UNIVERSITÄT
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ESSEN

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Anisotropic Polyconvex Free Energies

Motivation

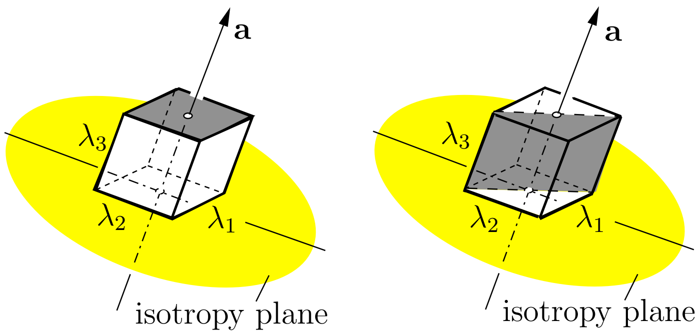
- Existence of minimizers of some variational principles in finite elasticity is based on the concept of quasiconvexity, see MORREY [1952].
- This integral inequality condition is rather complicated to handle. Thus, a more important concept for pratical use is the notion of polyconvexity in the sense of BALL [1977].

Definition of Polyconvexity

Let $W \in C^2(\mathbb{M}^{3\times3}, \mathbb{R})$ be a given scalar valued energy density. $\boldsymbol{F} \mapsto W(\boldsymbol{F})$ is polyconvex if and only if there exists a function $P : \mathbb{M}^{3\times3} \times \mathbb{M}^{3\times3} \times \mathbb{R} \mapsto \mathbb{R}$ (in general non-unique) such that

$$W(\boldsymbol{F}) = P(\boldsymbol{F}, \mathrm{Adj}\boldsymbol{F}, \det\boldsymbol{F})$$

and the function $\mathbb{R}^{19} \mapsto \mathbb{R}$, $(\tilde{X}, \tilde{Y}, \tilde{Z}) \mapsto P(\tilde{X}, \tilde{Y}, \tilde{Z})$ is convex for all points $\boldsymbol{X} \in \mathbb{R}^3$.



Material Symmetry Group

Transverse isotropy is characterized by one preferred unit direction $\boldsymbol{a}$. Material symmetry group:

$$\mathcal{G}_{ti} := \{\boldsymbol{I};\ \boldsymbol{Q}(\alpha, \boldsymbol{a}) \mid 0 < \alpha < 2\,\pi\}$$

Invariant Theory

Structural Tensor $\boldsymbol{M} := \boldsymbol{a} \otimes \boldsymbol{a}$ represents $\mathcal{G}_{ti}$

Polynomial Basis

$$I_1 := \mathrm{tr}\,\boldsymbol{C}\,, \quad I_2 := \mathrm{tr}[\mathrm{Cof}\boldsymbol{C}]\,, \quad I_3 := \det\boldsymbol{C}$$

$$J_4 := \mathrm{tr}[\boldsymbol{C}\boldsymbol{M}]\,, \quad J_5 := \mathrm{tr}[\boldsymbol{C}^2\boldsymbol{M}]$$

Problems

- The often used terms J_5 and $I_1 J_4$ are not elliptic and hence not polyconvex.
- Natural State is not met a priori.

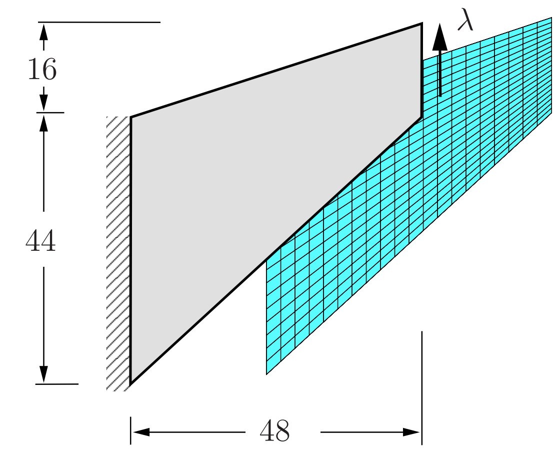
Solution: It seems reasonable from a physical point of view to construct a polynomial mixed invariant, which reflects the deformation of a preferred area element. Application of the Cayley Hamilton Theorem yields:

$$\boldsymbol{C}^2\boldsymbol{M} - I_1\boldsymbol{C}\boldsymbol{M} + I_2\boldsymbol{M} - \mathrm{Cof}[\boldsymbol{C}]\boldsymbol{M} = \boldsymbol{0}$$

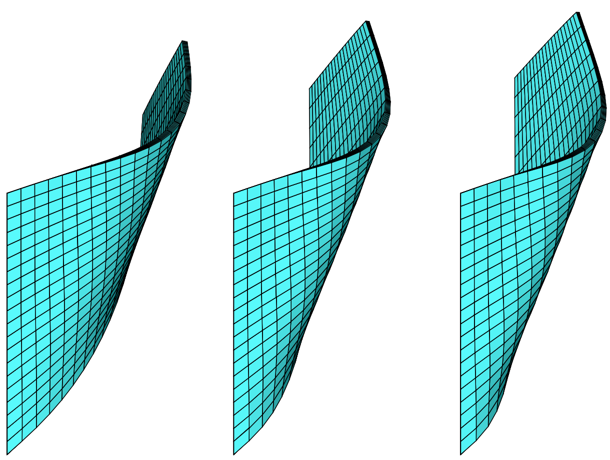
This leads to the polyconvex polynomial invariant

$$K_1 := \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{M}] = J_5 - I_1 J_4 + I_2 \bar{I}_M$$

3D-Simulation: Tapered Cantilever



Sequence of deformed configurations



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