Figures about the Studies of Mathematics in European Countries and the USA – A Nontrivial Research Analysis

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We are often confronted with questions like: *How many students are involved in a mathematical field of study? How many freshmen and graduates are involved in mathematics?*

2008 was the year of mathematics in Germany. During this time both the authors were involved in a project named "Numbers around Mathematics" and looked into these questions (see Dieter et al. (2008a,b,c,d)). Furthermore, they compiled and evaluated data concerning the German labour market (see Dieter & Törner (2009a,b)), gathering information from Eurostat and comparing German and other European figures. A country specific comparison seems necessary and in the following the first statistical evaluations of this work are presented.

Subsequent to writing this article, the authors became aware of a pipeline project (http://www.mathunion.org/icmi/other-activities/pipeline-project/), initiated by the IMU through the ICMI, with the objective "to look into the supply and demand of mathematics students and personnel in educational institutions and the workspace and to provide findings that will be useful for decision-making in different countries as well as providing a better understanding of the situation internationally" (see Holton et al. (2009)). In its current stage, data is being collected from Australia, Finland, France, Korea, New Zealand, Portugal, the UK and the USA. At this point, no comment will be made on the pipeline project; this will be picked up in an upcoming article.

The approach taken here is to present figures based on official data that originates from a single source, created for comparisons just like those being made here. Because of this the figures between the distinct countries become comparable. In contrast, the pipeline project collects data from many different authorities and there are difficulties in overcoming these inconsistencies. Nevertheless, both approaches have one important fact in common. International figures about mathematics have only just begun to be identified and it will take time to develop a better understanding of the international situation.

Eurostat – Terminology and offer of information

Eurostat is the statistical office of the European Union and is based in Luxembourg. It provides information and statistics for the European Union at a European level. Eurostat has statistics available that cover all issues of economy and society. These issues are divided into nine topics: general and regional statistics; economy and finance; population and social conditions; industry, trade and services; agriculture and fisheries; foreign trade; transport; environment and energy; and science and technology.

The national statistical offices of the EU member states, as well as the EFTA countries (Iceland, Liechtenstein, Norway and Switzerland), transmit their data to Eurostat. To make the data comparable they are harmonised by Eurostat. This gives an overview of the European Union as a whole, with insights into the situations of several countries or regions and, furthermore, comparisons are possible with other states (outside of the EU).

The majority of the data can be downloaded for free from the portal, which is located on the Eurostat homepage (http://ec.europa.eu/eurostat/). The following material is from this site and it will enable international comparisons to be made.

In Germany the four *fields of study*: mathematics, statistics, technomathematics and business mathematics, are subsumed into the '*area of studies*: mathematics' (for further information see Dieter et al. (2008a)). The equivalent at Eurostat to the German '*area of studies*: mathematics' is the field ef46 Mathematics/Statistics, which consists of mathematics, operations research, numerical analysis, actuarial science, statistics and other related fields.

There are different programmes that are relevant for mathematics. These programmes are:

- Diploma (university and university of applied sciences).
- Bachelors.
- Masters.
- Teacher training.
- Doctorate.

To make the data between distinct countries comparable, these programmes are structured by the *International Standard Classification of Education (isced)*. According to UNESCO (2006) "ISCED is designed to serve as an instrument suitable for assembling, compiling and presenting statistics of education both within individual countries and internationally". However, the systematics of Eurostat assigns the programmes that are important for this article on the basis of isced to two levels that are composed in the following way (see Eurostat (2003)).

- The *isced level 5* contains the first stage of tertiary education. These programmes normally have an edu-

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cational content but do not lead to the award of an advanced research qualification. These programmes must have a cumulative duration of at least two years. The *level isced5* is constituted of the relevant sub-categories *isced5a_d1* (bachelors, diploma and teacher training) and *isced5a_d2* (Masters and postgraduate studies).

- The *isced level 6* contains the second stage of tertiary education. These courses of study lead to the award of an advanced research qualification. The programmes are therefore devoted to advanced study and original research and are not based on coursework only. They typically require the submission of a thesis or dissertation of publishable quality that is the product of original research and represents a significant contribution to knowledge. In Germany the doctorate program belongs to the *level isced6*.

However, this classification of programmes as given by Eurostat is deficient. In the *level isced5a_d1*, diploma students and teacher trainees are not considered separately. Limitations also arise from the fact that the data of Eurostat does not allow the fields of study of the *level Mathematics/Statistics* to be viewed individually. As can be seen in Section 2, this procedure can distort results.

There are data from 37 countries present but some of these countries can be excluded from this comparison because there are no data available for the whole inquiry period from 1998 to 2007. The data from the following countries will be presented here: Germany, France, England, Sweden, Spain, Portugal, Italy, Turkey, Poland, Romania and the USA.

In the following sections the total figures of students, graduates and doctorates of these countries will be used.

Total figures of students

On the basis of the Eurostat data it is possible to calculate the share of the level *isced5a* for the field *ef46 Mathematics/Statistics*. The numbers of all mathematics and statistics students of the countries under consideration were compared with the total figures of all students of these countries. The absolute figures and the resulting quotas are shown in Table 1.

Comparing the data of the 11 countries over the inquiry period from 1998 to 2007 there are some astonishing facts. Germany is the *front runner* based on the proportion of all students that are *level Mathematics/Statistics*! In Germany, this share has always been between 2.2% and 2.6% over the last 15 years. Furthermore, since 2000 this quota has been increasing. Italy and Poland – countries that have just as many students as Germany – have quotas of less than 1%. The registered increase of the total number of students in Germany in the period from 1998 to 2007 is also noticeable in England, Sweden, Poland, Romania and Turkey.

A completely different tendency is recognisable in the states in the south of Europe. In Italy, the total number of students has increased by 8.8% from 1,823,210 to 1,983,005 but the figure of those who study mathemat-

		1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	*Total*	1,785,938	1,767,978	1,742,234	1,766,734	1,835,558	1,902,408	1,981,373	1,927,299	1,953,504	1,950,468
	Math/Stat	42,569	40,404	38,982	39,974	44,212	48,604	53,194	51,499	54,708	56,608
	Quota	2.4	2.3	2.2	2.3	2.4	2.6	2.7	2.7	2.8	2.9
France	*Total*									1,595,742	1,567,977
	Math/Stat									32,787	28,904
	Quota									2.1	1.8
England	*Total*	1,279,679	1,362,395	1,337,734	1,347,501	1,444,685	1,454,085	1,645,232	1,678,686	1,730,046	1,747,197
	Math/Stat	19,802	17,188	17,511	19,607	19,855	22,073	26,901	27,266	28,190	28,755
	Quota	1.6	1.3	1.3	1.5	1.4	1.5	1.6	1.6	1.6	1.6
Sweden	*Total*	263,760	302,985	312,287	323,840	348,100	378,570	389,390	386,656	380,846	371,307
	Math/Stat	16,927	7,529	7,564	7,007	6,529	7,069	7,117	6,683	6,773	5,636
	Quota	6.4	2.5	2.4	2.2	1.9	1.9	1.8	1.7	1.8	1.5
Spain	*Total*	1,596,644	1,602,653	1,603,743	1,571,639	1,541,743	1,519,599	1,507,520	1,484,962	1,472,127	1,468,942
	Math/Stat	24,368	23,451	21,670	19,213	16,764	15,020	13,439	12,132	10,734	10,134
	Quota	1.5	1.5	1.4	1.2	1.1	1.0	0.9	0.8	0.7	0.7
Portugal	*Total*	259,544		343,352	364,024	373,774	379,488	372,521	357,639	342,567	345,120
	Math/Stat	5,716		4,821	4,945	5,437	4,840	4,612	3,773	3,099	2,367
	Quota	2.2		1.4	1.4	1.5	1.3	1.2	1.1	0.9	0.9
Italy	*Total*	1,823,210	1,754,601	1,729,887	1,747,654	1,805,315	1,862,545	1,926,956	1,955,988	1,976,850	1,983,005
	Math/Stat	29,015	24,852	23,857	22,372	20,788	19,720	18,592	17,820	16,848	17,175
	Quota	1.6	1.4	1.4	1.3	1.2	1.1	1.0	0.9	0.9	0.9
Turkey	*Total*			777,726	830,174	869,780	898,328	1,385,094	1,463,424	1,625,337	1,702,182
	Math/Stat			23,391	25,044	26,010	27,188	28,723	30,924	33,330	34,475
	Quota			3.0	3.0	3.0	3.0	2.1	2.1	2.1	2.0
Poland	*Total*	1,157,935	1,362,269	1,539,312	1,731,554	1,858,502	1,931,543	1,989,889	2,062,612	2,089,762	2,092,162
	Math/Stat	10,341	9,836	14,778	16,591	15,937	17,342	17,106	16,219	16,862	16,798
	Quota	0.9	0.7	1.0	1.0	0.9	0.9	0.9	0.8	0.8	0.8
Romania	*Total*	339,569	380,476	416,593	484,072	527,151	565,664	621,501	673,266	785,403	887,526
	Math/Stat	8,632	9,367	11,336	11,440	12,651	11,951	12,174	12,482	18,112	21,286
	Quota	2.5	2.5	2.7	2.4	2.4	2.1	2.0	1.9	2.3	2.4
USA	*Total*								13,251,603	13,427,655	13,636,967
	Math/Stat								92,476	93,647	95,110
	Quota								0.7	0.7	0.7

Table 1: Proportion of the total number of students that are *level Mathematics/Statistics.*

ics has decreased by 40.8% from 29,015 to 17,175. Something similar can be observed in Portugal, too. Here there was an increase of the total number of students by 33% (from 259,544 to 345,120) and in the same period (1998-2007) a decrease of the mathematics students by 58.6 % (from 5,716 to 2,367). Probably the most serious change took place in Spain. Here, there was a decrease of the number of students by 8% and a simultaneous decrease of the mathematics students by 58.4%.

What led to such drastic changes in these countries? Is it the lack of mobilisation in school education or can reasons be identified in the job market? Only the figures from 2005 to 2007 are available for the USA but comparing these figures to the other results it can be seen that the USA, with a share of only 0.7%, has the lowest quota of all these countries.

It is important to pay particular attention to the *quota of women*, i.e. the proportion of all students of the *level Mathematics/Statistics* that are women. The necessary data set is available and the quotas of women of the 11 states over the period from 1998 to 2007 have been calculated. There are difficulties, however. It should be underlined that Eurostat has combined in the level isced5a the two quite differently structured programmes: diploma and *teacher training*. From previous studies (see Dieter et al. (2008a,b,c)) it can be seen that teacher training is preferred by women in Germany. This applies exceedingly in the area of primary education and there occurs, compared to the *diploma students*, a substantially higher quota of women. Furthermore, it is important to ask the question of whether these tendencies exist in all states or if it is a country-specific feature. Therefore, any conclusions should be drawn with care.

Table 2 shows the proportion of mathematics students of the particular countries that are women. Germany is about average in terms of the quota of women (note again that teacher training disciplines and prospective expert scientists are mixed). Since 1998, the proportion of women has increased continuously from 39.4% to 49.4% in 2007. For Sweden, over the same period, there was an increase from 35.5% in 1998 to 49.9% in 2007). England (average 38.5%), France (average 38%) and Turkey (average 41.9%) have the lowest quotas. The definite front runners are Portugal (average 60.2%), Italy (average 57.3%) and Poland (average 63.6%). It is reasonable to assume that the high quotas of women in these countries are due to a disproportionally high participation of women in the area of teacher training while the actual mathematics sector is poorly developed.

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	39.4	40.2	41.8	43.7	45.4	46.4	47.1	48.7	48.8	49.4
France									37.4	38.6
England	38.7	37.9	38.1	39.2	37.5	39.2	39.0	38.5	38.1	38.4
Sweden	35.8	42.2	44.1	45.6	44.5	46.2	46.2	46.8	49.0	49.9
Spain	52.0	52.2	51.7	50.9	51.7	51.6	51.8	50.3	50.9	50.6
Portugal	60.2		56.7	56.4	62.6	63.2	61.5	61.1	60.6	59.7
Italy	60.6	57.9	59.7	58.7	57.8	57.4	55.9	55.3	55.1	54.5
Turkey			42.1	42.1	41.3	41.3	42.0	41.9	41.8	43.0
Poland	62.5	60.0	66.7	65.2	64.2	64.8	63.4	63.4	63.5	62.6
Romania	50.7	49.1	52.8	47.7	46.6	48.1	48.0	45.5	44.4	39.9
USA								46.8	47.0	47.0

Table 2: Proportion of *level Mathematics/Statistics* that are women.

Figures of graduates

Thanks to Eurostat, beside the total figures of students there are *figures of graduates* available, too. These will be examined in the level *isced5a_d1* and afterwards in the level *isced6*.

Note that the *level isced5a_d1* of Eurostat subsumes the numbers for diploma (university and university of applied sciences), bachelors and teacher training. The important differentiation between teachers and specialist mathematicians is not possible because of the categorisation of Eurostat. Therefore conclusions are again made with care. The presented data does not permit answers to questions such as: *Is the reason behind the great number of graduates that many teachers passed their exams? Do the specialised mathematicians prevail? Is the ratio of both groups balanced?*

Concerning the data of Germany one thing is conspicuous: in 2006, the number of graduates is twice as high as in the previous year. As the value of 7,832 for 2006 cannot be confirmed with the values from the German Federal Statistical Office and as the figure seems to be far too high, it will be disregarded (and likewise for the 2007 value). If the figures of German graduates from 1998 to 2005 are compared with the figures of the remaining countries, it is noticeable that the numbers for Germany are again about average.

It has already been suggested that the number of women among the German graduates is so high because many of these women pursue teaching certification. However, can this conclusion be drawn for the other ten countries, too? It would be foolish to conclude this. Nevertheless, differences can be determined. Poland and

		1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	*Total*	3,951	3,750	3,335	2,932	2,953	3,027	3,349	3,904	7,832	8,688
	Female	1,722	1,683	1,503	1,402	1,443	1,574	1,887	2,237	5,364	5,825
	Quota	43.6	44.9	45.1	47.8	48.9	52.0	56.4	57.3	68.5	67.0
France	*Total*		8,610	9,392	7,606		8,077		5,206	5,802	6,033
	Female		3,828	4,286	3,360		3,573		2,089	2,323	2,483
	Quota		44.5	45.6	44.2		44.2		40.1	40.0	41.2
England	*Total*	4,599	4,865	4,569	4,799	4,575	5,266	5,422	5,271	5,500	5,645
	Female	1,273	1,320	1,824	1,988	1,858	2,209	2,322	2,195	2,264	2,302
	Quota	27.7	27.1	39.9	41.4	40.6	42.0	42.8	41.6	41.2	40.8
Sweden	*Total*	1,129	152	149	157	135	146	224	233	242	255
	Female	287	58	54	73	50	51	103	87	89	102
	Quota	25.4	38.2	36.2	46.5	37.0	34.9	46.0	37.3	36.8	40.0
Spain	*Total*	2,625	3,020	2,864	2,672	2,445	2,027	1,977	1,762	1,436	1,280
	Female	1,321	1,617	1,633	1,497	1,367	1,160	1,171	989	847	705
	Quota	50.3	53.5	57.0	56.0	55.9	57.2	59.2	56.1	59.0	55.1
Portugal	*Total*	313	593	583	725	676	706	1,028	902	866	627
	Female	186	430	425	529	467	504	723	635	634	437
	Quota	59.4	72.5	72.9	73.0	69.1	71.4	70.3	70.4	73.2	69.7
Italy	*Total*	2,986	3,123	2,806	2,972	3,343	3,401	3,281	2,895	2,496	1,936
	Female	1,976	2,015	1,770	1,883	2,014	2,091	2,003	1,731	1,426	1,094
	Quota	65.9	64.5	63.1	63.4	60.3	61.5	61.1	59.8	57.1	56.5
Turkey	*Total*		3,486	3,503	4,112	4,096	4,100	3,928	4,265	4,615	5,203
	Female		1,541	1,636	1,879	1,892	1,878	1,878	1,874	2,182	2,434
	Quota		44.2	46.7	45.7	46.2	45.8	47.8	43.9	47.3	46.8
Poland	*Total*	1,308	1,644	2,170	2,210	2,251	2,235	2,104	2,179	2,809	3,065
	Female	912	1,214	1,584	1,595	1,620	1,534	1,420	1,515	1,932	2,057
	Quota	69.7	73.8	73.0	72.2	72.0	68.6	67.5	69.5	68.8	67.1
Romania	*Total*	1,711	1,616	1,779	1,754	1,906	1,985	2,089	2,251	2,499	2,654
	Female	898	819	983	976	1,044	1,064	1,161	1,230	1,335	1,214
	Quota	52.5	50.7	55.3	55.6	54.7	53.6	55.6	54.6	53.4	45.7
USA	*Total*	12,820	12,328	12,070	11,674	12,395	12,863	13,327	14,351	14,770	14,954
	Female	5,912	5,732	5,688	5,567	5,787	5,817	6,124	6,414	6,655	6,594
	Quota	46.1	46.5	47.1	47.7	46.7	45.2	46.0	44.7	45.1	44.1

Table 3: Proportion of graduates (*isced5a*) that are women.

Portugal are unchallenged leaders with a share of women at an average of 70%. In Italy and Spain, the share of women is consistently high and amounts to more than 50%. Sweden, England, France, the USA and Turkey at no time achieve these values.

Figures of doctorates

Concerning the completed doctorates, the proportion of women is of peculiar interest. Unfortunately, Eurostat does not have figures from Poland and Romania available, leaving the nine remaining states. The proportion of women in the *level isced6* can be seen in Table 4.

		1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	*Total*	466	547	523	473	465	588	429	474	508	460
	Female	103	121	120	98	101	164	120	130	126	122
	Quota	22.1	22.1	22.9	20.7	21.7	27.9	28.0	27.4	24.8	26.5
France	*Total*	424	322	334	334		272		307	344	338
	Female	85	77	84	84		66		75	89	83
	Quota	20.1	23.9	25.2	25.2		24.3		24.4	25.9	24.6
England	*Total*	306	385	347	388	393	373	415	413	450	471
	Female	59	78	75	93	89	90	99	106	121	142
	Quota	19.3	20.1	21.6	24.0	22.7	24.1	23.9	25.7	26.9	30.1
Sweden	*Total*	222	81	85	91	87	106	133	60	116	120
	Female	37	9	16	16	14	17	41	10	27	28
	Quota	16.7	11.1	18.8	17.6	16.1	16.0	30.8	16.7	23.3	23.3
Spain	*Total*	153	158	191	165	189	228	176	149	162	142
	Female	64	73	74	65	70	92	69	64	71	55
	Quota	41.8	46.2	38.7	39.4	37.0	40.4	39.2	43.0	43.8	38.7
Portugal	*Total*	124	103	106	109	144	144	221	247	308	257
	Female	57	57	59	63	85	84	127	155	183	166
	Quota	46.0	55.3	55.7	57.8	59.0	58.3	57.5	62.8	59.4	64.6
Italy	*Total*	676	704			165	217	272	275	288	301
	Female	326	323			86	92	112	126	127	132
	Quota	48.2	45.9			52.1	42.4	41.2	45.8	44.1	43.9
Turkey	*Total*		55	42	43	77	91	71	77	73	99
	Female		23	17	19	28	26	22	29	29	34
	Quota		41.8	40.5	44.2	36.4	28.6	31.0	37.7	39.7	34.3
USA	*Total*	1,174	1,259	1,106	1,024	958	1,020	1,060	1,176	1,293	1,351
	Female	283	323	276	295	278	275	298	335	382	402
	Quota	24.1	25.7	25.0	28.8	29.0	27.0	28.1	28.5	29.5	29.8

Table 4: Proportion of postgraduates (*isced6*) that are women.

On closer inspection of the data, some interesting facts emerge. Taking the Alps as a natural border in Europe, north of the Alps the proportion of women is stable at a maximum of 25%. The average values for the relevant countries are: Germany 24.4%, France 24.2%, England 23.8% and Sweden 19%. The positive aspect in England is that the quota has been growing since 2002, rising to 30.1% in 2007. Outside of Europe, the USA is about the same with an average of 27.6%. However, looking at the states south of the Alps there are totally different values. In Turkey on average 37.1% of the doctorates are assigned to women. In Spain and Italy, the average values are 40.8% and 45.5%. However, the unchallenged front runner is Portugal with an average of 57.6% of the doctorates assigned to women.

There is a remarkable regional difference in these results. The following questions suggest themselves: *What is done in the south European countries to motivate the women into doctorates? Are there special promotional programmes that do not exist in the latitudinal lines of Germany? How else can this tremendous difference be explained? What have we missed in Germany so far?* There is evidence that these tendencies are conditioned by history.

Moving away from the proportions of women in the *level isced6* and looking at the absolute figures in Table

4, there are some more interesting facts. Germany is the country with most doctorates in the *level Mathematics/ Statistics* throughout Europe. Over recent years (1998-2007), the figures of doctorates were at an annual average of 500. There is a completely different situation in Turkey. Indeed, the figures of doctorates were constant here, too, but over the period from 1999 to 2007 there were only an average of 70 doctorates per year. While the figures of doctorates were constant in most of the states that are being used as the basis of this comparison over the inquiry period (1998-2007), two countries are exceptions. In Italy, there was a fall of graduate numbers of almost 55% to the year 1998. In contrast, the opposite took place in Portugal. In 1998, there were only 124 graduates and in 2006 there were 308.

Conclusions and further research

Because of the underlying data, no explanations can be offered for most of the observations outside of Germany, although it is believed that cultural influences explain some of the differences. Because of this, some of the questions have to remain unanswered and necessitate a more detailed country-specific and thereby more complex evaluation.

Eurostat mingles in its data "real" mathematicians and teachers but these two groups are not homogeneous. It is not known how many teachers are contained in the figures of the distinct countries. Surveys only make sense if the considered populations are comparable.

During this research, it was found that the figures of Eurostat rely on political agreements of the EU member states with the objective of drawing comparisons between these countries. As a matter of fact, a more detailed analysis based on these figures remains impossible. Despite this, this research portrays a first overview of figures concerning the studying of mathematics at universities. As a consequence of this, the mathematical associations may be encouraged to expand on this preliminary approach of collecting international figures.

The reader is invited to confer with us if questions arise. Further suggestions are also welcomed.

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