# Manifestation of a complex edge excitation structure in the quantum Hall regime at high fractional filling factors

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We experimentally study a transport across the integer incompressible strip with local filling factor $\nu_c=1$ at the sample edge at high imbalances across this strip. The bulk is in the quantum Hall state at the integer ($\nu=2,3$) or high fractional ($\nu=5/3,4/3$) filling factors. Unlike the integer case, for the fractional bulk filling factors, we find a lack of the full equilibration across the edge even in the situation where no potential barrier survives in the integer incompressible strip with $\nu_c=1$. We interpret this result as the manifestation of complicated edge excitation structure at high fractional filling factors.

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## I. INTRODUCTION

An interest to the fractional quantum Hall effect (FQHE) is determined by its many-particle nature,[1] which gives rise to unique physical properties. Particularly, collective gapless edge excitation modes, unlike usual edge magnetoplasmons, becomes to be very sensitive to the interaction. They thus can be described by the one-dimensional strongly-interacting Luttinger liquid picture.[2–4]

For the smooth edge potential profile, it is well established now for integer[5] and fractional[6–8] quantum Hall-effect regimes that there is a structure of strips of compressible and incompressible electron liquid at the sample edge, see Fig. 1. For the FQHE regime (at fractional bulk filling factor $\nu$), the strip structure emerges,[9,10] if the edge potential variation is wider than five to six magnetic lengths, i.e., for most of real edge potentials. Every incompressible strip can be characterized by a local filling factor $\nu_c<\nu$, which corresponds to the integer or fractional quantum Hall state in the strip. The number of strips with integer $\nu_c$ equals to the number of the filled Landau levels in the bulk. The number of the strips with fractional $\nu_c$ is determined by the FQHE hierarchical structure,[11,12] the magnetic field, and the sample quality.

Gapless collective modes are predicted[9] to exist both at the edges of the every incompressible strip with fractional local filling factor $\nu_c$ and at the edge of the bulk incompressible state. Physically, they can be understood as variations of the strip's borders, while moving along the strip. The structure of the edge modes follows the structure of the ground state at $\nu_c$: there should be several excitation branches for the bulk filling factors $\nu_c$ that are not of the principal Laughlin sequence.[2,13]

Experimentally, it is hardly possible to distinguish between excitation of multiple branches of a single strip and the simultaneous excitation of edge modes from different strips[14–17] because of the narrowness of the fractional incompressible strips. For this reason, experimental investigation of the excitation structure can better be performed at the edge of the bulk incompressible state at the high fractional filling factor such as $\nu=4/3,5/3$. In this case, for a sample of low carrier density and medium mobility, it is possible to have only one incompressible strip with integer $\nu=1$ at the sample edge. A special experimental environment[18] allows to eliminate the potential jump in the integer incompressible strip, so the transport across the sample edge should be sensitive to the edge properties of the bulk incompressible state only.

Here, we experimentally study a transport across the integer incompressible strip with local filling factor $\nu_c=1$ at the sample edge at high imbalances across this strip. The bulk is in the quantum Hall state at the integer ($\nu=2,3$) or high fractional ($\nu=5/3,4/3$) filling factors. Unlike the integer case, for the fractional bulk filling factors, we find a lack of the full equilibration across the edge, even in the situation, where no potential barrier survives in the integer incompressible strip with $\nu_c=1$. We interpret this result as the manifestation of several gapless edge excitation branches for $\nu=4/3,5/3$ high fractional quantum Hall states.

## II. SAMPLES AND TECHNIQUE

The samples are fabricated from two GaAs/AlGaAs heterostructures with different carrier concentrations and mobilities grown by molecular-beam epitaxy. One of them (A)

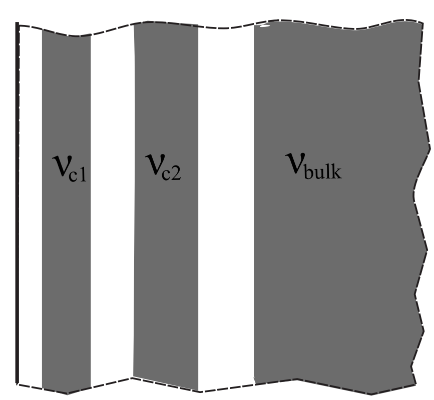


FIG. 1. Schematic of the smooth sample edge in the quantum Hall regime. Light gray areas are the incompressible strips at local filling factors $\nu_{c1}, \nu_{c2}$ and the bulk incompressible state at the filling factor $\nu_{\text{bulk}}$. White areas are the compressible regions.

 

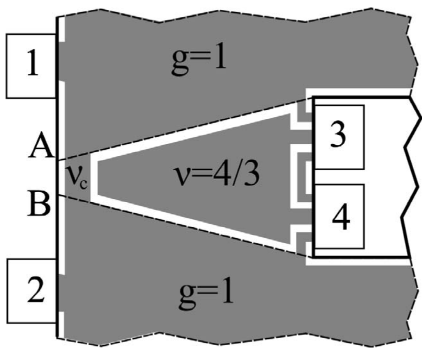


FIG. 2. Schematic of the active sample area. The etched mesa edges are shown by solid lines, the dashed lines represent the split-gate edges. The gate-gap region at the outer mesa edge is denoted as AB. Light gray areas are the incompressible regions at filling factors $\nu$ (in the bulk) and $\nu_c = g < \nu$ (under the gate and the incompressible strip at the mesa edge). Compressible regions (white) are at the electrochemical potentials of the corresponding Ohmic contacts, denoted by bars with numbers.

contains a two dimensional electron gas (2DEG) located 210 nm below the surface. The mobility at 4K is $1.93 \times 10^6$ cm$^2$/V s and the carrier density $1.61 \times 10^{11}$ cm$^{-2}$. For heterostructure B, the corresponding parameters are 70 nm, 800 000 cm$^2$/V s, and $3.7 \times 10^{11}$ cm$^{-2}$. FQHE states are not achievable for the wafer B. It is used to obtain results for integer filling factors at higher magnetic fields because of the high carrier concentration. Standard magnetoresistance and magnetocapacitance measurements were performed to characterize the electron system in the ungated area and under the gate.

The samples are patterned in the quasi-Corbino sample geometry.[18] Each sample has an etched region inside, providing a topologically independent inner mesa edge (Corbino topology). A split-gate is used to connect these two edges in a controllable way (see Fig. 2). A structure of compressible and incompressible strips is present at every edge at the bulk filling factor $\nu$ (see Fig. 2). An incompressible quantum Hall state under the gate (with filling factor $g < \nu$) is chosen to coincide with one of the incompressible strips at the outer mesa edge (with local filling factor $\nu_c$), $g = \nu_c$. In these conditions, some of the compressible strips (white in Fig. 2) are redirected from the inner to the outer mesa edge along the split gate. Compressible strips are at the electrochemical potentials of the corresponding Ohmic contacts.[5] The gap in the split gate at the outer edge (the gate-gap region, denoted as AB in the figure) has no Ohmic contacts inside and is much narrower ($L_{AB} = 5$ $\mu$m in the present experiment) than at the inner one (about 1 mm). As a result, applying a voltage between Ohmic contacts at outer and inner edges leads to the electrochemical potential imbalance across the incompressible strip at local filling factor $\nu_c = g$ in the gate-gap region AB at the outer edge (see Fig. 2).

For the proposed experiment, it is crucial to establish the systematics of the incompressible strips at the outer sample edge. For a smooth edge potential, the decrease of the electron concentration to the sample edge is similar to decreasing the electron concentration in the bulk by the gate potential. By lowering the gate voltage, different quantum Hall states

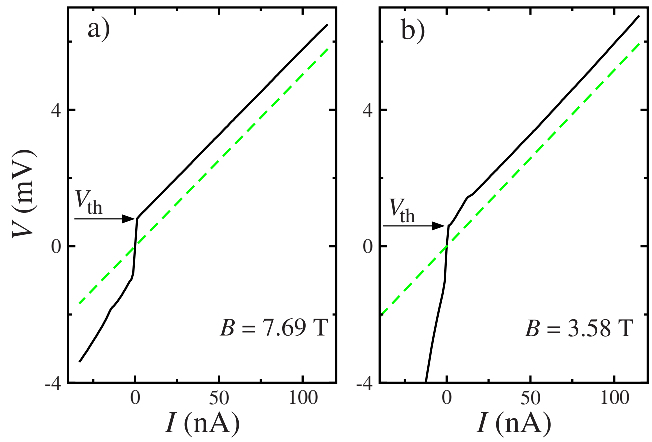


FIG. 3. (Color online) Examples of $I$-$V$ curves (solid) for the filling factor combinations $\nu = 2, g = 1$ for samples with different electron concentrations [(a) and (b) are for wafers B and A, correspondingly]. Calculated equilibrium $I$-$V$'s are shown by the dashed lines ($R_{eq} = 2h/e^2$). The positive branch ($V > V_{th}$) of the experimental curve is linear and parallel to the equilibrium line in both cases. The threshold voltage $V_{th}$ is also denoted. Normal magnetic fields equal to (a) 7.69 T and (b) 3.58 T.

are arising under the gate. Thus, in magnetocapacitance measurements, we obtain not only gate voltage values, corresponding to integer or fractional filling factors $g$ under the gate, but also verify the structure of incompressible strips at the sample edge. We can confirm in this way that there is only $\nu_c = 1$ incompressible strip for bulk filling factors $\nu = 2, 5/3, 4/3$ and there are two integer $\nu_c = 1, 2$ strips for $\nu = 3$. This fact is because of moderate mobility of 2DEG and low magnetic fields, in which $\nu = 5/3, 4/3$ are realized for the wafer A.

In the present paper, we study $I$-$V$ curves in four-point configuration, by applying dc current between a pair of outer and inner contacts and measuring dc voltage between another pair of inner and outer contacts. Four-point configuration is used to eliminate possible contact influence on the experimental traces, allowing the most accurate measurements (contact resistance is typically below 100 $\Omega$ for the present samples). The contact behavior is still tested separately by two-point magnetoresistance measurements to exclude the possibility of Corbino-type or nonlinear contacts. All measurements are performed in a dilution refrigerator with base temperature of 30 mK, equipped with a superconducting solenoid. The results, presented here, are independent of the cooling cycle.

## III. EXPERIMENTAL RESULTS

Typical $I$-$V$ curves for transport across the integer incompressible strip $\nu_c = 1$ are presented in Figs. 3 and 4 for the integer and fractional bulk filling factors $\nu$. The experimental $I$-$V$ curve is strongly nonlinear and asymmetric: the positive branch starts from the finite threshold voltage $V_{th}$ and is linear after the threshold; the negative branch continuously goes from zero and is strongly nonlinear. The linear behavior of the positive branch above the threshold is demonstrated in

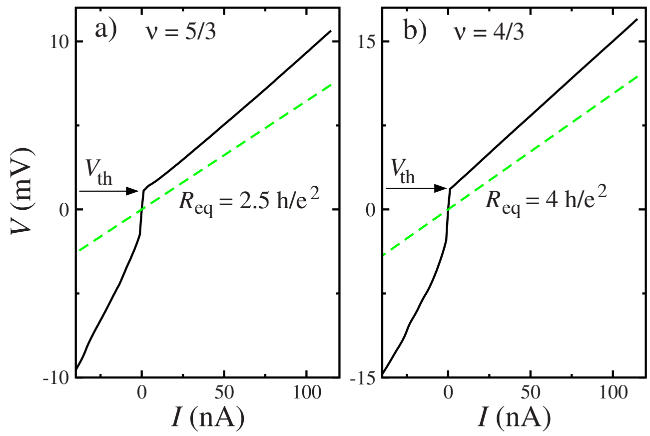


FIG. 4. (Color online) Examples of $I$-$V$ curves (solid) for the filling factor combinations (a) $\nu=5/3, g=1$ and (b) $\nu=4/3, g=1$ for wafer A. Calculated equilibrium $I$-$V$'s are shown by the dashed lines. The positive branch ($V>V_{th}$) of the experimental curve is linear and has a higher slope than the equilibrium line, $R>R_{eq}$. The threshold voltage $V_{th}$ is also denoted. Normal magnetic field equals to (a) 4.29 T and (b) 5.36 T.

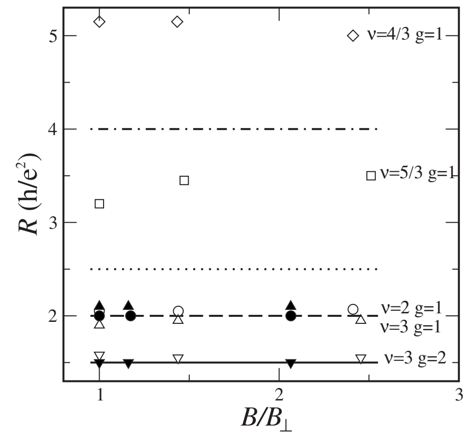


FIG. 5. Slopes $R$ of the linear positive branch of the experimental $I$-$V$s for different filling factor combinations as a function of the total magnetic field. Open symbols are for the wafer A: $\nu=3, g=2$ (down triangles, normal field is $B_\perp=2.38$ T), $\nu=3, g=1$ (up triangles, $B_\perp=2.38$ T), $\nu=2, g=1$ (circles, $B_\perp=3.58$ T), $\nu=5/3, g=1$ (squares, $B_\perp=4.29$ T), $\nu=4/3, g=1$ (diamonds, $B_\perp=5.36$ T). Filled symbols are for the corresponding $\nu$ and $g$ for wafer B ($B_\perp=5.18$ T for $\nu=3$, 7.69 T for $\nu=2$). Lines indicate the equilibrium slopes $R_{eq}$: solid line is for $\nu=3, g=2$, dash is for $\nu=2, g=1$ and $\nu=3, g=1$, dotted one is for $\nu=5/3, g=1$, dash-dot is for $\nu=4/3, g=1$.

a wide voltage range in Figs. 3 and 4. A linear fit allows to determine the slope $R$ of the positive branch for $V>V_{th}$ with high accuracy.

This behavior is characteristic[18] for the transport across the integer incompressible strips at small $L_{AB}$ (see Fig. 2), in comparison to the equilibration length[20] $l_{eq}$, $L_{AB} \ll l_{eq}$. In the opposite case, for $L_{AB}>l_{eq}$, the $I$-$V$ curve is linear[18] and coincides with the equilibrium one. Its slope can be determined by the relation:[6,19]

$$R_{eq} = \frac{h}{e^2}\frac{\nu}{g(\nu-g)}. \qquad (1)$$

In Figs. 3 and 4, the main difference between the integer and fractional bulk filling factors is present. In the integer case, the slope of the positive branch above the threshold is strictly equals to the equilibrium one, $R=R_{eq}$, while it is much higher than the equilibrium for fractional $\nu=5/3, 4/3$: $R>R_{eq}$.

Experimental slopes $R$ are presented in Fig. 5 for different integer and fractional filling factor combinations $\nu, g$. The data are obtained in normal and tilted magnetic fields. Each data set is presented as a function of the total magnetic field at the fixed perpendicular one. Equilibrium slopes, calculated from Eq. (1), are denoted by lines.

The experimental $R$ values are independent of the in-plane magnetic fields for integer filling factor combinations (see Fig. 5). They coincide with the calculated equilibrium lines very well.[22] This behavior is also supported by the results for the sample B (filled symbols in Fig. 5), obtained at twice higher normal magnetic fields. We can conclude that the present behavior is independent of the magnetic-field value and is specific to the filling factors only. The data are temperature independent below 1K.

The experimental situation is more intriguing for the fractional filling factor combinations $\nu=5/3, g=1$ and $\nu=4/3, g=1$. Slopes of the positive branch are much higher than the calculated equilibrium values (see Fig. 5). This fact is also demonstrated directly in Fig. 4. It can also be seen from Fig. 5 that there is a weak dependence of $R$ on the in-plane magnetic field for fractional bulk filling factors. The dependence is different in sign for $\nu=4/3$ and $\nu=5/3$ and is more pronounced for the latter one. The data are temperature independent below 0.4 K. At higher temperatures $R$, values are diminishing with increasing the temperature.

## IV. DISCUSSION

Let us start the discussion from the simplest situation of $\nu=2, g=1$. Equilibration takes place between two spin-split sublevels of the first Landau level[21] [see Fig. 6(a), where the energy diagram is shown]. It is well known[20,21] that the equilibration length $l_{eq}$ is determined by both the potential barrier at local filling factor $\nu_c=1$ and the spin-flip rate. This length is about 1 mm at low imbalances,[20] which provides high differential resistance $R \sim l_{eq}/L_{AB} \gg R_{eq}$ at $V<V_{th}$. A high positive imbalance is diminishing the potential barrier at $\nu_c=1$, so that the flat-band situation is achieved at $-eV=-eV_{th}=-\Delta_c$ [see Fig. 6(b)]. Thus, there is no potential barrier between compressible strips, allowing electron diffusion along the level for $V \geq V_{th}$. Spin-flip can easily be obtained in this case by photon emission.[21] The equilibration length drops below $L_{AB}=5$ $\mu$m for these reasons,[15] allowing the full equilibration $R=R_{eq}$ across the sample edge. The corresponding resistance slope $R_{eq}$ can easily be calculated,[6,19] see Eq. (1). At negative voltages, the potential profile at $\nu_c=1$ is increasing, giving rise to the complicated tunnel branch of the $I$-$V$ curve.

This picture is verified by the observation of the equilibrium slope of the positive $I$-$V$ branch for $\nu=2$ and $g=1$ (see

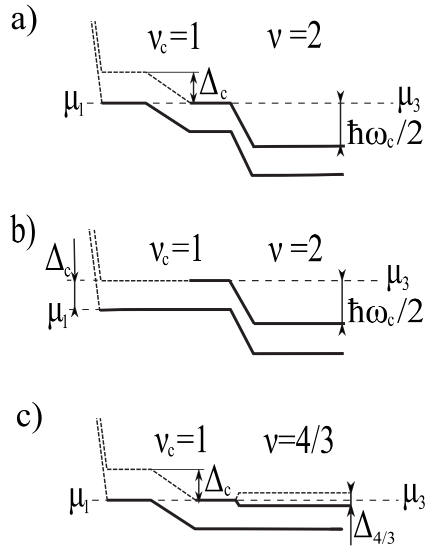


FIG. 6. Schematic of the energy levels in the active sample area. Solid lines represent the filled electron states. Dashed lines are for the empty ones. $\Delta_c$ is the potential jump in the $\nu_c=1$ incompressible strip. Pinning of the Landau sublevels to the Fermi level (shot-dash) is shown in the compressible regions at electrochemical potentials $\mu_1$ and $\mu_3$. (a) Integer filling factors $\nu=2,g=1$. Equilibrium situation $\mu_3=\mu_1$, no voltage $V$ is applied to Ohmic contacts 1 and 3. (b) Integer filling factors $\nu=2,g=1$. Flat-band conditions for $-eV=-eV_{\text{th}}=\mu_1-\mu_3=-\Delta$ ($V>0$, $e$ is the absolute value of the electron charge). (c) Fractional filling factors $\nu=4/3,g=1$. Equilibrium situation $\mu_3=\mu_1$, no voltage $V$ is applied to Ohmic contacts 1 and 3.

Figs. 3 and 5). It is also valid[18,22] for any integer filling factors, e.g., for $\nu=3$ and $g=1,2$. We cannot expect any influence of the in-plane magnetic field on the transport in the flat-band situation, at $V>V_{\text{th}}$, as it is also confirmed in the present experiment.

The edge structure is very close to ones discussed above for the filling factor combinations $\nu=4/3$, $g=1$ and $\nu=5/3$, $g=1$ [see Fig. 6(c)], except for the energy gap at the Fermi level in the bulk. It is a many-particle fractional energy gap[1] in this case. As it is tested from the magnetocapacitance measurements, there is only one incompressible strip with $\nu_c=1$ at the sample edge, even for $\nu=5/3$ in the bulk. It seems to be a result of the moderate mobility, that does not allow fractional strips in low magnetic fields. Despite the similarity of the edge structure to the $\nu=2$ case, experimental slopes of the positive branch are always significantly higher than can be expected from Eq. (1) for fractional filling factors in the bulk.

Experimental slopes, exceeding the equilibrium ones, indicate partial equilibration for $V>V_{\text{th}}$. In general, it can be caused by two reasons: (i) the transport across the $\nu_c=1$ incompressible strip and the adjacent compressible one is different; (ii) the edge of the bulk FQHE state is responsible for the partial equilibration.

The first reason cannot explain our results. The structure of both the $\nu_c=1$ incompressible strip and the adjacent compressible one is the same for the integer $\nu=2$ and the fractional $\nu=4/3,5/3$. We observe a full equilibration as well in lower magnetic fields (at $\nu=2$ for the sample A), as in approximately twice higher ones (at $\nu=2$ for the sample B). Thus, it is the edge of the bulk FQHE state at $\nu=4/3,5/3$ that is responsible for the partial equilibration.

The edge of the FQHE incompressible state is characterized by the gapless collective excitation modes.[2,9] A charge transfer into the edge of the FQHE state is accomplished by their excitation,[2–4,9,17] so they govern the charge redistribution across the FQHE edge.[9,17] There should be several[2,13] excitation modes for high fractional filling factors $\nu>1$. In the case of their common excitation, however, the equilibrium resistance is given by the simple relation [Eq. (1)] (see, e.g., Refs. 23 and 24). Thus, partial equilibration ($R>R_{\text{eq}}$) in the flat-band conditions in our experiment can be achieved only for selective excitation of the gapless modes (*cp*. Ref. 23). The linear behavior of the *I*-*V* curve at $V>V_{\text{th}}$ can be easily understood in this case: in the flat-band conditions, electrons are always leaving the fractional bulk incompressible state at the same energy, leading to the same excitation schema at any applied voltage. For this reasons, our experimental result can be regarded as the manifestation of several gapless excitation branches for $\nu=4/3,5/3$ high fractional quantum Hall states.

This conclusion is also supported by the small but different dependence of the equilibration on the in-plane magnetic-field component. The $\nu=4/3$ FQHE state can be regarded as the quasielectron state at completely filled first Landau level, while the $\nu=5/3$ state belongs to the quasihole state at *two* filled Landau levels.[12] The difference in the ground-state structure leads to the difference in the structure of edge excitations. That could be responsible for the different in-plane field dependencies.

## V. CONCLUSION

In summary, we experimentally study transport across the integer incompressible strip with local filling factor $\nu_c=1$ at the sample edge at high imbalances across this strip. The bulk is in the quantum Hall state at the integer ($\nu=2,3$) or high fractional ($\nu=5/3,4/3$) filling factors. Unlike the integer case, for the fractional bulk filling factors, we find a lack of the full equilibration across the edge even in the situation, where no potential barrier survives in the integer incompressible strip with $\nu_c=1$. We interpret this result as the manifestation of several gapless edge excitation branches for $\nu=4/3,5/3$ high fractional quantum Hall states.

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