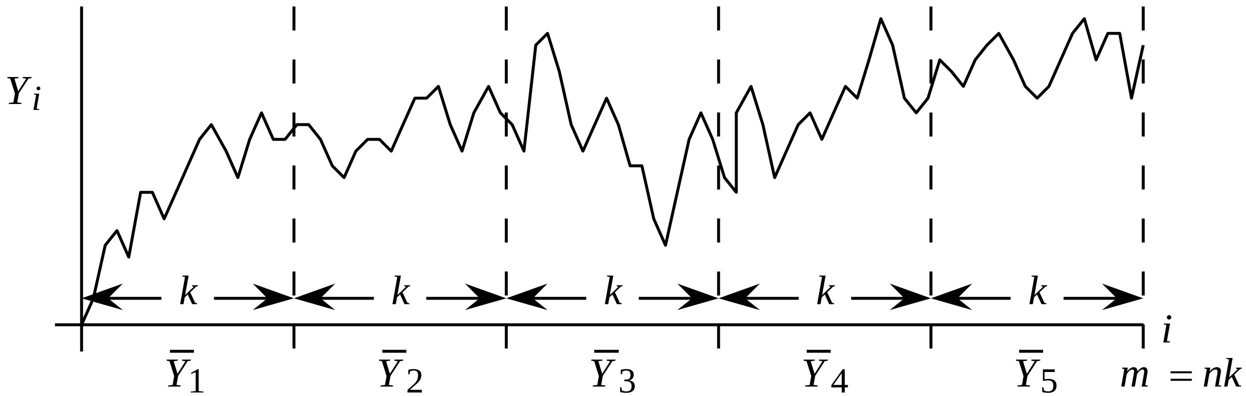
- First determine initialization bias and cutoff m using Welch method.
- Run k independent replications each of length n observations, and If possible, make use of runs from previous bias determination phase.
- Discard m observations from each replication.
- Calculate the average of each replication.
- Calculate the mean of replications.
- Calculate the confidence interval.

Batch mean method

- One disadvantage of the replication deletion method is that data must be deleted on each replication and cause lost of information, However, a single replication design has the problem to compute the standard error of the sample mean.
- The method of batch mean attempts to solve the problem by dividing the output data from one replication (after appropriate deletion) into a few large batches, and then treating the means of these batches as if they were independent.

Batch means

- Divide a run of length m into n adjacent “batches” of length k where m = nk.
- Let Ȳj be the sample or (batch) mean of the jth batch.



- The grand sample mean is computed as

Grand mean ⇨ $$\bar{\bar{Y}} = \frac{\sum_{j=1}^{n} \bar{Y}_j}{n} = \frac{\sum_{i=1}^{m} Y_i}{m}$$

Batch means

- The sample variance is computed as

$$S_{\bar{Y}}^2(n) = \frac{\sum_{j=1}^{n} (\bar{Y}_j - \bar{\bar{Y}})^2}{n-1}$$

- The approximate 100(1 – a)% confidence interval is

$$\bar{\bar{Y}} \pm t_{n-1,1-\alpha/2} \frac{S_{\bar{Y}}(n)}{\sqrt{n}}$$

Batch mean

- **Advantages:**
- Simple (relatively)
- Often works fairly well (in terms of coverage)

- **Problems with batch means**:
- Choose batches big enough so that $\bar{Y}(k)j$'s are approximately uncorrelated.
- Otherwise, $S^2{}_n Y$ can be biased (usually low) for $Var(\bar{Y}(k)j)$, causing under coverage
- How to choose batch size k? Equivalently, how many batches n?
- It may never to have more than n = 20 or 30 batches
- Because: Due to autocorrelation, splitting run into a larger number of smaller batches, while increasing degrees of freedom, degrades the quality (variability) of each individual batch

Summary

- The goal was to understand method for statistical analysis of simulation output data.
- Distinguish the Simulation as Terminating and non terminating.
- Inital Bias, welch method Batch mean and replication deletion method.
- A care must be taken to use appropriate statistical methods, since simulation output data are usually non-stationary, auto correlated.
- A failure to recognize, deal with randomness in simulation output can lead to serious errors, misinterpretation, bad decisions.

References

- **Simulation and Modelling Analysis** (Law , Kelton)

- **Simulation Modeling and Analysis with ARENA** (Tayfur Altiok and Benjamin Melamed)

Numerical part will be solved in Numerical example lecture

THANK YOU