Microstructural behaviour of transversal isotropic material

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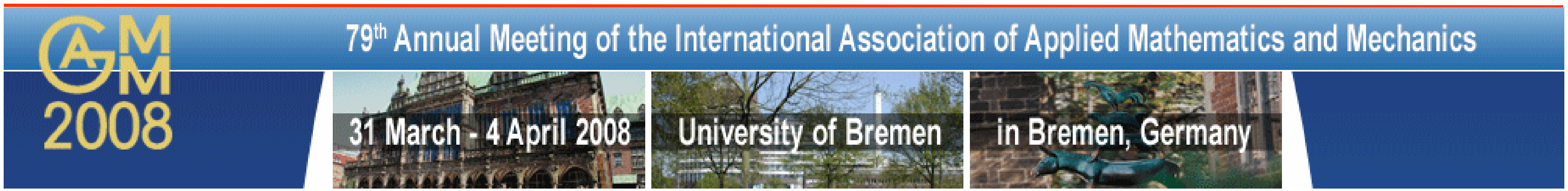
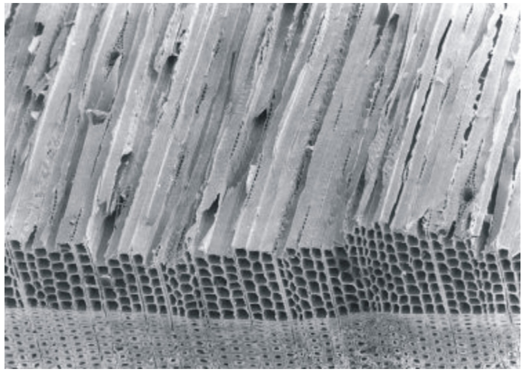


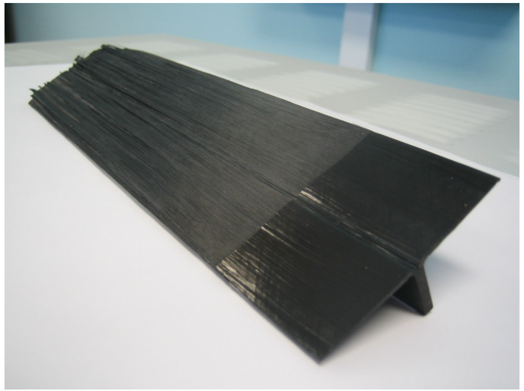
- Introduction
- Classical results of a transversal isotropic tension test
- Experimental results of a discrete reinforced model
- Results of a discrete reinforced finite element model
- A Cosserat model and its results
- Conclusions

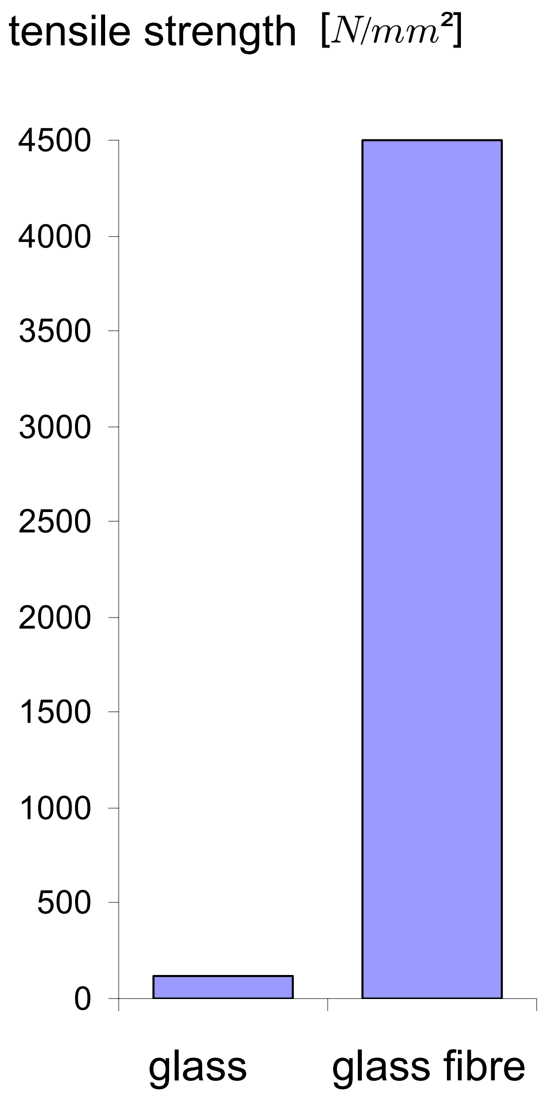
transversal isotropic materials

naturally grown:
e.g. wood

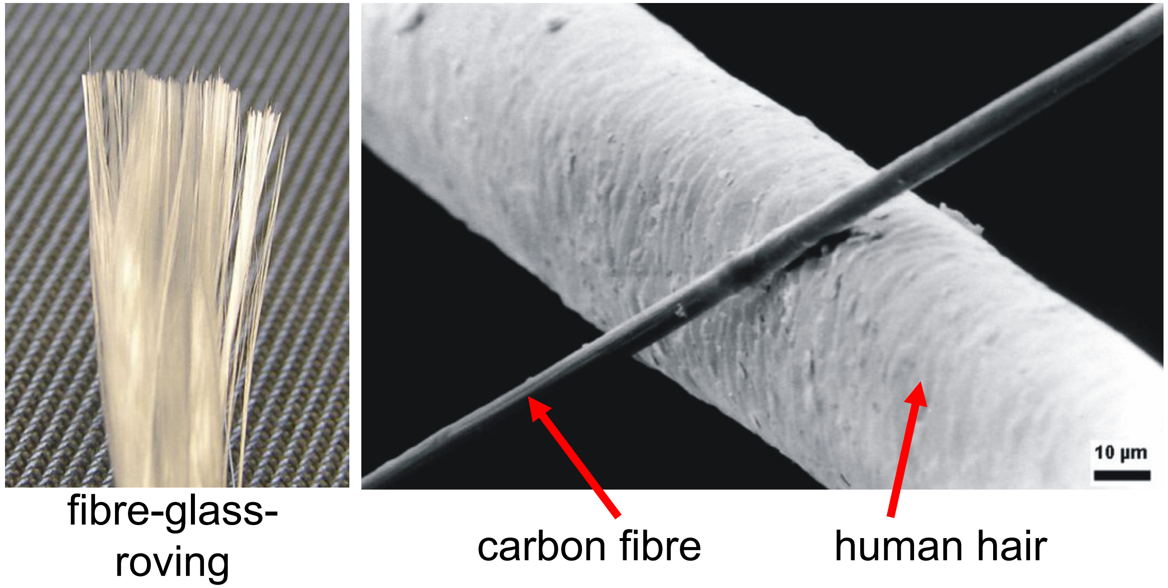


technically produced:
e.g. uniaxial-fiber-reinforced-plastics

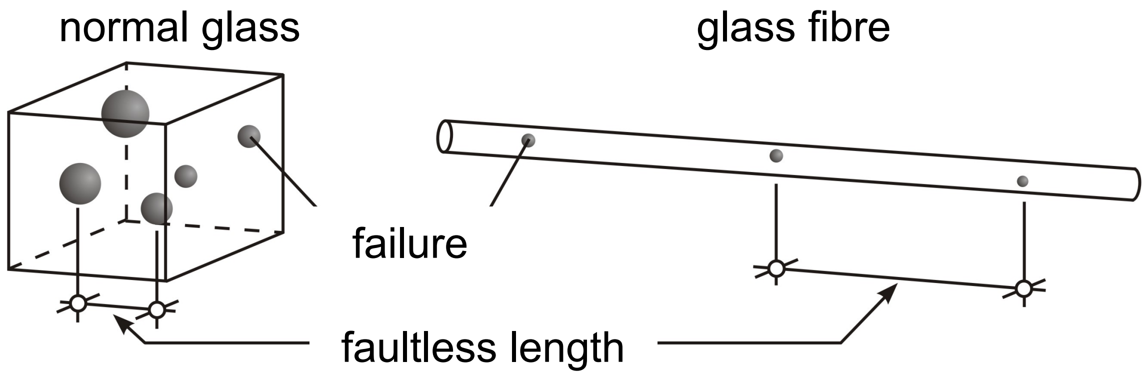




tensile strength $[N/mm^2]$
4500
4000
3500
3000
2500
2000
1500
1000
500
0
glass
glass fibre



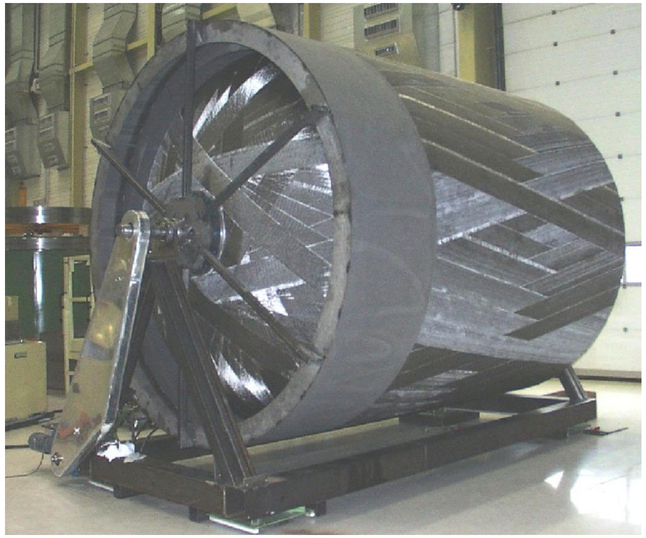
10 µm
fibre-glass-roving
carbon fibre
human hair



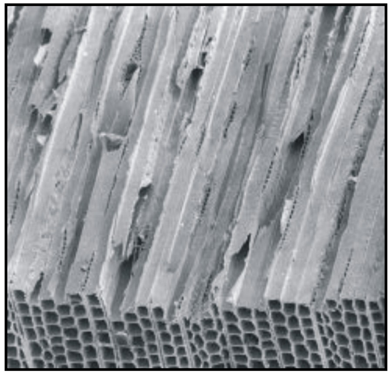
normal glass
glass fibre
failure
faultless length

load bearing parts of the Airbus A380

pilot study of a fiber-reinforced cylindrical shell for external power rockets of Ariane V



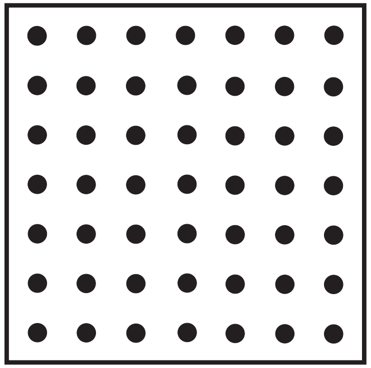
Classical continuum



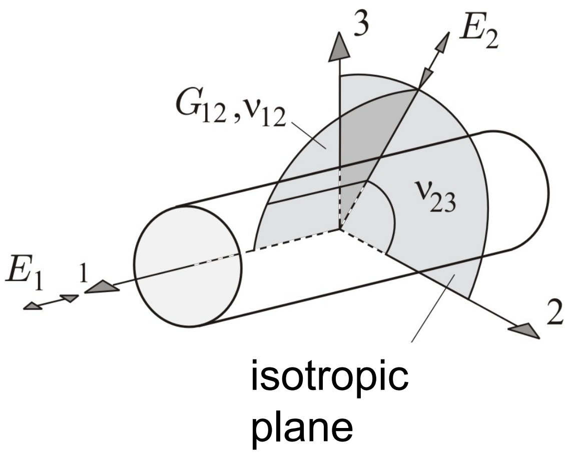
homogeneous model without directed points

→ kinematic of points

$$\epsilon = f(\mathbf{F})$$



Boltzmann-continuum



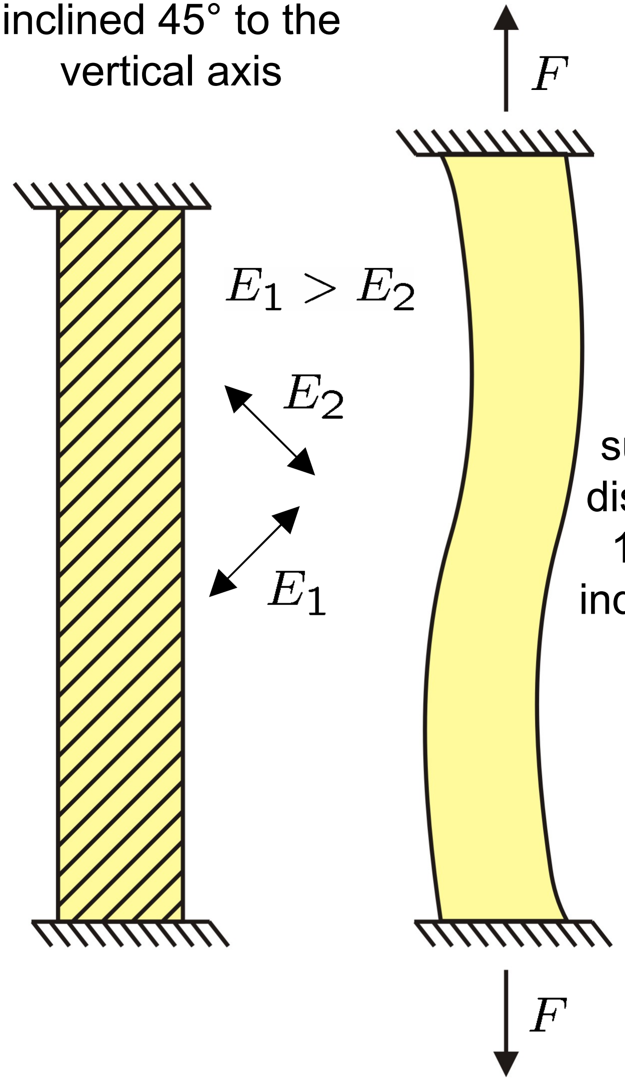
$$\boldsymbol{\sigma} = \mathbb{C}\,\boldsymbol{\epsilon}$$

constitutive stress-strain relationship for transversal isotropic material

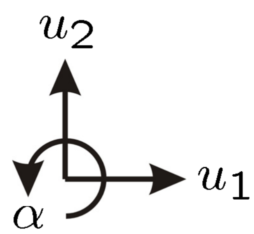
$$E_1^* = E_1/\Delta\,, \quad E_2^* = E_2/\Delta\,, \quad \Delta = (1+\nu_{23})(1-\nu_{23}-2\nu_{12}^2\frac{E_2}{E_1})$$

$$\mathbb{C} = \begin{bmatrix} E_1^*(1-\nu_{23}^2) & E_2^*\nu_{12}(1+\nu_{23}) & E_2^*\nu_{12}(1+\nu_{23}) & 0 & 0 & 0 \\ & E_2^*(1-\nu_{12}^2\frac{E_2}{E_1}) & E_2^*(\nu_{23}+\nu_{12}^2\frac{E_2}{E_1}) & 0 & 0 & 0 \\ & & E_2^*(1-\nu_{12}^2\frac{E_2}{E_1}) & 0 & 0 & 0 \\ & & & G_{12} & 0 & 0 \\ & \mathrm{S\ \ Y\ \ M} & & & G_{12} & 0 \\ & & & & & \dfrac{E_2}{2\,(1+\nu_{23})} \end{bmatrix}$$

tension test, plain strain, fibers inclined 45° to the vertical axis

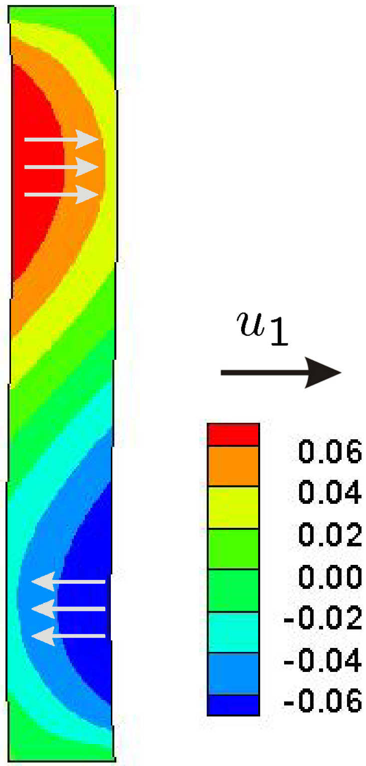


superelevated displacement in 1-direction to indicate s-shape

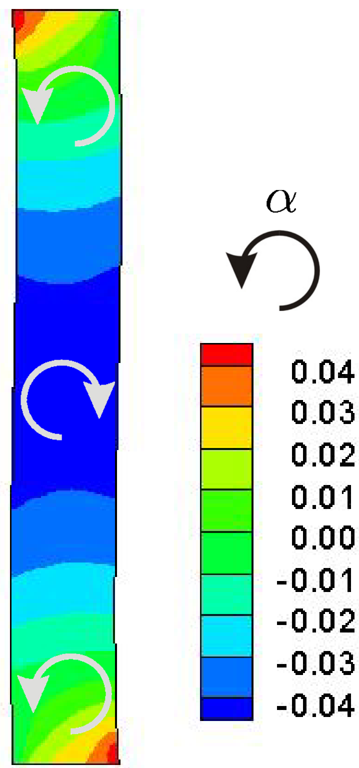


finite element computation with transversal isotropic Boltzmann continuum

displacement in 1-direction

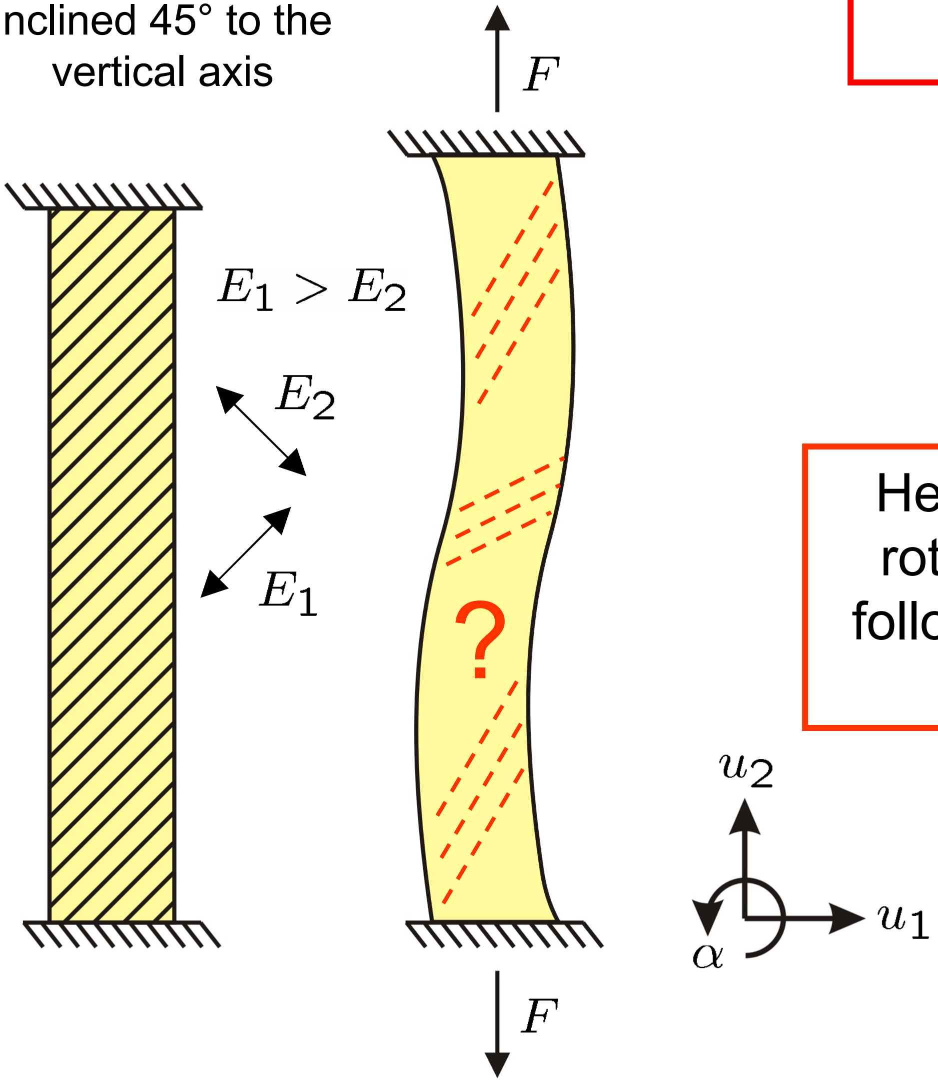


macro-rotation

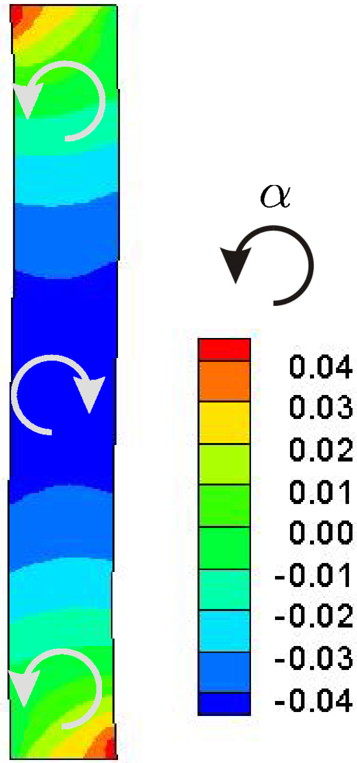


tension test, plain strain, fibers inclined 45° to the vertical axis

What is the direction of the fibers in the deformed state?

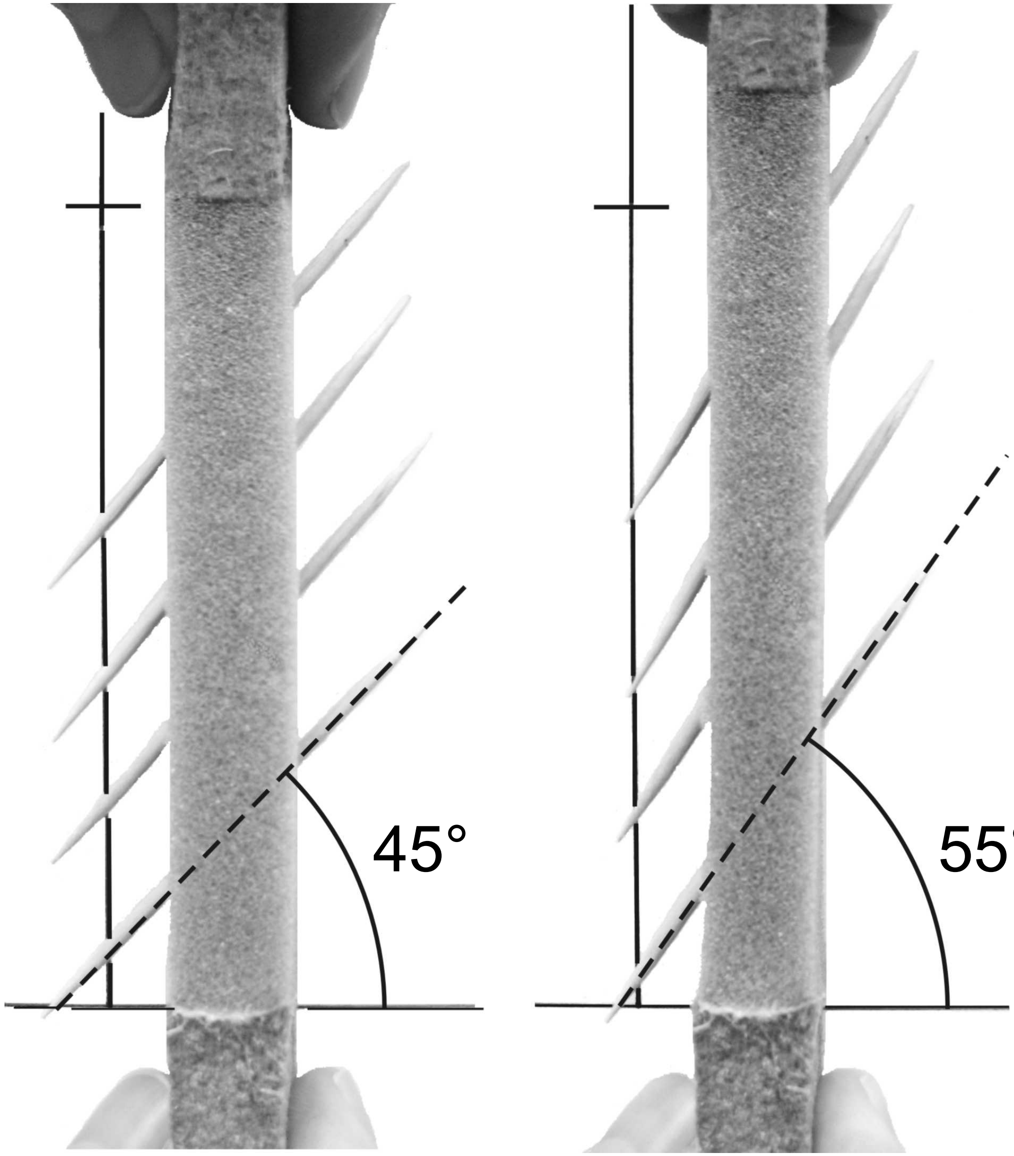


Here clockwise rotation, if they follow the macro-rotation!



Experimental model for microstructural behaviour

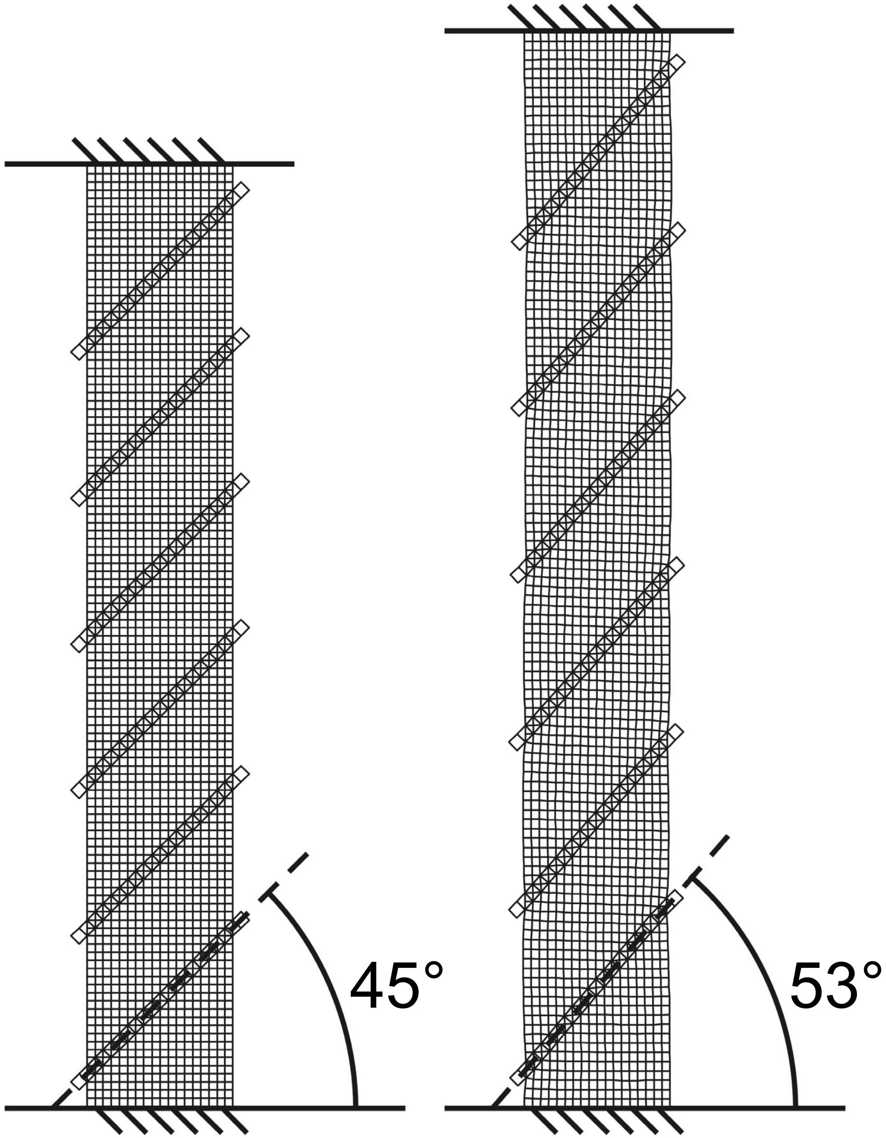
tension test, cellular rubber with toothpicks inclined 45° to the vertical axis, good bonding between both materials



anticlockwise rotation of the toothpicks everywhere

tension test, discrete reinforcement inclined 45° to the vertical axis reinforce the isotropic matrix material, full bonding between both materials

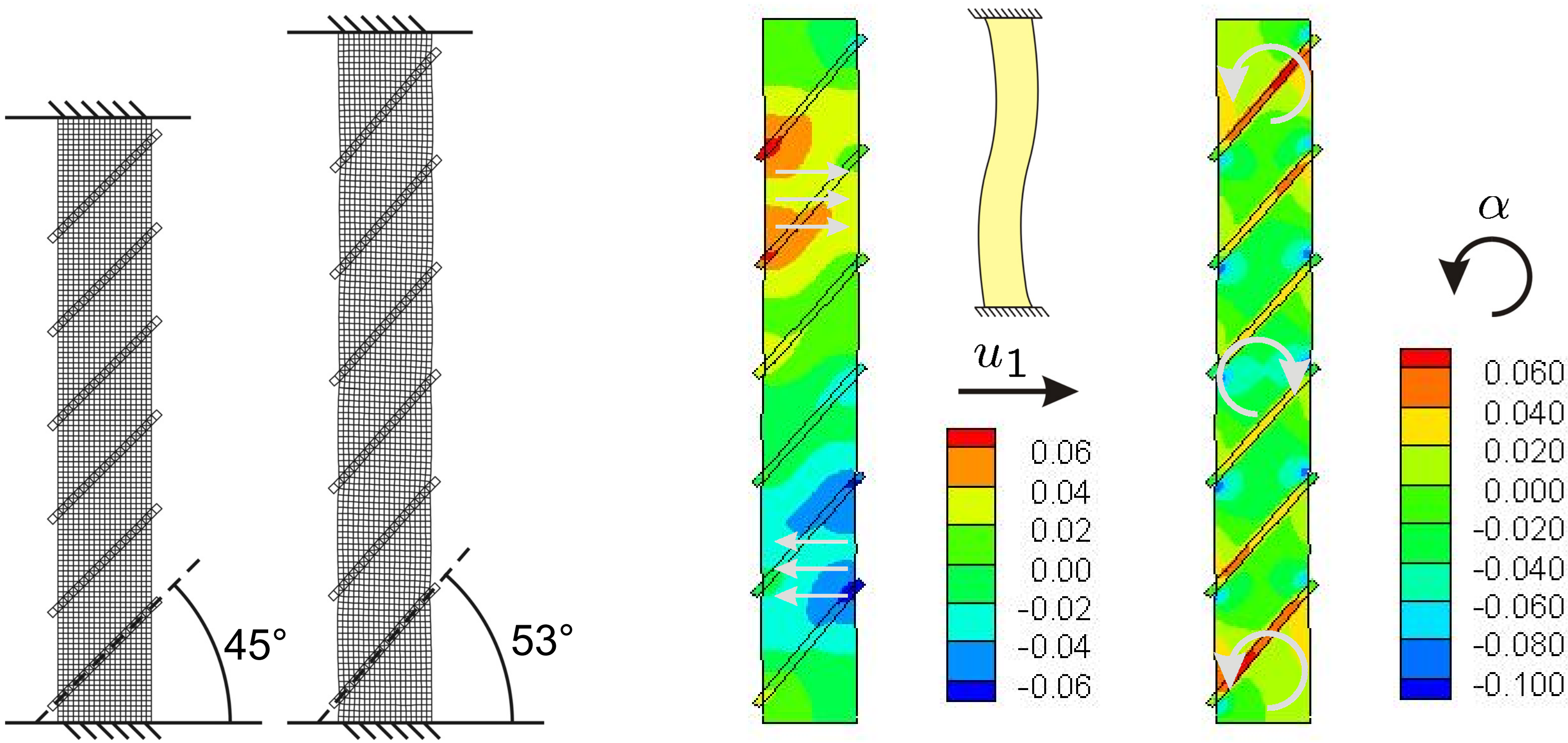
anticlockwise rotation of the reinforcement!



the discrete reinforced FE-model covers

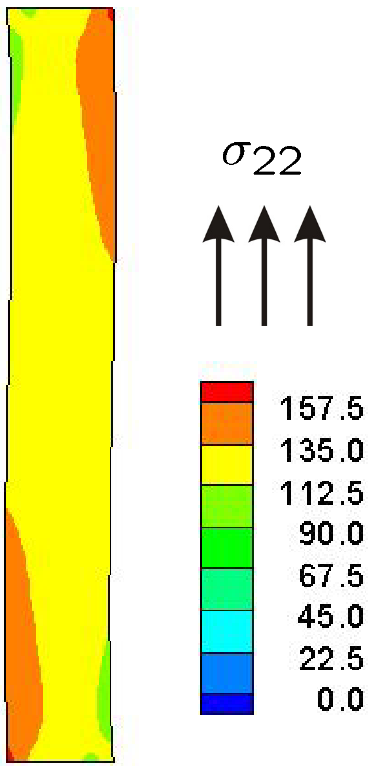
experimental results for the reinforcement

classical results for the matrix material

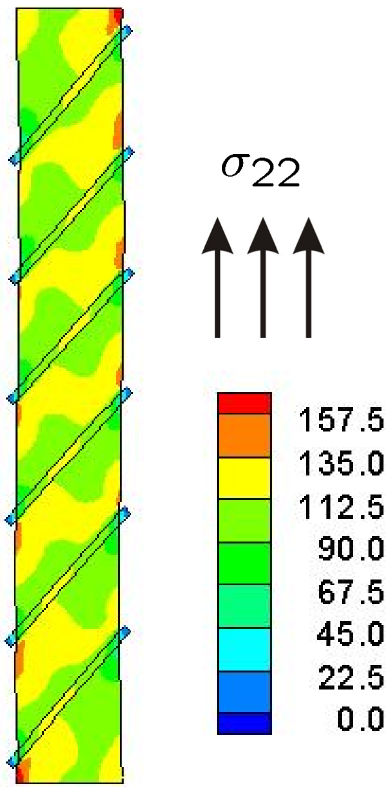


compare stress in 2-direction

transversal isotropic Boltzmann continuum



discrete reinforced Boltzmann continuum



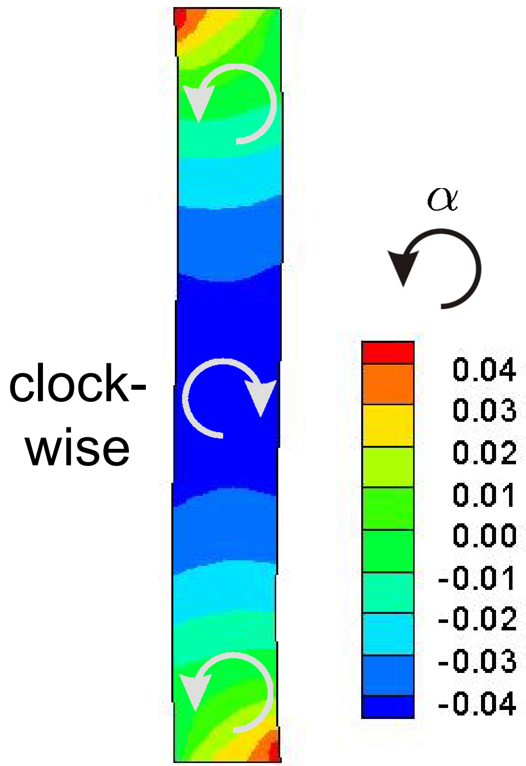
Comparison of the models

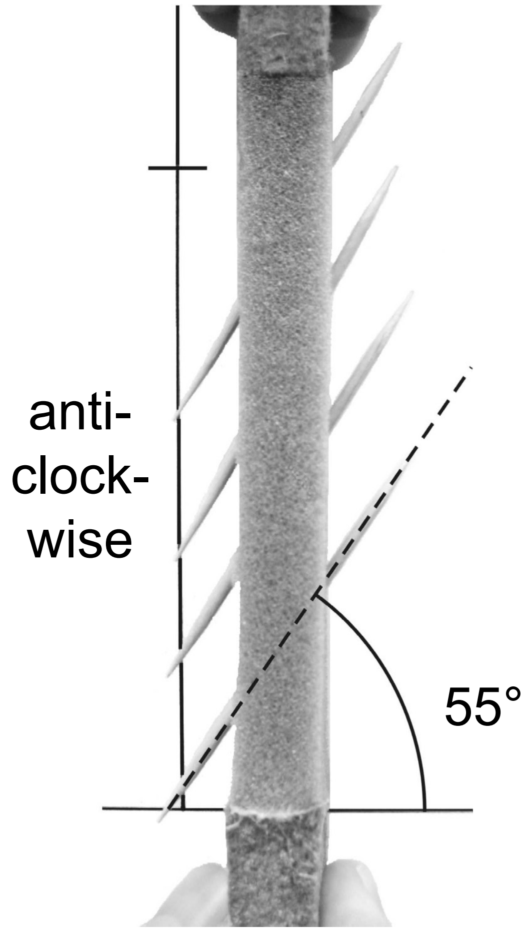
transversal isotropic Boltzmann continuum

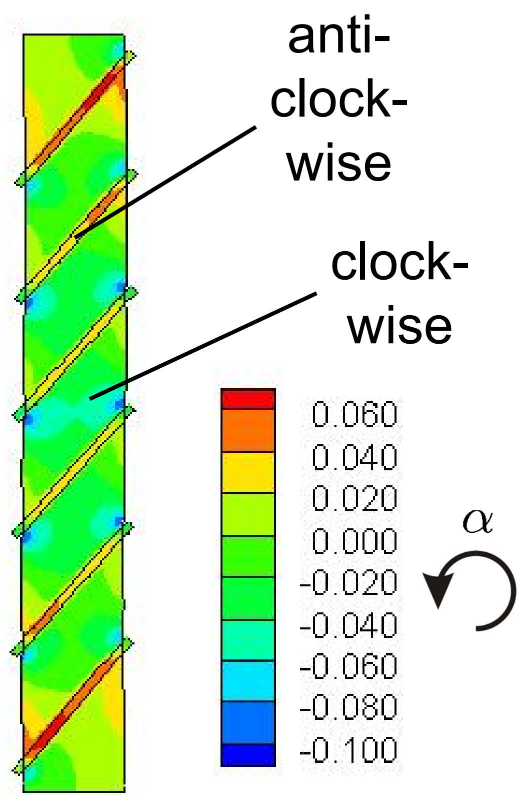
experimental model

discrete reinforced Boltzmann continuum

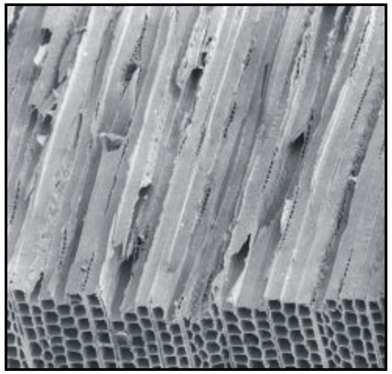
→ cover both results





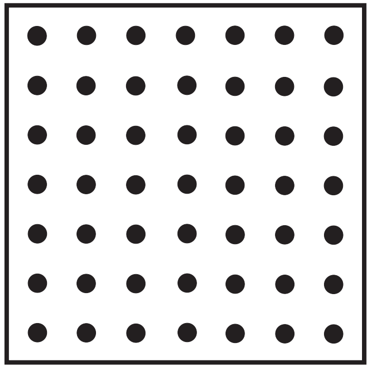


Cosserat continuum



homogeneous model without directed points

→ kinematic of points

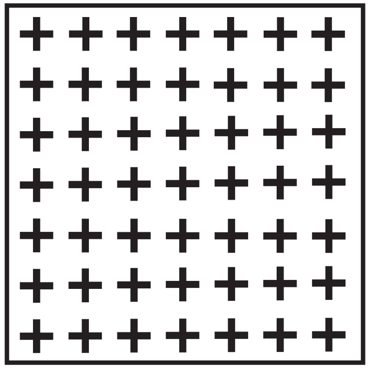


Boltzmann-continuum

homogeneous model with directed points

→ extention of the kinematic through Cosserat-rotations

Cosserat-continuum



constitutive relations

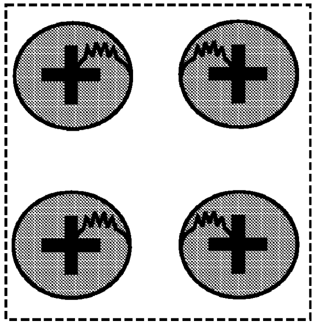
we use two Cosserat-parameters in this work:
$\mu_c\,,\ L_c$

regularization (stress-concentration, plasticity, shear-bands …)

non-symmetric strain as known in dislocation theory

size-effects through strong inner structure

μ_c acts like a „direct spring“ between Cosserat- and macrorotations

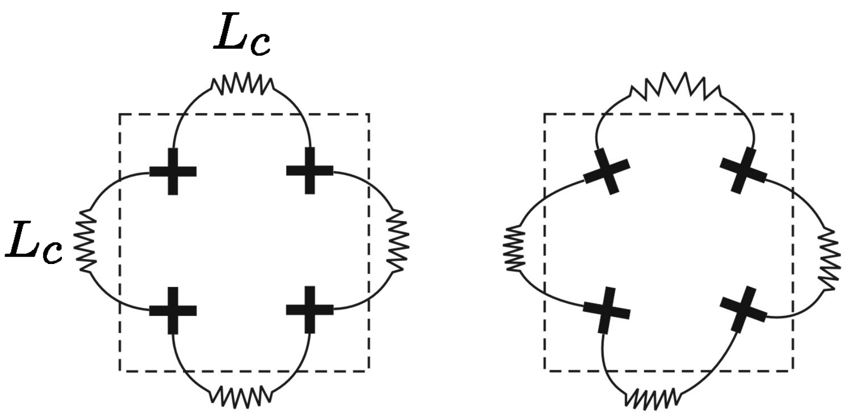


→ Biot‘s model for

$$\mu_c = \mu$$

$$L_c = 0$$

L_c acts like a spring between the fibers of the matrix



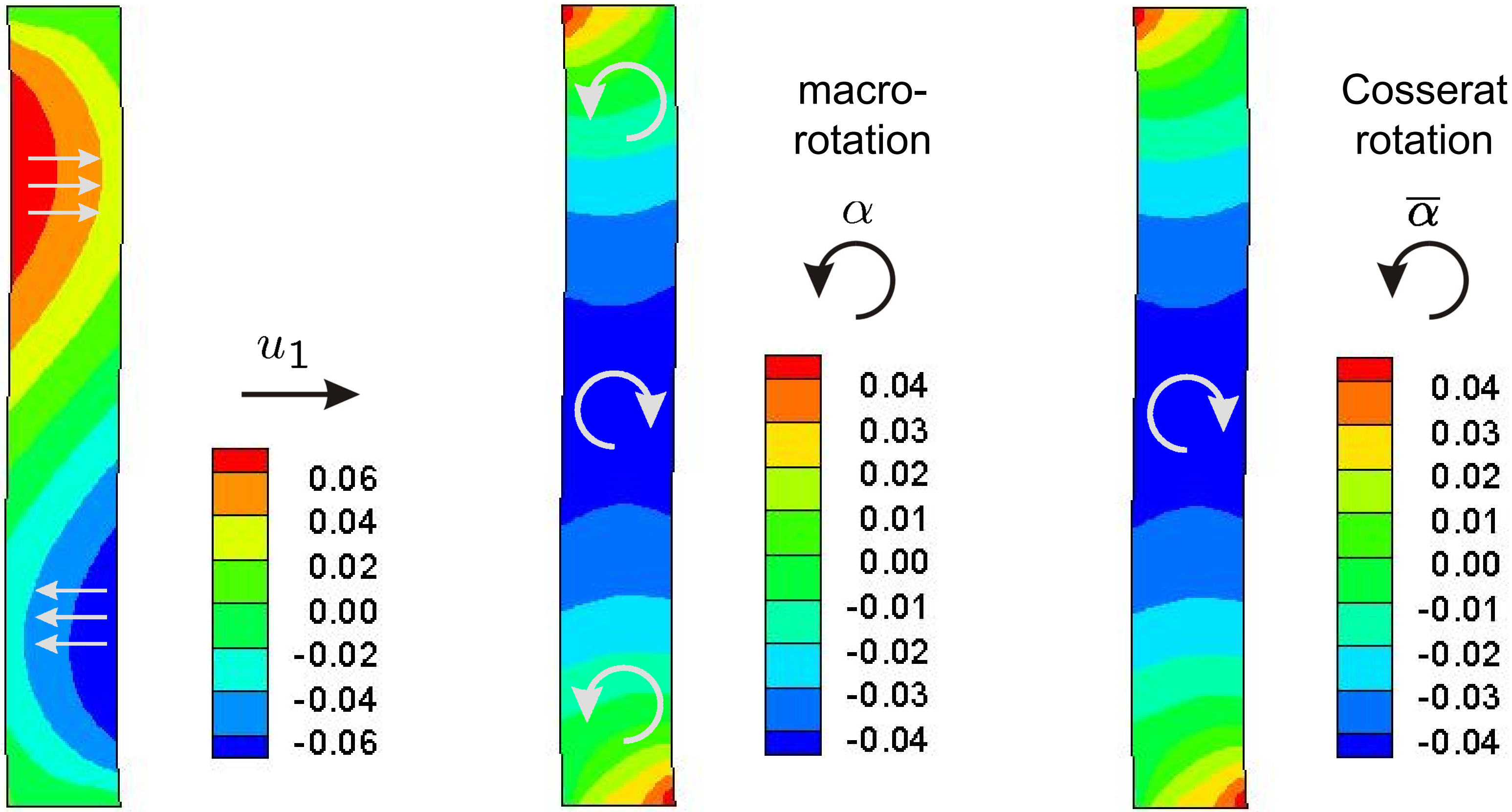
$\mu_c = 0$ and small L_c

→ fibers feel free to optimize their direction

Transversal isotropic Cosserat continuum

$$\left.\begin{aligned} \mu_c &= \mu \\ L_c &= 0 \end{aligned}\right\} \text{Biot's model}$$

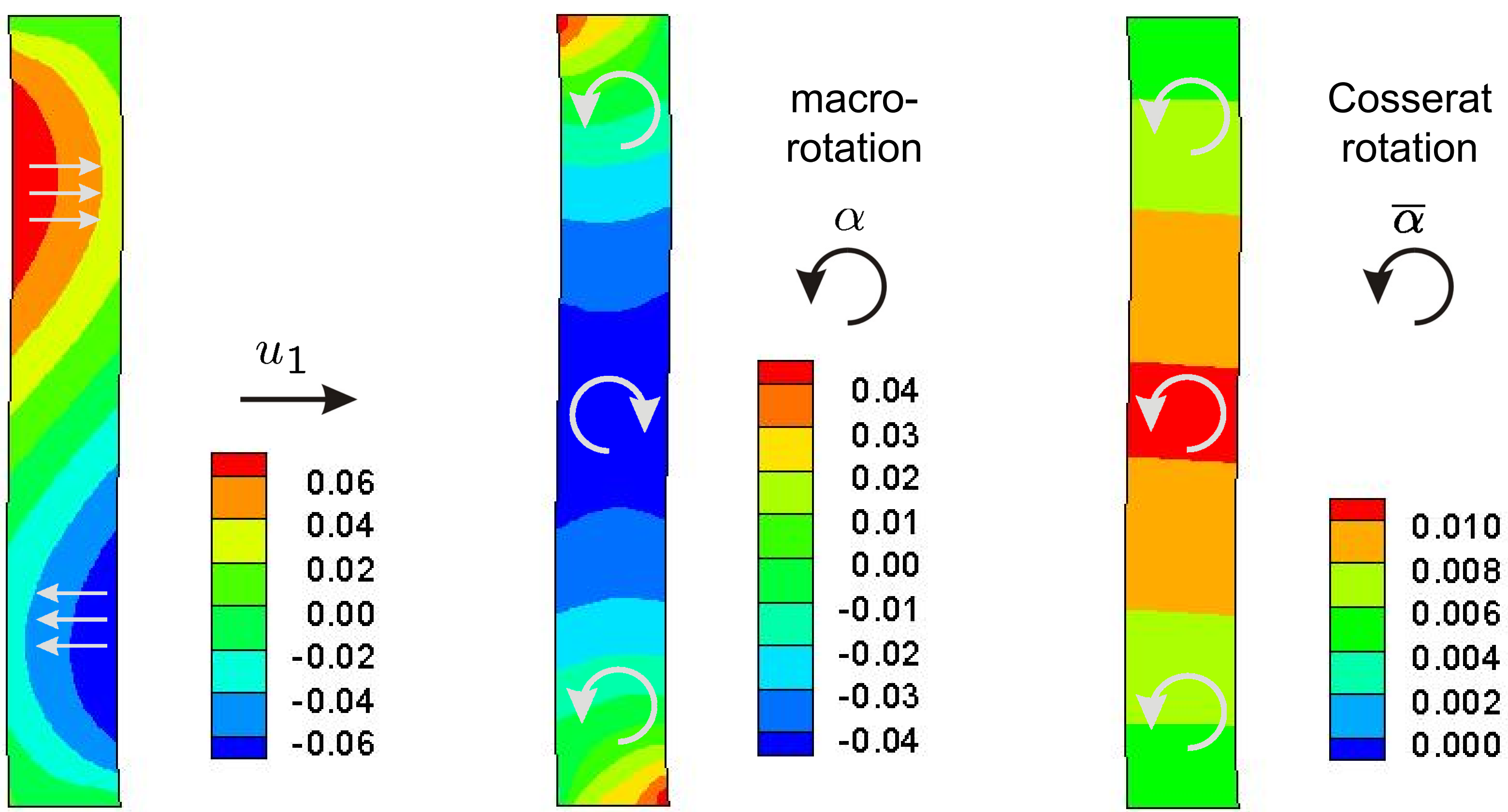
→ as transversal isotropic Boltzmann continuum
→ Cosserat rotation couples directly to the continuum rotation → no microstructural information



Transversal isotropic Cosserat continuum

$$\left.\begin{aligned}\mu_c &= 0\\ L_c &= 3\end{aligned}\right\}$$ cover classical and experimental results

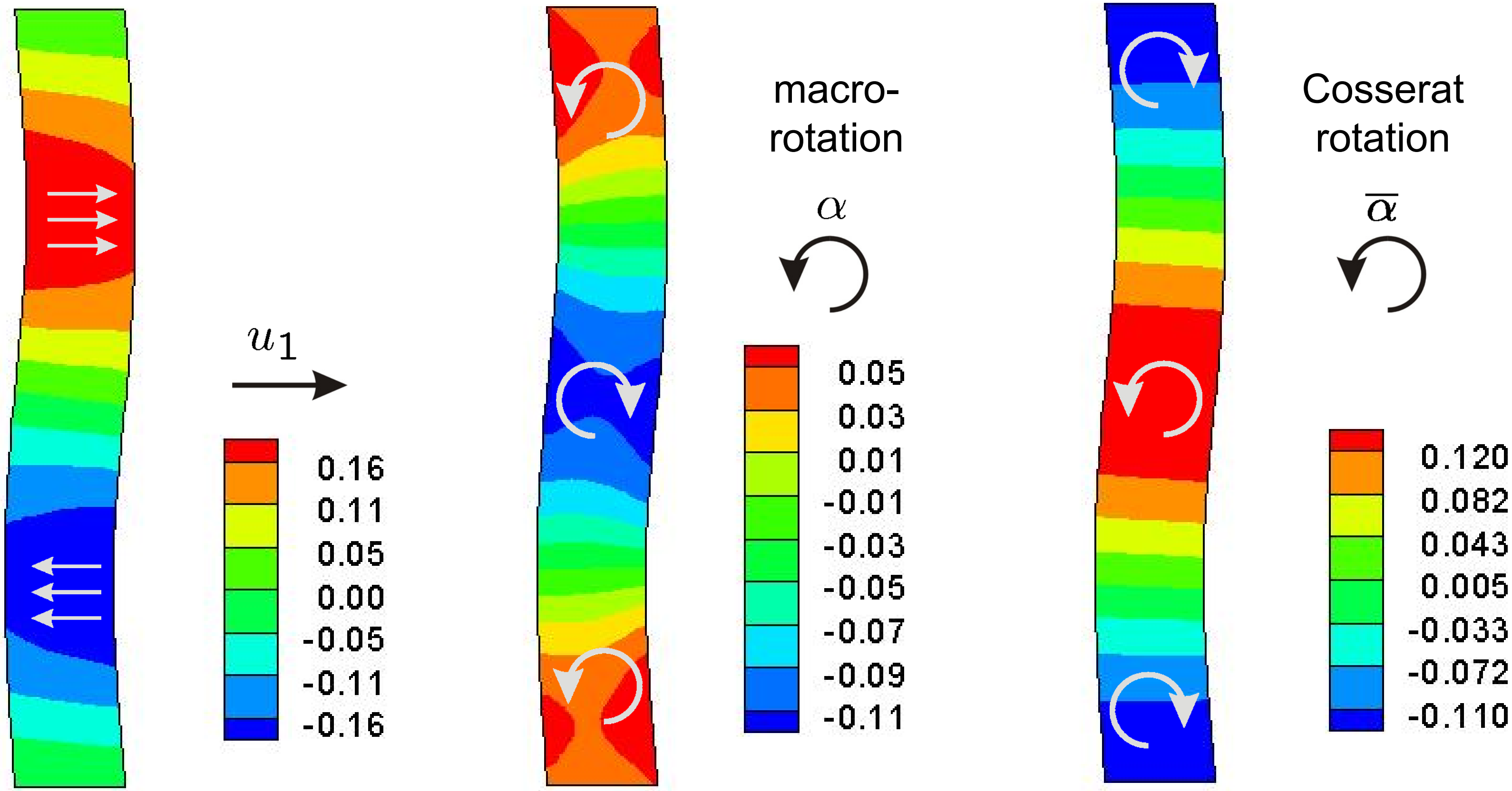
→ Cosserat rotation independent to continuum rotation
→ microstructural information



Transversal isotropic Cosserat continuum

$\left.\begin{aligned} \mu_c &= 0 \\ L_c &= 1 \end{aligned}\right\}$ increasing s-shape

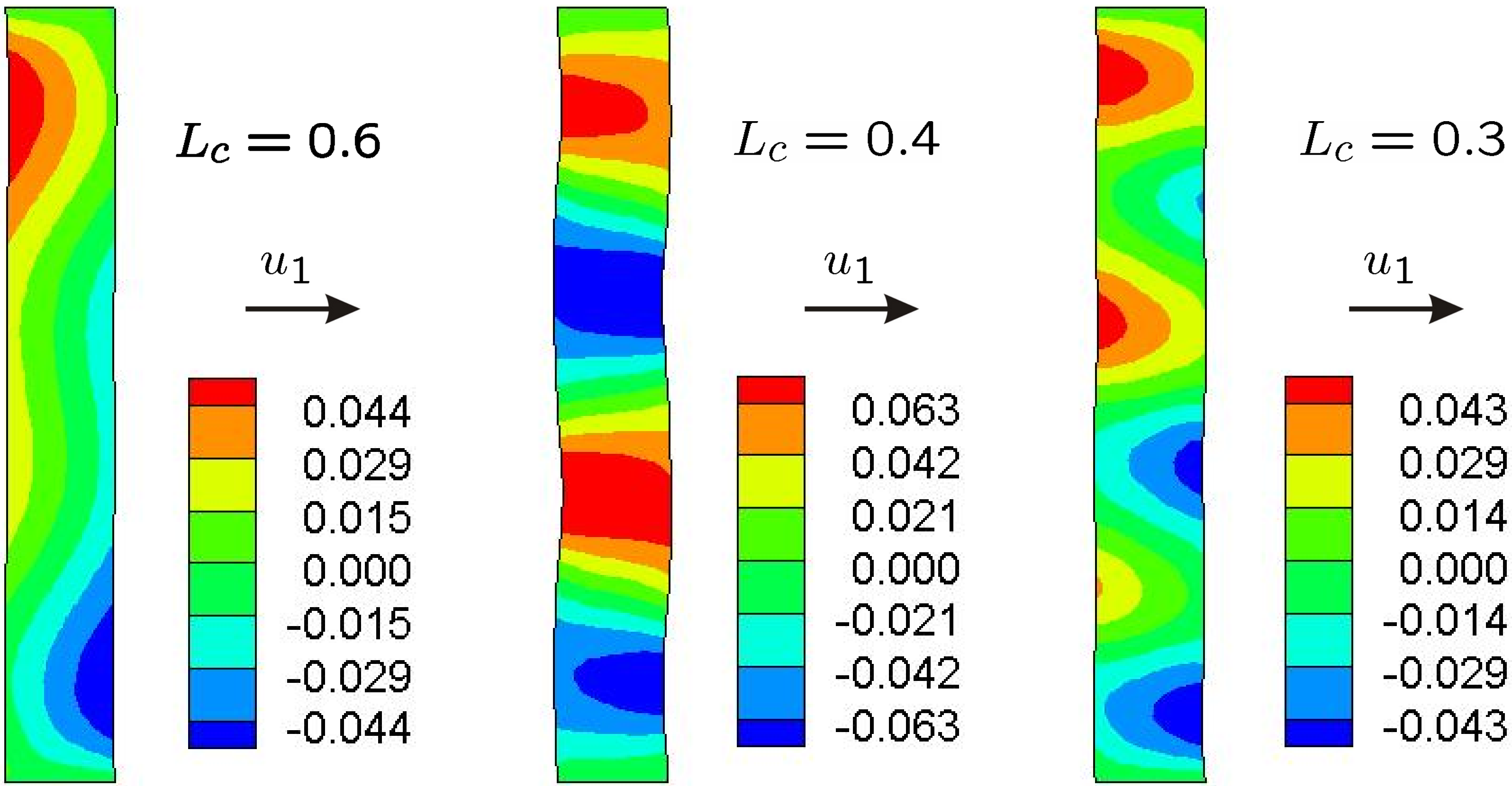
→ Cosserat rotation independent of macro-rotation → microstructural effects



Transversal isotropic Cosserat continuum

$\mu_c = 0$, L_c small } non-classical solutions appear

→ Cosserat rotation independent to continuum rotation
→ microstructural effects



Conclusions

Transversal isotropic Boltzmann continuum does not / can not contain information about microstructural behaviour

The discrete reinforced finite-element-model covers classical and experimental results

Transversal isotropic Cosserat continuum covers classical and experimental results

Depending on parameters the transversal isotropic Cosserat continuum shows non-classical microstructural solutions

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