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Editor’s Statement

The papers in this volume – which was prepared by the Finnish-German research group MAVI (MAthematical VIews on Beliefs and Mathematical Education) – contain the abstracts of talks given at the second workshop on ›Current State of Research on Mathematical Beliefs‹. The conference took place at the Gerhard-Mercator-University of Duisburg on March 8–11, 1996. The aim of this research group, being the initiative of my colleague Erkki Pehkonen and myself, is to study and examine the mathematical-didactic questions that arise through research on mathematical beliefs and mathematics-education.

The next workshop will take place at the University of Helsinki on August 23–26, 1996. Further, a preprint volume containing more than 600 titles around mathematical beliefs is in preparation.

Again, the initiators would like to encourage all interested colleagues to join our network and to participate in our activities.

Duisburg, March 1996 *Günter Törner*

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Peter Berger

Computers and Affectivity

Aspects of the Computer Worldviews of German Mathematics and Computer Science Teachers

Background

The aim of our study is to investigate the hypothesis that, by analogy with *mathematical worldviews*, there are also *computer science worldviews* to be found among German mathematics and computer science teachers and that those views are multi-dimensionally structured. In this context the changing of the views during teaching practice, as well as possible effects of mathematical views on computer science views will be of particular interest. To be more precisely, we should better use the German term *Informatik* instead of *computer science*, since as one result of the study it turned out, that German teachers actually do not see Informatik as a mere science of the computer.

The methods of our study have already been described in detail in the proceedings of the first MAVI Workshop (see Berger 1995). So we may confine ourselves here to a summary. The investigation is mainly based on qualitative methods, with the empirical material deriving from 30 in-depth video taped interviews, made with 28 teachers at secondary schools (›Gymnasien‹) and comprehensive schools (›Gesamtschulen‹) in North Rhine-Westphalia, and a further 2 who are employed in the school administration as mathematicians and computer scientists.

Included, there has been a preliminary study of 9 interviews. It provided some empirically founded suggestions, of how the main study should be designed to obtain more and deeper information from the interviews. One idea was that a small questionnaire, developed on the basis of the empirical results of the pre-study, could prepare or ›incubate‹ the teachers for the interviews. This questionnaire was constructed around some terms or problems that played a dominant part in the statements and remarks of our interview partners in the pre-study.

In this paper, we focus on one important aspect of the complex scope of Informatik worldviews, that is a teacher's *computer worldview*. Most of the German Informatik teachers started out as mathematics teachers. In Germany, Informatik classes are mainly to be found at the Gymnasium and – to a lesser degree – at comprehensive schools. So computer worldviews of German Informatik teachers are actually those of mathematics and computer science teachers employed at a Gymnasium. Those teachers for the most part have made great efforts to extend their qualifications by doing thorough in-service trainings or university studies in computer science, while already being employed as teachers. This should be taken as an indication for the fact that they are highly committed to Informatik as a new school subject. However, for the most part they do not overestimate the rank of Informatik in the context of the other, classical subjects, as the interviews showed.

1. Three social roles – three fields of experience

If we are going to investigate the computer worldviews of mathematics and computer science teachers, we should regard teachers not only as teachers. There are essentially three social roles relevant in this context: besides the role of a *teacher,* it is that of an *expert* (of mathematics and computer science), and last but not least the important role of a *private individual* as a part of society. Corresponding to these social roles, there are three scopes of experience forming a teacher's views on the computer: *school*, *science*, and *society*. Based on experiences in these fields, each social role may have separately shaped the computer worldview of the individual, resulting in specific facets and in overlapping and sometimes even inconsistent partial views.

In the interviews and the questionnaire, we modelled three thematical fields corresponding to the three scopes of experience, in which the partial views should manifest themselves like specific colours of a complex picture. Thus, we should be able to investigate the worldviews by investigating the partial views, and to investigate the partial views by looking closer at what the interviews will reveal about the teachers' individual views on the role of computers in the specific thematical field – the information may be given explicitly or implicitly, conscious or unconscious to the interview partners themselves.

1.1. Computers and school

To the thematical field school, a detailed survey of results is given in Berger (1995). For better understanding the context, we should recapitulate one main aspect here.

In the pre-study three terms turned out to be central in the teachers' statements concerning Informatik as a school subject: *computer*, *programming language* and *algorithm*. The teachers had been asked in the questionnaire to give their individual assessments of the ›centrality‹ of the three items by dividing 100 points to them. The result is shown in Diagram 1 (represented by barycentric coordinates, with thick points depicting the actual positions and the ›pinpoints‹ the position at the beginning of their employment as Informatik teachers).

The orientation of the field towards *algorithm* indicates a change, which is also confirmed by the interviews. In-depth analysis of the teachers' statements concerning this aspect shows that the change can be understood as a general tendency *from phenomenons to essentials*: turning away from the earlier emphasized technical aspects of hard- and software, toward the fundamental concept of algorithms. During the period of introducing computers into schools, a widespread ›pioneer spirit‹ had been favouring the more technical skills of mastering the challenge of a new technology. In contrast to this, we note at present that the teachers increasingly focus on a didactical reflection of the aims and requirements of general education. Faced with an inflationary hard- and software innovation, making the laboriously acquired know how becoming faster and faster obsolete and antiquated, most of the interviewed teachers incline towards fundamental and lasting educational objectives.

Particularly with respect to the tasks of all-around education, detailed and specialized knowledge is rated increasingly lower than general insight into and attitudes toward computers. According to the interviewed teachers, the computer at school should no longer play the predominant part of a ›magic machine‹. They rather understand it as a tool, to be seen in perspective to other central topics as programming languages and algorithms.

Diagram 1. Teachers' views on central concepts of Informatik as a school subject.

1.2. Computers and the science Informatik

While in the field school the role of the computer mostly is seen at a restricted, but still central position, the interview statements regarding Informatik as a scientific discipline turned out to be thoroughly marked by a distinct ›computer distant‹ point of view. This is also confirmed by a plain qantitative observation: In the statements describing the essentials of the science Informatik, the frequency of the terms ›computer‹ (German ›Rechner‹) or ›machine‹ is significantly low (cf. Table 2).

number of occurrences	frequency (%)
0	19,0
1	42,9
2	9,5
3	23,8
4	4,8

Table 2. Occurrences of the terms ›computer‹ or ›machine‹.

More than 60% of the interview partners do not use the terms at all, or at most once. And if so, it often is done with the intention of restriction and distancing:

- »All has somehow got to do with computers. Nevertheless, computers are not the point, but all those theoretical foundations.«

- »For me, the computer is not really a characteristic feature of Informatik.«
- »The technical know how, the machine, is one aspect – but that I would rather regard as engineering, not as Informatik.«

One teacher turns out to be remarkably unsure with regard to the essentials of his subject: »I think, the crux of the matter is, that we do not exactly know, what we are actually doing and what sort of science that really might be, which is copying a little bit here and there [from other sciences].« But for the most part the teachers have clear and distinct ideas of what Informatik essentially is. They come up with a broad spectrum of individual characterizations:

- »Informatik is: Given a problem, how to find a solution?«
- »Informatik is quite mathematical – but here mathematics can be tested. You can try it out and see it.«
- »Basically, Informatik is to deal with the outside world in our heads – that's actually the same as Philosophy does.«
- »Informatik is to handle complexity. It is itself complicated, and it must be, just because the world is complicated.«
- »Informatik has become a rival to mathematics. Mathematics now is annexing subjects of Informatik.«

Most of the interview partners see Informatik as a dominantly formal science. And even some who regard Informatik as science of the computer emphasize its foundational aspects (cf. Table 3).

Informatik essentially is ...	%
information science	23,8
structural science, as mathematics	14,2
computer science, oriented towards applications	14,2
theory of algorithms	14,2
computer science, oriented towards foundations	9,6
theory of formal languages and machines	4,8
system analysis (science of systematically solving problems)	4,8
formal philosophy	4,8
science of complexity	4,8
a ›hotchpotch‹ of other sciences	4,8

Table 3. Teachers' characterizations of Informatik as a scientific discipline.

1.3. Computers and society

To the computer's role in society and every day life, the interview statements in many ways form a reasonable contrast to the field science. Most of the interviewed teachers attach great importance to the computer in that field and see its role here as central. Significantly frequent individual assessments are made, and the comments are often emotionally charged.

The assessments are highly individual, forming a wide range from euphoric agreement to vehement disapproval, from confidence to extreme worry. For illustration, we quote from six different interviews:

- »The computer is the central medium – it secures our standard of living.«
- »I think, we could not survive without the computer.«
- »At the moment we are living in a time where the computer is being overestimated, simply because it is a time of radical change. In 50 years time it would be a dead-normal thing, like a kitchen appliance today.«
- »The technological instrument computer has infiltrated us.«
- »A radical change is going on. How it will develop – there are so many tendencies – it's a horror. One could barely describe it in words – it will crash.«
- »It depends on what man makes from it – it's another kind of atomic bomb coming our way.«

There is a widely held opinion, that being familiar with computers for a long time, an Informatik teacher's attitude to computers could not be characterized by fear nor fascination. This plausible view is taken by most teachers, as two quotations from a discussion between teachers indicate:

- »Informatik teachers are surely not afraid of computers, but on the other hand they are not thrilled by it any longer.«
- »At most some older colleagues may have still fear to make a fool of themselves at the computer.«

Some of the interviews and the quotations above, however, reveal a totally different sight. We cite two examples. First, a female probationary teacher (aged 26):

> I have no ***fear*** of computers ... [but] there are many people who are simply ***afraid*** of them. The concepts behind computers were created by humans, and – I don't know – well, I am not ***afraid*** of them.
>
> (Did you previously think differently about that?) Yes, maybe because people could ***blame me*** for things that I couldn't do. But that I don't ***fear*** now. However, yesterday I had been here in the computer centre, and I acted a little bit ***stupid***. I only needed some information, and I immediately said that I were totally ***stupid***, because the person there always acts a little bit arrogant and so. So I didn't necessarily want him to know that I had studied this [Informatik]. I ***fear*** that somebody would ***blame me*** for not being able to work with MS-DOS or Unix commands.

In her view on the computer, fear seems to be a central aspect. Although the interview questions did not refer to that topic, it is the first she mentions, and she elaborates on it: fear experienced by others, by herself, fear of being blamed. She spontaneously makes fear to a subject of discussion, even in denying it. To her, fear is a category of assessment of the phenomenon ›computer‹. Strictly speaking, it is her individual main view on the computer.

To the assertion that Informatik teachers could no longer be fascinated by computers, we cite a teacher (aged 55) for mathematics, physics and Informatik with many years of experience of running in-service-trainings for Informatik teachers:

> Internet – a ***crazy possibility*** to communicate with a gigantic public. Global village. A ***fascinating*** thing. ...
>
> When it breaks free, it would become a ***gigantic danger*** of totally losing yourself – with typical withdrawal-symptoms. That is a ***gigantic danger*** – a ***gigantic danger***. You are no longer sitting facing one another, but you are separated. We will no longer be communicating with all that non-verbal signals, but only by computers. That's a ***gigantic danger***. This should not be allowed to

> happen. ... In such a case, the computer would become an instrument ***damaging the whole society***. Then it will be ***critical***. One has to be careful.
>
> On the other hand, I can fully understand that naively euphoric usage. ... ›Piazza-rituals‹ as at the fairs and so on. ... I can understand that people play with it, that one would like to get to know all the possibilities. That I find positive. It's a ***totally new experience***, you can discover some totally new aspects of your own personality. Thus, it is a ***widening of one's own self –*** definitely. ... I find it a ***fascinating thing***. It is ***fun***. ... This central communicating-machine, I think, will come. But that, yes, that will be ***fun***.

He exposes two contrary positions: euphoric agreement and extreme worry. It may seem, that he is not able or willing to decide between those extreme positions. But an analysis of the whole interview reveals that he actually is not undecided. Not two antagonistic views of the computer's role are to be seen here, rather different judgements on two classes of computer users, regarding their competence in managing risks: the small class of professionals and the huge class of non-professionals. Towards himself, he takes a view of optimism and self-confidence, seeing himself as a competent and experienced user, who in spite of all risks will be able to manage a creative and wise usage of the new technology. Most of the unprofessional users, however, he views from a mainly pessimistic and critical perspective. To him, the problems do not arise from the new technology, but from its unreflected usage by the masses, being left alone with it.

2. Computer worldviews and affectivity

Due to its subject-oriented paradigm, qualitative research is mostly interested in *local* observations, owed to a two-phase evaluation of description and interpretation. On this basis, establishing of sound *global* results will only be possible by means of ›generalisation-instruments‹ that will not construct their own realities (cf. Lincoln & Guba 1985, pp. 70).

To obtain a more global point of view, after local in-depth analyses of the individual interviews, we took as an appropriate generalisation-instrument a sort of interview-profile, by using the parameters

- *contents:* what does the teacher say about the role which the computer plays in the specific field?
- *assessments:* how (and how often) does he assess this role?
- *affectivity:* how does he colour his words, and what is the extent of emotion (›affectivity‹) in his presentation of this role?

If we distinguish only three items of interpretative characterization (*low*, *medium*, *high* or similar), which different profiles would the interviews bring up? As a rather unexpected result, it turned out that there is one global interview-profile fitting for the whole series of the 21 standardized interviews of the main study (see Table 4).

fields of experience

	science	*school*	*society*
role of the computer	peripheral	central, but qualified	central

importance assigned to the computer	little	medium	high
frequency of assessments	low	medium	high
affectivity of the statements	none to low	medium	medium to very high
	expert	*teacher*	*private individual*

social roles

Table 4. Global interview profile.

There is a significant qualitative correlation between the importance assigned to the computer, the frequency of assessments, and the affectivity colouring the corresponding statements. Passing through the social roles from *expert* over *teacher* to *private individual*, the interview partners' computer views take an increasingly emotionally charged perspective, from which at the same time they assign a more and more central and relevant part to the computer. According to the hypothesis that higher affectivity refers to deeper layers of personality, we may depict the situation in a ›shell model‹ (see Figure 5).

In terms of *fields of experience* we can summarize: The more the theoretically specialized character of the field declines and social aspects and everyday experiences become determining for the field, the greater importance – approving or disapproving – is attached to the role of computers. The more the human being is involved, the more the phenomenon computer is seen as ›explosive‹. Not the view of the computer scientist is computer-centred, but the view of the private individual.

References

Berger, P. 1995. Teachers' Beliefs about Computers and Computer Science. In: Törner (1995), 15–22.

Lincoln, Y.S. & Guba, E.G. 1985. Naturalistic Inquiry. Beverly Hills: Sage Publications.

Törner, G. (ed.) 1995. Current State of Research on Mathematical Beliefs; Proceedings of the MAVI Workshop, University of Duisburg, October 4-5, 1995. Schriftenreihe des Fachbereichs Mathematik. Gerhard-Mercator-Universität Duisburg. Preprint Nr. 310.

José Carrillo

Relationships Between Teachers' Conceptions of Mathematics and of Mathematics Teaching and their Problem Solving Modes

Background

Within the vast field of teachers' conceptions (particularly concerning case studies, under the umbrella of a qualitative research methodology), it is easy to find some studies dealing with the feasible relationships between mathematics and mathematics teaching conceptions, or even between the espoused teaching conceptions and those in practice. The so called consistency problem has led to the recommendation of the use of classroom observations when investigating teachers' conceptions, or to compare the espoused conceptions with some facets of mathematical knowledge (see Pajares (1992) and Thompson (1992) to get an overview).

Owing to the fact that our goals should be translated into improvements in teaching, and because of the so relevant role played by teachers in this process, it is extremely important to obtain meaningful keys to professional development. In fact, professional development or change has emerged as one of the most attractive current research lines (Pehkonen, 1994; Tillema, 1994).

I locate my inquiries in teachers' conceptions in the search for the beforementioned keys and it is at that point where problem solving comes to play a main role.

A large amount of research has been devoted to show the advantages of a problem solving based methodology in order to achieve meaningful learning among students (as an example, we have recently had the opportunity of reading some articles in the ZDM (Zentralblatt für Didaktik der Mathematik) under the general title *Using open-ended problems in mathematics,* Pehkonen (1995), Nohda (1995), Stacey (1995), Silver (1995)).

In relation to mathematics teachers, I am convinced (in agreement with Hart (1991)) that training in problem solving, delving into metacognitive aspects, is useful in challenging teachers' conceptions and *»in promoting a possible alternative to them«* (Carrillo & Contreras, 1994, p. 153). When the solver, whether a teacher or a student or even a person in general, is approaching a problem, their conceptions come into play. It is obvious that mathematics knowledge influences the way they try to solve the problem, but what I want to express here is a different theory. Based on my experience as a teacher trainer, I strongly believe that problem solving is a good training strategy. In order for that to be achieved, it is necessary that one reflects about the metacognitive aspects that have appeared through the resolution process, and also about the inherent conceptions of the problem and of the different solutions. Conceptions influence knowledge and viceversa, and with problem solving we have the possibility of directly influencing conceptions.

That is why one needs to analyse the connections between the teachers' conceptions and their problem solving modes. Thus we previously need some instruments to make possible the aforementioned analysis.

1. Analysis instrument (conceptions)

Our research in Huelva has brought about the development of some instruments to analyse the teachers' conceptions on mathematics and mathematics teaching. They are outlined in Carrillo & Contreras (1994) and described in depth in Carrillo & Contreras (1996). Summarising, we consider 4 types of mathematics teaching conceptions (traditional, technological, spontaneous, and investigative, in accordance with Porlán (1989)) characterised by a remarkable amount of descriptors, which are divided into 6 categories and 21 subcategories:

methodology • praxis • objectives • programme	***subject significance*** • focus • goal	***learning conception*** • type and way • type of grouping • motive force • aptitude • attitude
student's role • participation in syllabus and lesson design • key to teaching-learning transfer • what do they usually do?	***teacher's role*** • what does he/she usually do? • how does he/she do it? • why does he/she do it? • coordination with colleagues	***assessment*** • character • criteria • instruments

Similarly, we consider 3 types of mathematics conceptions (instrumentalist, platonist and problem solving, in accordance with Ernest (1989)) with the correspondent descriptors being classified into 3 categories and 4 subcategories:

type of knowledge • what is it composed of? • what is it like?	***aims*** (of mathematical knowledge)	***means of development*** (of mathematics) • construction process • type of reasoning

2. Analysis instrument (problem solving)

We have also developed an instrument with the objective of getting solvers' profiles. It is a problem solving assessment instrument already partially described in Carrillo (1995) and above all (exemplifying its use) in Carrillo & Guevara (1996). It is divided into 3 categories and 18 descriptors:

I. *Personal Characteristics*

- I.1. Repercussion on behaviour due to being observed
- I.2. Behaviour with regard to daily problems
- I.3. Habits for facing Mathematical problems
- I.4. Usual behaviour in problem solving
  - I.4.1. Predisposition
  - I.4.2. Self-confidence
- I.5. Knowledge organization. Mathematical training for problem solving
- I.6. The role that is given to the memory in resolution of problems

II. *Tactic characteristics of the process (effectiveness of the action)*

- II.1. Obtaining a meaningful representation
- II.2. Effectiveness and adjusting of the planning
- II.3. Effectiveness and adjusting of the execution
- II.4. Effectiveness in the use of revision
- II.5. Finished level of the solution

III. *Regulator characteristics of the process (control of the action)*

- III.1. Importance granted to the obtaining of a significant representation.
- III.2. Importance granted to the obtaining of a viable plan.
- III.3. Importance given to the explanation of the execution state.
- III.4. Coherency and control of the process (It includes the control of process towards the established aims, the kind of decisions of control and importance granted to the control of the process)
- III.5. Temporal organization
- III.6. General metacognitive knowledge (It includes the knowledge of strategies of problem solving and the characteristics that an expert must have about them)

But the major contribution of this instrument, from my point of view, is its scale of valuation. Each descriptor has 5 levels, precisely described, that refer to different degrees of achievement. As an example, I make explicit the scale of III.1.:

1. Nothing, the important thing consists of knowing more or less about what the problem is and coming to a quick execution. The solver does not make a contrast between the foreseeable requirements of the problem and his/her knowledge situation; the approach is immediate and blind.

2. Scant, to spend part of the resolution time takes him/her a lot of effort. The contrast between the requirements and knowledge is too poor, consisting of testing if the subject is familiar.

3. Some, although he/she becomes impatient if it takes a long time, and enters in another stage. The contrast between the requirements and knowledge exists, but distributed within the process, although more intensively at the outset.
4. Quite, although it is not developed immediately at the beginning, existing the possibility of the lack of some details. A detailed contrast at the adequate moment exists between requirements and knowledge.
5. He/she is conscious that the spending of time and effort in the attainment of a meaningful understanding is worthy and consequently he/she does not enter in a different stage without having reached it. A very detailed contrast, between requirements and knowledge, and a searching for adequate information exist.

All the 5 levels of the scale give shape to what I call the *Good Problem Solver Profile*, very useful in order to define goals in problem solving training (whether with students or with teachers).

In my study, I have considered the first category as something additional, because I have not aimed at investigating personal characteristics in depth – that would require the assistance of information gathering instruments that have not been used.

3. The study

I have analysed the mathematics and mathematics teaching conceptions of 9 teachers (7 giving classes in highschool and 2 giving classes now in the first course of a technical degree, but before in highschool), having gathered information from 2 open-ended questionnaires, 1 open-ended interview and the final consensus stage. They have also approached 3 problems and answered questions in 3 open-ended interviews (one after every problem) and in 1 open-ended questionnaire about their features as problem solvers. From these data I have hypothesised their problem solving modes.

Although the ultimate goal of this work is not, in any way, to establish general relationships, this fact does not diminish the importance of researching possible coincidences or connections between different aspects of the human intellect and behaviour, because our aim consists of knowing better the motive forces that impel a teacher to behave in a certain way.

At this point, I would like to quote the work of Chorao (1994). She applies Kirton's theory of creativity styles (Kirton, 1989) to pedagogical practice, thereby distinguishing 2 types of teachers according to their creativity styles, adaptors and innovators, and associating to them some feasible characteristics concerning their pedagogical practice. Of course, at the moment, we can not link both pieces of research (research into the relationships between teachers' creativity and ways of teaching, and into the relationships between teachers' problem solving modes and teaching styles and mathematics conceptions), but it appears reasonable to think of some links arising that could strongly contribute to obtaining some desirable keys to professional development.

4. Relationships

The teachers' conceptions inferred throughout our research can be outlined in Table 1 (where TR stands for traditional, TE for technological, S is spontaneous, I investigative, IN instrumentalist, P means platonist and PS problem solving, noticing also that * stands for predominance). Below (Table 2), I summarise the teachers' problem solving modes writing the levels that they have got in each descriptor belonging to the second and third categories (in general, levels 1 and 2 mean low achievement, and 4 and 5 high).

	Conceptions	
Individuals	Math. Teaching	Mathematics
AP	TR-TE	P
CG	TR-TE	P
JR	TE	IN
KN	TE	P-PS
LD	S-I	P-PS*
PG	TR*-TE	IN-P
PN	I	PS
RC	I	PS
RV	TR-TE*	P

Table 1. Teachers' Conceptions.

I have not looked for direct relationships, as they can not arise from logical deductions. It is obvious that they do not exist (we have had, as students, some traditional, or instrumentalist teachers, for example, who were quite good as problem solvers), but, if the adjective *good* (regarding problem solvers) is developed, described in depth, I am convinced (similarly as done by Chorao) that we will find rich relationships. I believe that I have already discovered some of them, but this research line has just begun.

I will avoid commenting the relationships that one can see between mathematics teaching and mathematics conceptions, because we would need a wider description (based, at least, on the categories; a general description like TR is not enough), too wide for the length conditions of this paper, also because they have been extensively studied by researchers, and above all owing to the fact that the main goal of this paper consists of making explicit other kinds of relationships.

As an overview, we could say that a certain degree of correlation exists between mathematics teaching conceptions and mathematics conceptions, and between mathematics teaching conceptions and problem solving modes, but especially between mathematics conceptions and problem solving modes, although with some exceptions.

Problem Solving Mode	
Second Category	Third Category

Indiv.	II.1.	II.2.	II.3.	II.4.	II.5.	III.1	III.2	III.3	III.4	III.5	III.6
AP	4	3	3	2	1	2	2	2	2	2	2
CG	4	3	3	2	2	4	5	2	3	4	4
JR	2	2	2	2	1	2	2	2	2	2	2
KN	3	3	3	4	2	3	2	5	3	1	3
LD	5	4	5	4	4	5	5	5	5	5	4
PG	3	2	3	2	1	2	1	2	2	1	2
PN	4	3	3	3	4	4	3	5	4	4	5
RC	3	3	2	3	1	3	3	4	2	2	3
RV	3	3	2	2	2	2	2	3	3	3	3

Table 2. Problem Solving Modes.

A first glance at the tables allows us to state the following groups: A = JR, PG , characterised by (TE, IN, levels 1–2); B = AP, CG, RV , characterised by (TR-TE, P, levels 2–4); C = LD, PN , characterised by (I, PS, levels 3–5); and KN and RC as the aforementioned exceptions. However, the real exception is RC, characterised by (I, PS, levels 2–3), given that KN could belong to B, because the weight of platonist features in this case is relevant:

TEchnological-**IN**strumentalist-levels **1–2**
TRaditional-**TE**chnological-**P**latonist-levels **2–4**
Investigative-**P**roblem **S**olving-levels **3–5** (except **RC)**

It is quite meaningful that IN is associated with a deficient control; in fact, JR and PG have got the subband 1–2 of the third category (also AP has got the level 2 in this category, but with a better performance concerning the second category). It is also remarkable that the most deficient case of the second category (subband 1–2 with only one 1) is TE and IN (JR).

However, the subband 2–3 is heterogeneous (as we could think that it is likely to correspond to intermediate bands), with 2 individuals TR-TE-P, 1 TR*-TE-IN-P, 1 I-PS and 1 TR-TE*-P, among which only the individual characterised by I-PS breaks the line of the group, but I would like to reiterate that a *good* conception is not always matched with *correct* problem solving. In addition, 1 TE-P-PS (KN) is in the subband 2–4 and, finally, in the subband 3–5 one finds individuals with a great proportion of I and PS (associations that we could label as desirable).

If we make the precision of our microscope bigger, we can analyse the second and third categories descriptor by descriptor, or by groups of descriptors with a common goal. Again, PG and JR are in the lowest subband regarding II.1. and II.2. (subband 2–3), and they are the unique solvers in it, from which one can state that

IN →	DEFICIENT CONTROL (subb. 1–2 of III)

DEFICIENT UNDERSTANDING AND PLANNING (subb. 2–3 of II.1.–II.2.)

This relationship can not be generalised because of the characteristics of this work, but it seems to be quite actual. In fact, in the previous relationship one can include the TE tendency, but I think that TE is not so determinant as IN (other TE individuals (without IN) exist with better levels).

On the contrary, the best couples II.1.–II.2, subband 4–5, come from a wider range: AP and CG are TR-TE-P, LD is S-I-P-PS* and PN is I-PS. But this fact is a claim to the absence of IN.

The lowest subband of II.3. (level 2) is composed of individuals with nearly no coincidences, showing in this way the lack of a link that can exist between efficiency as problem solver and the adequacy of a certain mathematics or mathematics teaching conception.

The lowest levels of II.4. (level 2) belong to AP, CG, JR, PG and RV, who only have in common the majority of student's role (TE) and the thought that the best descriptor of mathematics in relation to the type of knowledge is *useful* (IN3). However, the coincidences with KN (II.4. level 4) are the same, except IN3, that it is why one can establish the following relationship:

USEFULNESS [IN] → DEFICIENT USE OF REVISION (level 2 of II.4.)

The conclusions with respect to II.5. are similar to those of II.3.

Moving to the third category, the lowest level (2) of III.1. corresponds to AP, JR, PG and RV, who also have the lowest general profile. This coincidence highlights the importance of the metacognitive aspects in problem solving, in this case the importance of the attainment of a good understanding. Furthermore, they are the individuals with the largest proportion of TE (together with KN and CG) and IN (except RV, who only has IN3). On the other hand, AP is P, but he has an instrumentalist category (Aim, IN4), besides IN3. CG is different, because he has IN3, but all his categories are platonist in general. Coincidences exist also in the student's role and in some descriptors of assessment, but IN3 appears again as the most relevant one. In addition, LD and PN have the best profile also in this case, together with CG. In short, CG distorts the results, because, as TR-TE, he could belong to a lower level. However, it is likely that a conception with a big weight of PS puts aside the lowest levels (KN, who is P-PS, and RC, who is PS, are in the level 3, PN, who is PS, is in the level 4, and LD, who is P-PS*, is in the level 5). So we can conclude, not without any doubts, the existence of the following relationships:

Some **TE** features + **USEFULNESS** [IN] → SCANT IMPORTANCE TO UNDERSTANDING (level 2 of III.1.)
Large proportion of **PS** → At least, ACCEPTABLE IMPORTANCE TO UNDERSTANDING (levels 3–5 of III.1.)

Concerning III.2., one can only conclude that it depends strongly on factors that have not been analysed in this work, and again that

IN → SCANT IMPORTANCE TO PLANNING (levels 1–2 of III.2.)

but we can not conclude the association from right to left.

At the level 2 of III.3. we find AP, CG, JR and PG, who have very little in common in relation to specific descriptors; RV has the level 3, coinciding these 5 individuals in TR or TE, without coincidences with respect to their mathematics conception, except that none is PS and all of them have IN3. The highest levels are for RC (4) and LD, KN and PN (5), whose intersection is a large proportion of PS. Thus, one can state the following relationships:

TR o TE + PS (with IN3) → DEFICIENT EXPLANATION OF THE EXECUTION STATE (levels 2–3 of III.3.)

IMPORTANT WEIGHT OF PS → RIGHT EXPLANATION OF THE EXECUTION STATE (levels 4–5 of III.3.)

I have not got any association of S or I with PS. Therefore, the following relationship (possible summary of the 2 previous relationships) must be tested hereafter:

IMPORTANT WEIGHT OF PS ⟷ RIGHT EXPLANATION OF THE EXECUTION STATE (levels 4–5 of III.3.)

With respect to the descriptors III.4., III.5. and III.6., I have clearly only detected that

IN →	DEFICIENT COHERENCY AND CONTROL OF THE PROCESS (2–3 of III.4.)
	DEFICIENT TEMPORAL ORGANIZATION (1–2 of III.5.)
	SCANT METACOGNITIVE KNOWLEDGE (2 of III.6.)

In conclusion, the instrumentalist view of mathematics seems to be associated with a lack of awareness of the importance of some relevant aspects in problem solving (deficient metacognition and control of the process). The problem solving view of mathematics could be on the opposite pole. Moreover, the instrumentalist individuals of the sample have shown deficiencies concerning understanding and planning, and, in general, the third descriptor (usefulness) appears as highly determinant (as, for example, with respect to the use of revision). The technological teaching conception, and, in some cases, the traditional one, together with an instrumentalist view, has been associated with low problem solving levels, but it is likely that the instrumentalist view is more determinant.

In my opinion, attention should be paid to discovering other relationships and to confirming the aforementioned. Problem solving is an intersection point of mathematical knowledge and conceptions, and problem solving training gives us the possibility of going a long way towards professional development.

References

Carrillo, J. 1995. La resolución de problemas en Matemáticas: ¿cómo abordar su evaluación? [Problem solving in Mathematics: how to tackle its assessment?].Investigación en la Escuela, 25, 79–86. [In Spanish]

Carrillo, J. y Contreras, L.C. 1994. The relationship between the teachers' conceptions of mathematics and of mathematics teaching. A model using categories and descriptors for their analysis. In Ponte, J.P. & Matos, J.F. (Eds.) Proceedings of the 18th PME Conference. Vol. II, pp. 152–159. Lisbon: Departamento de Educaçao Universidade de Lisboa.

Carrillo, J. y Contreras, L.C. 1996. Un modelo de categorías e indicadores para el análisis de las concepciones del profesor sobre la matemática y su enseñanza [A model of categories and descriptors for the analysis of teachers' conceptions of mathematics and mathematics teaching]. Educación Matemática (in press). [In Spanish]

Carrillo, J. y Guevara, F. 1996. Un instrumento para evaluar la resolución de problemas [A problem solving assessment instrument]. Uno (in press). [In Spanish]

Chorao, M.F. 1994. Teachers' Creativity Styles and Pedagogical Practices. In Carlgren, I.; Handal, G. & Vaage, S. (Eds.) Teachers' minds and actions. Research on teachers' thinking and practice. London: The Falmer Press.

Ernest, P. 1989. Beliefs Influence in Mathematics Teaching. Mathematics Education and Society. Document Series 35. Unesco, 99–101.

Hart, L.C. 1991. Monitoring change in metacognition. Proceeding of the 15th PME Conference. Vol II, 141–148. Assisi (Italy).

Kirton, M.J. (Ed.) 1989. Adaptors and Innovators. Styles of Creativity and Problem-Solving. London: Routledge.

Nohda, N. 1995. Teaching and evaluating using ›Open-ended problems‹ in classroom. Zentralblatt für Didaktik der Mathematik, 2, 57–61.

Pajares, M.F. 1992. Teachers' Beliefs and Educational Research: Cleaning Up a Messy Construct. Review of Educational Research, 62(3), 307–332.

Pehkonen, E. 1994. On Teachers' Beliefs and Changing Mathematics Teaching. Journal für Mathematik-Didaktik, 15 (3/4), 177–209.

Pehkonen, E. 1995. Introduction: Use of open-ended problems. Zentralblatt für Didaktik der Mathematik, 2, 55–57.

Porlán, R. 1989. Teoría del Conocimiento, Teoría de la Enseñanza y Desarrollo Profesional [Knowledge Theory, Teaching Theory and Professional Development]. Unpublished Ph. Thesis. Departamento de Didáctica de las Ciencias. Sevilla University. [In Spanish]

Silver, E.A. 1995. The nature and use of open problems in mathematics education: Mathematical and pedagogical perspectives. Zentralblatt für Didaktik der Mathematik, 2, 67–72.

Stacey, K. 1995. The challenges of keeping open problem-solving open in school mathematics. Zentralblatt für Didaktik der Mathematik, 2, 62–67.

Thompson, A.G. 1992. Teacher's Beliefs and Conceptions: a Synthesis of the Research. En Grouws, D.A.(Ed.) Handbook on Mathematics Teaching and Learning. New York: McMillan.

Tillema, H.H. 1994 . Training and professional expertise: bridging the gap between new information and pre-existing beliefs of teachers. Teaching & Teacher Education, 10(6), 601–615.

Iris Kalesse

Beliefs and Change of Beliefs of First Year Students

A First Overview

Background

Up to date there has been some material published about the mathematical worldviews of students and teachers. About the mathematical worldviews of mathematics students, especially those interested in mathematics teaching not that much has been published up to now. At many universities in Germany, the quote of mathematics students quitting their studies is rather high. The question is, why? Which concepts of mathematics teaching at university, and which concepts about mathematics in general do the highschool graduates (›Abiturienten‹) have, who start studying mathematics?

What are the future mathematics teachers' worldviews on mathematics at the beginning of their studies? To which extent do their mathematical worldviews change in the course of their studies? My investigation is based upon these questions. Through interviews held with some future mathematics teachers I am going to try to compare their mathematical worldviews at the beginning of their studies with those at the end of this semester.

Before presenting some first results of my investigation, I should say something about the way in which the future mathematics teachers are being educated in the subject mathematics at German universities. At ›Gymnasium‹ or comprehensive schools there are teachers for the grades 5–10 (›Sekundarstufe I‹-teachers), for the grades 11–13 (›Sekundarstufe II‹-teachers), or for Grades 5–13 (›Sekundarstufe I+II‹-teachers). Of the future teachers (›Lehramt‹-students) only those for Sekundarstufe I have their own specific foundation courses at university. The other students start their studies in mathematics together with the diploma students. That implies that the beginners lectures of the future Sekundarstufe I-teachers take place at a lower standard and – as the lectures are attended by fewer students – in a more intimate atmosphere.

1. The methods of the study

The study is based on two series of videotaped interviews with the same seven mathematics Lehramt-students, the first of which already has taken place in the first four weeks of winter semester 1995/96, from mid-October until mid-November. The second interview series will follow at the beginning of the lectures of summer semester 1996, in mid-April. The interviews were held at the location of Gerhard-Mercator-University of Duisburg.

When working out the interviews I have being lead to a great extent by the questionnaires that Sander has introduced at the first MAVI workshop (H.J. Sander 1995).

The interviews were devided in four parts. In the first short part I obtained some formalities for example age, education, mathematical ›Grundkurs‹ or ›Leistungskurs‹. The second and largest part consist of questions, to which the interviewed persons should freely react. It essentially entails the following questions:

1. Do you have any concrete memories of your mathematical education in the ›Grundschule‹, ›Sekundarstufe I‹ or ›Sekundarstufe II‹?
a) What did you find typical of mathematical education?
b) What feeling accompany these memories?

2. a) What is the most important thing with regard to a good mathematics teacher?
b) Did you have a good mathematics teacher, what did you appreciate about him?
c) Do you have an ideal teacher?
d) Since when did you want to become a mathematics teacher?

3. What did you essentially expect from your mathematical studies? What should you be taught at university?

4. What are your fears
a) now at the beginning of your studies,
b) during your studies,
c) with regard to your later occupation as mathematics teacher?

5. What is your attitude towards mathematics (rather/very positive; neutral; rather/ very negative)?

6. Why do you study mathematics?

7. What does it actually mean to ›teach mathematics‹?

8. What is ›x‹?

9. Why is it not allowed to divide by zero?

The third part consists of the task of arranging thirteen cards with terms according to their importance with regard to mathematics, so that the term most strongly associated with mathematics will be in the first position. Finally, the following question is asked: »What is mathematics according to your view?« In addition, the students should decide whether mathematics is a human or god given concept, where the example ›politics is human‹ is given. The interviews took between thirty and forty minutes.

2. Selection of the interview partners

In total, about eight persons should be interviewed. In this regard the interpretation and not the representation is of importance. The only requirements that the interviewed persons should meet, were that they should be

1. Lehramt-students with the subject mathematics;
2. in the first semester and should not have done previous studies.

During winter semester 1995/96 only about ten students have registered for ›Lehramt Sekundarstufe I‹. Some of them had changed their studies by implication they were either registered as ›Lehramt Sekundarstufe II‹ or for another subject. Thus only three students came into question for my study, namely two women and one man, who agreed to take part in the interviews. To select some first semester Lehramt-students of Sekundarstufe II, the following was done: From a exercise group list some eight persons were selected, four women and four men. This request was put in the first lecture. Five of the eight previously mentioned students were willing to take part in the interview. However, one of these students could not take part because he had already started another course. That left me with four first semester Lehramt-students, one man and three women, which I could interview.

3. Some first results

I will mainly focus on the third part of the interviews, where the students were asked to arrange the above mentioned cards with regard to their individual assessment of how strong the particular term is associated with mathematics. The results are shown in Table 1, with rankings from 1 (first position, highest assessment) to 13 (last position, lowest assessment).

terms	Stefan (S)	Katha-rina (K)	Anja (A)	Lars (L)	Christi-ane (C)	Martina (M)	Eva (E)
motivation	1	3	2	1	1	1	1
to follow the process of obtaining the proof or development (›Nachvollziehen‹)	5	4	8	2	2	3	4
pleasure	2	1	6	11	10	4	2
knowledge or competence	4	2	4	7	6	8	9
good memory	7	10	3	8	3	7	3
independent work and self responsibility	6	9	7	3	9	2	5
feeling of success	11	5	1	6	7	5	8
understanding or mechanical learning	12	11	5	4	4	9	10
creativity	10	7	10	5	8	11	6
fantasy	9	6	9	10	12	10	7
feelings	8	8	12	9	11	12	11
learning by heart without thinking	13	13	13	13	5	6	12
fear	3	12	11	12	13	13	13

Table 1. The students' arrangements of the terms.

To establish an overall measurement of assessment, I assigned 13 points to every term at position 1, 12 for every term at position 2, etc., and took as an ›assessment score‹ of each specific term the sum of all points assigned to it. The outcoming was the following ranking from ›motivation‹ (highest assessment) to ›fear‹ (lowest assessment):

1. motivation

2. to follow process of obtaining the proof or development (›Nachvollziehen‹)
3. pleasure
4. knowledge or competence
5. good memory
6. independent work and self-responsibility
7. feeling of success
8. understanding or mechanical learning
9. creativity
10. fantasy
11. feelings
12. learning by heart without thinking
13. fear

The specific scores are given in Diagram 3.

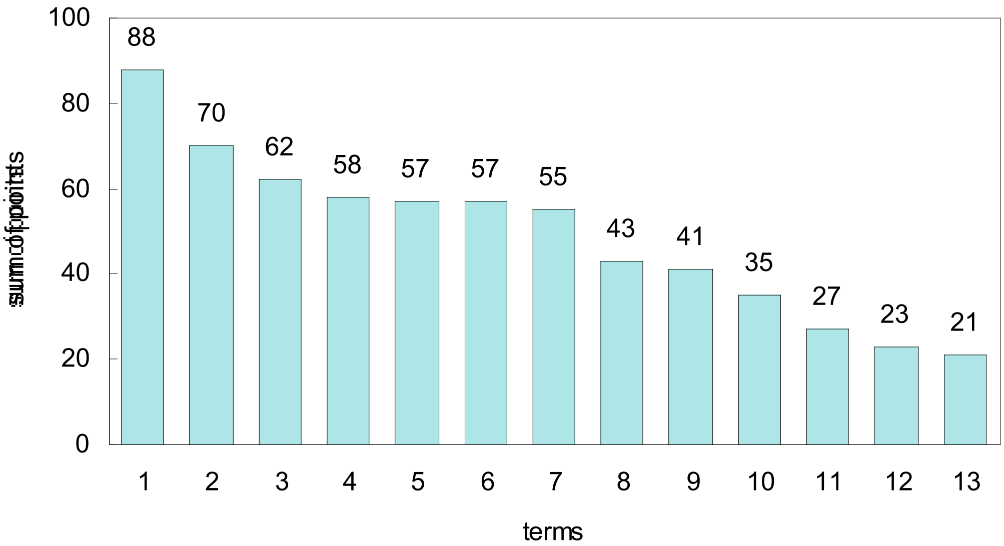

Diagram 3. The ›assessment scores‹ of the 13 terms.

It is quite clear that ›motivation‹ obtained the first position over all. Then ›to follow the process of obtaining the proof or development‹ is clearly at second position. Only Anja rated this term lower and she said: »Thus I somehow had to learn to proof and to develop these things, I have now moved in the direction of ›fantasy‹ because ›proof‹ and ›creativity‹ are closely related. Without that it wouldn't work. I then find, that it partly has nothing to do with mathematics.«

In contrast to the first two terms, the terms ›pleasure‹, ›knowledge/competence‹, ›good memory‹, ›independent work/self responsibility‹ and ›feeling of success‹ are situated closely together, somewhat distanced from the following terms ›understanding/mechanical learning‹, ›creativity‹, and ›fantasy‹.

It should be mentioned that already while videotaping the interviews it turned out, that the terms ›understanding‹ and ›mechanical learning‹ should better have been put on separate cards. Many times the interviewed students asked, which term I actually did refer to. A lot of them chose ›mechanical learning‹, others did not exactly know what term to chose. That's why it has been placed so low in the ranking. This I recognize mostly from the reasons they gave for the placing of the term. I decided to include the original remarks, because the expressions would somehow loose the intended meaning.

- »*Mechanisches Lernen (mechanical learning)* ist ja dieses so wie Auswendiglernen gemeint, find ich ziemlich blödsinnig, weil, gewisse Sachen muß man eigentlich auswendig lernen, aber ich find's blöd, weil wenn man irgendwas stur auswendiglernt, dann lernt man das, aber man weiß gar nicht, was damit gemeint ist.« (E)
- »Da hab ich mir *mechanisches Lernen* ausgesucht, das finde ich ein bißchen dumm, das kann man bei Mathe nicht anwenden, man sitzt davor und lernt alles auswendig, das ist Unfug.« (Wo würdest du das Verständnis einordnen?) »Das Verständnis würde ich weiter vorne einordnen. [...] hinter Spaß, man muß das *verstehen*, dann kommt auch die Kompetenz und das Wissen.« (K)
- »*Verständnis (understanding),* um das halt auch zu verstehen, was man nachvollzieht.« (L)
- »*Verständnis oder mechanisches Lernen*, ich würd' nicht unbedingt sagen, daß Mathematik mechanisch ist, aber man muß doch ein gewisses Verständnis dafür haben und irgendwie so 'nen Draht dazu, hat vielleicht nicht so jeder.« (A)
- »Zuerst kommt das *Verständnis* und dann kann man sich vielleicht das, was man gelernt hat noch mal anders aufschreiben oder kürzer und dann muß man diese handliche Formel, die man dann verstanden hat, *stur auswendiglernen* und daraus ergibt sich dann das *Wissen*, das ist dann praktisch die Summe von diesen einzelnen Dingen.« (C)
- *Verständnis oder mechanisches Lernen*: »ja, mechanisches Lernen, ja vielleicht sollte man das, was man lernen soll, schon ein bißchen einteilen, ehm, daß man halt Schritt für Schritt vorgeht und nicht versucht, alles auf einmal [...].« (S)
- »*Verständnis, mechanisches Lernen* ist im Prinzip nicht nur Mathe, sondern überhaupt Naturwissenschaften, würd' ich sagen.« (M)

The remaining terms (›feelings‹, ›learning by heart without thinking and fear‹) relatively little differ from each other with regard to their ratings.

It is strange that Christiane and Martina do not agree with the rating of the term ›learning by heart without thinking‹ on the lowest level. Christiane actually has the same opinion as the others. She says: »[...] dieses *sture Auswendiglernen* bringt da eigentlich wenig, weil, man muß es zwar auswendig können, aber man muß auch verstehen, wieso das so ist. [Man] muß [...] schon stur auswendiglernen, aber mit Verstand.«

She actually contradicts herself, when she says ›learning by heart without thinking but with understanding‹. Martina points out, that she likes to learn by heart, she actually regards it to be the basis of further mathematical development. With respect to the term ›fear‹ Stefan made a totally different rating (3 instead of 11, 12 or 13). According to him ›fear‹ has a lot to do with mathematics and could also be a positive aspect. He says: »Fear increases motivation.« To the question, if fear is inevitable in mathematics, he emphasizes ›fear of mistakes‹.

Anja also says something similar to Stefan, she also mentions ›fear of mistakes‹, but it seems as if she does not relate it so much to mathematics than Stefan. It seems that she has not encountered it in mathematics, because she adds: »I fear suddenly experiencing problems with mathematics, although you are not used it.«

It is not possible to present further results of the study at the present stage. After finishing the second series of interviews a deeper analysis of the mathematical beliefs and their changes of the interviewed students will be feasible.

References

Sander, H. J. 1995. ›What is a Good Mathematics Teacher?‹ – Results of an Inquiry of Students at a Gymnasium in North Rhine-Westphalia. In: Törner (1995), 57–62.

Törner, G. (Ed.) 1995. Current State of Research on Mathematical Beliefs; Proceedings of the MAVI Workshop, University of Duisburg, October 4–5, 1995. Schriftenreihe des Fachbereichs Mathematik. Gerhard-Mercator-Universität Duisburg. Preprint Nr. 310.

Pekka Kupari

Changes in Teachers' Beliefs of Mathematics Teaching and Learning

Introduction

Since now mathematics has been considered, due to the assumptions of universality, as neutral subject – i.e. culture-free, societally-free and value-free. Bishop (1993, 7) has stated that »we are in danger of taking the ideas and values of universally applicable mathematics so much for granted that we fail to notice them or to question them or to see the possibility of developing alternatives«.

A full analysis of what is happening in mathematics education in the most of countries must include an examination of more than form, content and surface features. According to Neyland (1995) »it must look at the beliefs and values and power relations, which underpin both teaching practice and the structure and composition of curriculum«.

But as we know it is very difficult to undertake this kind of analysis because the world of beliefs, values and power relations is complex and dynamic. The research literature on teachers' mathematical beliefs points clearly out the difficulty in overcoming these ›coloured‹ beliefs developed during previous school experiences. However, teachers are key to the success of current reform movement.

In my earlier research on beliefs I have studied the structure of mathematics teachers' beliefs and conceptions. In addition, I have discussed the connections between teachers' mathematical beliefs and their teaching practices (Kupari 1995). In this paper I discuss the changes taken place in the beliefs of the Finnish comprehensive school mathematics teachers about mathematics teaching and learning in the first half of 1990s.

1. On the study

1.1. Some theoretical issues

The underlying grounds of my study lie on the theoretical frameworks presented by Ernest (1991), Conroy (1987) and Thompson (1991). Quite a lot of research has been made concerning teachers' beliefs and changing the teacher (e.g. Hoyles 1992, Pehkonen 1994, Pehkonen & Törner 1994, Clark & Peterson 1986, Middleton et. al 1990, Nespor 1987, Thompson 1992, Neyland 1995). This research has produced for instance the following results:

- The development of a given teacher's beliefs of mathematics teaching and learning is influenced by the personal experiential background of that teacher, including his/her professional and educational experiences, and how these are interpreted and internalized by the teacher.

- Curriculum changes may have consequences on teachers' belief systems.
- Change (in teacher) must occur from within and cannot be imposed from above: not trying to change beliefs in order to have the ›right‹ effect but rather as a means to throw light on beliefs, beliefs-in-practice and on the innovation itself. This would mean helping teachers and prospective teachers become reflexive and self-conscious of their beliefs and presenting objective data on the adequacy or validity of these beliefs.
- The mechanism for the change in beliefs and values should not be described as conversion – it is more like absorption. Teachers are compelled into changing their language and practices; their beliefs and values change accordingly.

Studies of teachers' conceptual change that provide detailed and insightful analyses of such changes are necessary to improve our understanding of the mechanisms that bring about the restructuring and development of teachers' conceptual schemes. A better understanding of those mechanisms is critical to the design of strong and truly successful teacher education and enhancement programs (Thompson 1992).

1.2. The methods of the study

The nature of the study is survey-type and the empirical data has been collected in two phases. The Institute for Educational Research has conducted two national assessment studies during the last six years: the first one in 1990 and the second one in 1995. In both assessment studies versatile information about mathematics teachers and their instruction was collected using the teacher questionnaire. The questionnaires consisted also the belief inventory, which was originally developed by Erkki Pehkonen and Bernd Zimmermann (Pehkonen & Zimmermann 1990). The teachers produced also free answers, which gave in some degree a possibility to validate the information of the inventory. The description of the empirical study is presented in Table 1.

Phase of study	Measurement of beliefs	Sample of teachers
1990	Belief inventory 29 Likert-type statements Free answers (written)	68 teachers of grade 9
1995	Belief inventory 30 Likert-type statements 23 same statements as in 1990 Free answers (written)	68 teachers of grade 9 15 same teachers as in 1990

Table 1. The description of the empirical study.

The following three research questions were presented:

1. What were the overall changes in teachers' mathematical beliefs from 1990 to 1995 like?
2. How could we closely analyse changes in teachers' mathematical beliefs?
3. What were the results like when changes were analysed using teachers' individual belief profiles?

2. Some results

2.1. What were the overall changes in teachers' mathematical beliefs from 1990 to 1995 like?

Changes in teachers' mathematical beliefs were first assessed within the whole teacher samples and through the means of the common belief variables (see Diagram 2).

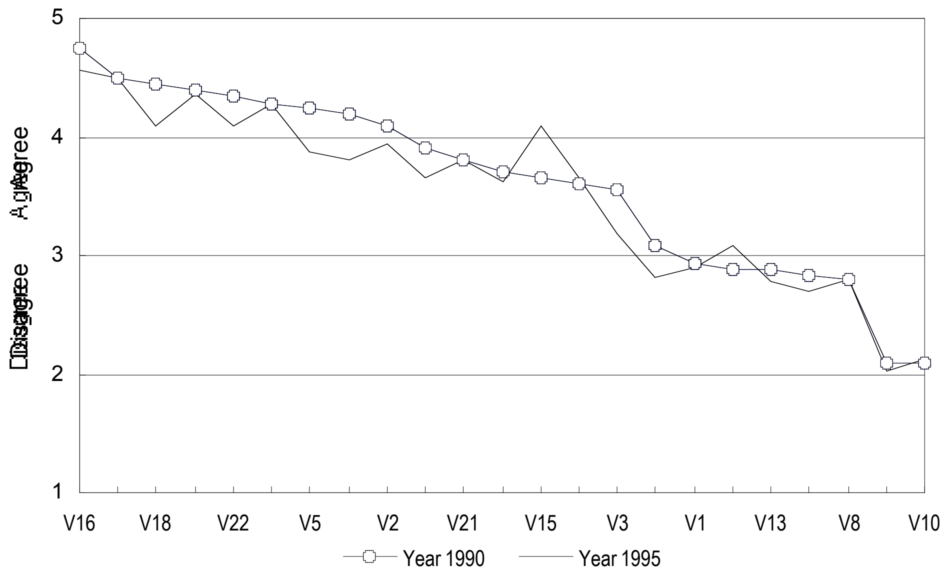

Diagram 2. Comparison of the means of the common belief variables in 1990 and 1995.

In both studies the teachers were very unanimous as to the following statements (the six first variables in Diagram 2):

- »During math lessons one should emphasize the importance of mathematical thinking.«
- »The student should have the possibility to experience that the same result can be achieved in different ways.«
- »One should rather cover fewer contents thoroughly than many contents superficially.«
- »The teacher should encourage the students to find different strategies for solving problems, and to discuss these strategies.«
- »One should use as often as possible problems where the student has to think first, and where the mastery of numerical calculations alone will not lead to the solving of the problem.«

- »In the teaching one should clearly point out that mathematics is an essential factor in our culture playing an important role in society and everyday life.«

Correspondingly, the teachers were most in disagreement in the statements (the two last variables in Diagram 2):

- »The most important task for the teacher is to maintain good order in the class.«
- »When solving problems it is most important that the students get right answers.«

The overall observation was that the teachers' mathematical beliefs were quite stable and changes very small. But we certainly know that when we examine results based on the averages, quite a lot of information will be lost.

2.2. How could we closely analyse changes in teachers' mathematical beliefs?

Using the data of the 1995 study phase belief profiles of all 68 teachers were produced. On the basis of these profiles two criterion teachers were defined: *the traditional type* and *the innovative type*. For example, Middleton with his colleagues (1990) have used a similar method (cluster analysis and profile technique) in describing elementary teachers' beliefs.

The traditional teacher type could be said to represent the behaviourist or neo-behaviourist learning and teaching tradition (Neyland 1995). In this tradition mathematics is approached from an absolutist viewpoint. The content has broken down to a sequence of tasks to be mastered and facts to be learned. The focus tends to be on what students can do, rather than on what understandings and meanings have been achieved. If teachers overemphasize rote reproduction of knowledge and skills, then understanding suffers. Students are not really worried if they fail to understand what the teacher is getting at – they believe that if they can get right answers, then they understand.

As regard to *the innovative teacher type* it would be typical the orientation towards the alternative approach of teaching, i.e. the social constructivism. It views mathematics as fallibilist and knowledge as emancipatory. The teacher has the responsibility of aiding this reconstructive process, which involves learning the concepts, orientations, values and processes of the expert community as well as seeing none of these as beyond examination and revision (Neyland 1995).

Concerning the beliefs measured in the empirical study these teacher types had much to common but also many separating features. The most important deviations between these two types were connected to the features like

- the role of calculation techniques,
- the amount of routine problems,
- the amount of students' (rote) exercise,
- the role of mathematically correct language,
- the need of understanding in learning,
- students' possibility to formulate their own problems and solve them.

On the basis of these criteria a trial was made to categorize all the teachers of the 1995 study. The result was not clear at all. Certain groups of teachers which were more traditional or more innovative in nature were found. However, a part of teachers (perhaps 20% of all) could not be properly located in either group.

2.3. What were the results like when changes were analysed using teachers' individual belief profiles?

The profile method was used to closely examine the changes in beliefs of those 15 teachers which were same in both studies. Analysing the differences of the teachers' profiles in 1990 and 1995 the following observations were made (cf. Table 3):

- There were ten teachers whose beliefs were close to the beliefs of the traditional teacher type. In the beliefs of five teachers no essential changes had been taken place. Then there were five teachers whose beliefs had changed either to the traditional or to the innovative direction (cf. Thompson 1991).
- There were five teachers whose beliefs were close to the beliefs of the innovative teacher type. From 1990 to 1995 these teachers' beliefs had changed more to the innovative direction.

Teacher	Direction of change	Nearest teacher type
1	→ traditional	traditional
2	±	traditional
4	→ innovative	traditional
5	±	traditional
6	±	traditional
8	→ innovative	traditional
9	→ innovative	traditional
11	±	traditional
12	→ traditional	traditional
14	±	traditional
3	→ innovative	innovative
7	→ innovative	innovative
10	→ innovative	innovative
13	→ innovative	innovative
15	→ innovative	innovative

Table 3. Comparison of the teachers' profiles in 1990 and 1995.

3. Conclusions

In this paper I have discussed the questions concerning changes in mathematics teachers' mathematical beliefs. The empirical data were collected from the Finnish comprehensive schools in 1990 and in 1995. On the basis of the results some conclusions could be presented.

First, mathematics teachers seemed to hold multidimensional beliefs as to the nature of mathematics teaching and learning (cf. Middleton et. al 1990).

Second, the results showed that teachers' beliefs tend to change in some degree also as the result of the ›natural development‹ and without any special interventions or development projects. On the other hand, the results revealed a real need for the teachers' pre- and in-service training, where teachers could become aware of their own beliefs of mathematics learning and teaching. Simultaneously, teachers must also be helped to construct new beliefs which could replace the (old) rejected beliefs (cf. Nespor 1987). These interventions for teachers' professional development are really demanding and quite long-term. However, there exist more and more examples of these interventions (e.g. Cobb et al. 1990, Battista 1992).

Third, there were some certain features in teachers' beliefs, in which change could be especially seen. In 1995 teachers seemed more strongly to believe that it is important to pay attention to the students' processes in solving problems and not only to having (right) answers. Furthermore, teachers tended to notice the highly important fact that students must necessarily understand the arguments and procedures presented by the teacher in order to meaningfully construct his/her own learning.

Fourth, mathematics teachers seemed to have some strong ›core beliefs‹ (cf. Kaplan 1991), in which they absolutely persisted. These beliefs were connected to mathematically exact use of language, the emphasis of calculation techniques and students' extensive exercise (drill). Although these issues are important in learning mathematics, they in their extremely forms seem anyhow to automatically exclude many as important issues (cf. the relationship between the normal and formal language, the calculation techniques vs. the use of mathematical knowledge, the meaningfulness in exercise).

References

Battista, M.T. 1992. Teacher beliefs and the reform movement in the mathematics education. Phi Delta Kappan. 462–470.

Bishop, A.J. 1993. Influences from society. In A. Bishop, K. Hart, S. Lerman & T. Nunes (Eds.) Significant influences on children's learning of mathematics. Paris: UNESCO, 1–26.

Clark, C.M., & Peterson, P.L. 1986. Teachers' thought processes. In M. C. Wittrock (Ed.) Handbook of research on teaching (3rd ed.). New York: Macmillan, 255–296.

Cobb, P., Wood, T., & Yackel, E. 1990. Classrooms as learning environments for teachers and researcher. In R. Davis, C. Maher, & N. Noddings (Eds.) Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education Monograph. Reston, VA: National Council of Teachers of Mathematics, 125–146.

Conroy, J. 1987. Monitoring school mathematics: teachers' perceptions of mathematics. In T.A. Romberg & D.M. Stewart (Eds.) The monitoring of school mathematics: Background papers. Volume 3: Teaching and future directions. Madison: Wisconsin Center for Education Research, 75–92.

Ernest, P. 1991. The philosophy of mathematics education. London: Falmer Press.

Hoyles, C. 1992. Mathematics teaching and mathematics teachers: A meta-case study. For the Learning of Mathematics 12 (3), 32–44.

Kaplan, R.G. 1991. Teachers' beliefs and practices: A square peg in a square hole. In R.G. Underhill (Ed.) Proceedings of the thirteenth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Blacksburg, VA: Virginia Tech, 119–125.

Kupari, P. 1995. Mathematics teachers' beliefs and conceptions of mathematics education. In E. Pehkonen (Ed.) NORMA-94 Conference. Proceedings of the Nordic Conference on Mathematics Teaching in Lahti 1994. University of Helsinki. Department of Teacher Education. Research report 141, 110–115.

Middleton, J.A., Webb, N.L., Romberg, T.A. & Pittleman, S.D. 1990. Teachers' conceptions of mathematics and mathematics education. Wisconsin Center for Education Research. School of Education. University of Wisconsin-Madison.

Nespor, J. 1987. The role of beliefs in the practice of teaching. Journal of Curriculum Studies 19(4), 317–328.

Neyland, J. 1995. Neo-behaviourism and social constructivism in mathematics education. In A. Jones, A. Begg, B. Bell, F. Biddulph, M. Carr, M. Carr, J. McChesney, E. McKinley & J.Y. Loveridge (Eds.), SAMEpapers 1995. Science and Mathematics Education Papers – 1995. Hamilton: Centre for Science, Mathematics and Technology Education Research, 114–143.

Pehkonen, E. 1994. On teachers' beliefs and changing mathematics teaching. Journal für Mathematik-Didaktik 15 (3/4), 177–209.

Pehkonen, E. & Törner, G. 1994. Development of teachers' conceptions about mathematics teaching: Which are key experiences for the change of conceptions? Schriftenreihe des Fachbereichs Mathematik. Gerhard-Mercator-Universität Duisburg. Preprint Nr. 270

Pehkonen, E. & Zimmermann, B. 1990. Probleemakentät matematiikan opetuksessa ja niiden yhteys opetuksen and oppilaiden motivaation kehittämiseen. Osa 1: Teoreettinen tausta and tutkimusasetelma. Problem fields in mathematics teaching and their connection to the development of teaching and pupils motivation. Part 1: Theoretical background and research design. University of Helsinki. Department of Teacher Training. Research report 86. English abstract.

Thompson, A.G. 1991. The development of teachers' conceptions of mathematics teaching. In R.G. Underhill (Ed.) Proceedings of the thirteenth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Blacksburg, VA: Virginia Tech, 8–14.

Thompson, A. 1992. Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grows (Ed.) Handbook of research on mathematics teaching and learning. New York: Macmillan, 127–146.

Marja-Liisa Malmivuori

Self-Confidence as a Framework for Mathematics Learning

Introduction

The study on mathematical attitudes have traditionally represented a domain, where students' affective responses and affectively toned values or preferences in mathematics learning have been considered as influential factors in their mathematical performances. In this, maybe the most influential are students' understandings of themselves in relation to mathematics. Usually these features have been considered under the title of students' self-concept of ability in mathematics. But through the attitudinal studies performed especially by Fennema and her colleagues (see e.g. Fennema, 1989), and the studies of self-efficacy in mathematics learning – based on Bandura's (1986) social learning theory –, self-confidence in mathematics has been acknowledged as an important single determinant behind students' mathematical performances and their motivation to study mathematics, as well as the gender differences in these.

»Confidence in one's ability to learn and to perform well on mathematical tasks« is the most commonly used definition of students' self-confidence in mathematics, given by Fennema & Sherman (1976). When we classify self-confidence among students' beliefs about self in mathematics, the construction can be defined as »a belief about one's competence in mathematics« (McLeod 1992). As a part of one's evaluative self – i.e. self-esteem – in mathematics, students' self-confidence can be seen to form a central framework, through which they approach mathematics learning situations and try to do mathematics. – Self-confidence can then be defined as the functional part of students' mathematical self-concept, i.e. »self-concept at work« in mathematics learning situations.

1. The object of the study

The aim of this study was to clarify the role that students' self-confidence in mathematics have in the prediction and explanation of their mathematics achievements in Finland, in relation to some other features in their mathematical belief structures, that has been suggested to have an important impact on mathematics achievements and motivation in resent studies of mathematical beliefs and attitudes. Further objects of the study were to adjust the gender difference between females' and males' self-confidence levels in mathematics usually found in foreign attitudinal studies, and to consider the predictive effect of self-confidence levels on mathematical performances for females and males separately, both generally and within separate mathematical domains.

2. Subjects of the study

The sample of the study consisted of 182 female and 193 male students from 19 ordinary lower secondary schools and 25 classes over the country in Finland. The data of the study was collected in connection with the Finnish part of an international research project called KASSEL-Project (Blum & al., 1992) and designed to study the development of students' mathematical skills at lower secondary school level in different countries. The data was based on students' scores in three mathematics tests of the KASSEL-project in 1994 and 1995, and on their responses to a self-report questionnaire in 1994, designed to measure students' mathematical beliefs in the Finnish schools of the project.

3. Measurements

The belief data of the study was obtained through the structured parts of a self-report questionnaire, that measured students' self-confidence in mathematics and their beliefs about mathematics, mathematical problem solving, and mathematics learning and teaching with a continuous scale, ranging from –5 (fully disagree) to +5 (fully agree). The items for the self-confidence scale was mainly adapted from Fennema & Sherman's (1976) Confidence in Learning Mathematics Scale, including statements as »I think I could learn more difficult mathematics« or »I am not the type to do well in mathematics«. Responses to the negative statements were revised, and in all 9 items were added up to measure consistently ($\alpha = .76$) the degree of students' self-confidence in mathematics.

The rest of the constructed sum variables ($.41 \leq \alpha \leq .70$) measured the strength in students' beliefs about: their weakenesses and strengths in mathematics (Clear View of Self), mathematics usefulness (Usefulness), mathematics as based on innate mathematical ability and on faultless performances (External View of Mathematics), mathematics learning as based on own high effort and use of self-regulative strategies (Effort), mathematics teaching and learning as based on active interaction and help from teacher (Teacher Dependence) or on co-operation with other students (Co-Operation). The measures of mathematical achievements consisted of students' scores in Number Test, Algebra Test, and Geometry Test, and sum totals of these scores in 1994 and 1995. In all 40 minutes was allowed for the completion of each test, with problems arranged in order of increasing difficulty and resulting in maximum 50 points.

4. Results

4.1. General findings

The correlational considerations of the studied belief variables and mathematics scores for all students showed clearly the highest (positive) correlations between students' self-confidence and their achievements, with the same statistically highly significant correlation of .526 for total achievement scores both in year 1994 and 1995. Examination of the correlations further revealed, that most of the connections between the achievements and the other studied belief variables were strengthened within a year, but still the statistically significant correlations between the total test scores in 1995 and the other belief variables ranged only from .133 (for Co-Operation) to .266 (for Effort). The same tendency could be seen in these correlations for each mathematics test (i.e. Number Test, Algebra Test, Geometry Test) separately.

Support for the separate considerations of females' and males' belief structures and achievements was gained through a performed t-test, that confirmed females significantly ($p <$

.001) lower confidence in their ability to learn and perform well in mathematics compared with males. Instead, no apparent gender difference was found in the correlations between students' mathematics scores and their self-confidence levels. The performed additional correlational studies for females and males separately further revealed, that the positive connections between the test scores and students' view of mathematics as useful, or preference for own effort or co-operation in mathematics learning were stronger among males than among females. Instead, slightly stronger negative connections were found between females' test scores and their tendency for external view of mathematics.

4.2. Self-confidence as a predictor of mathematics achievements

Linear simple and stepwise regression analyses were performed in order to examine the predictive impact of students' self-confidence levels on their mathematical performances against the other studied belief variables.

The first obtained regressions of students' total sums of mathematics test scores, with the studied seven belief variables as independent variables, confirmed students' self-confidence as the best predictor of their mathematics performances, with the explained variances of 27.2% (in 1994) and 26.9% (in 1995) for females, and 24.7% (in 1994) and 27.9% (in 1995) for males. The other significant but weaker predictors after Self-Confidence were External View of Mathematics for females (with additional 1.7%) both in 1994 and 1995, and for males Usefulness (with additional 2%) in 1994, and Effort (3.4%), Co-Operation (1.7%), and External View of Mathematics (1.8%) in this order in 1995. As single predictors these other influential belief variables accounted not more than 5% of the total variances in females' test scores and 11.4% of the total variances in males' test scores. After the three most efficient predictors of these variables (i.e. Effort, Usefulness, External View of Mathematics) were forced to the equation, still 20.6% of the variance in total test scores in 1995 could be explained by Self-Confidence for females and 16.1% for males.

The examination of the predictive power of students' self-confidence for their mathematics achievements in 1995 within different mathematical domains produced some further results. The proportions in the variances of mathematics test scores, explained by males' self-confidence levels, were slightly larger than those of females', within each of the three mathematics tests. The largest proportion was found in students' scores in the Number Test, for both females (26.4%) and males (26.9%), but gender differences turned up in the regression equations for the test scores. Self-Confidence was the only significant predictor of females' scores in the Algebra Test, whereas males' scores in this test could be explained with three other belief variables besides Self-Confidence (i.e. Effort, External View of Mathematics, Co-Operation). Only one significant predictor besides Self-Confidence (i.e. Co-Operation) was found for males' scores in the Number Test, where also the predictive power of Self-Confidence for males was the largest.

Consideration of the regression equations and partial correlations in these regressions further revealed, that females' self-confidence levels were significantly mediated by their preference for own effort and self-regulation in mathematics learning, and their view of mathematics as useful. Instead, the negative effect of the external view of mathematics on their scores was rather independent from their self-confidence levels. Males' self-confidence levels again operated more as an independent predictor of their mathematical performances, besides which also their beliefs about own effort and self-regulation in mathematics learning, about co-operation, about mathematics usefulness, or about the nature of mathematical ability and faultless performances affected their performances - regardless of their self-confidence levels.

5. Conclusions

Even if the predictive power of students' self-confidence levels was about the same amount for both females' and males' performances, for males the proportion explained by the other significant belief predictors than self-confidence was consistently larger than that of females'. Moreover, the impacts of these other significant variables on mathematics scores were mediated more effectively through females' self-confidence levels, than the self-confidence levels of males'. Only females' view of mathematical ability and the role of mistakes in performances could be clearly detached from their self-confidence levels. Again, such central belief constructions represented by Effort, Co-Operation, and Usefulness variables affected males' mathematical performances also independently of their self-confidence levels. (see also Malmivuori, 1996).

In addition to the obtained gender differences in the general patterns of self-confidence and test scores, variation was found also in the roles that students' self-confidence levels played in different mathematical domains. Females' self-confidence represented an especially important factor in predicting their performances in algebra – in relation to the other belief variables, whereas males' performances in calculation were more clearly than in the other mathematical domains predicted by their self-confidence levels. These results strengthen the view, that students' self-confidence levels have a special role in their mathematical belief structures and that this role is importantly intertwined with their mathematical performances. Gender differences in these patterns of self-confidence may further represent the key to the explanation of the variation in females' and males' mathematical experiences and performances.

References

Bandura, A. 1986. Social foundations of thought and action: A social cognitive theory. Prentice-Hall, Inc., Englewood Cliffs, NJ.

Fennema, E. 1989. The study of affect and mathematics: A proposed generic model for research. In: Affect and mathematical problem solving (eds. D.B. McLeod & V.M. Adams), 205–219. New York: Springer-Verlag. Fennema, E. & Sherman, J.A. 1976. Fennema – Sherman mathematics attitudes scales. JSAS catalog of selected documents in psychology, 6, 31 (Ms. Mo. 1225).

Malmivuori, M.L. 1996. Self-confidence in students' mathematical belief structures. To be published in ZDM, 4 /1996.

McLeod, D.B. 1992. Research on Affect in Mathematics Education: A Reconceptualization. In: Handbook of research on mathematics teaching and learning (ed. D.A. Grouws), 575–596. New York: Macmillan.

Christoph Oster

Details about Coaching Mathematics as Seen by an Active Teacher Involved

Background

Private coaching is an international educational phenomenon which has been recognized for decades. There have been scientific studies on this subject in Germany from the mid-fifties onwards. The studies have shown that in Germany nearly every second pupil needs some coaching while he attends school. In the last few years more and more pupils, even of the ›Grundschule‹ from grades one to four, have private lessons. All empirical studies prove that ›Mathematik‹ and ›Englisch‹ are the most common school subjects for coaching in Germany. Nearly 30 % of all private lessons are lessons in mathematics. Therefore it seems necessary to conduct more quantitative and even qualitative research to describe and understand the global concepts of coaching mathematics more precisely.

In this article, I am referring to an interview I made with a coaching teacher at the end of 1995. First of all some information about the coach:

This teacher has been giving private lessons in form of individual tuition for more than 7 years. He has taught 30–40 pupils in the last few years and he teaches 3 pupils at the moment. Most of them attend a ›Gymnasium‹ (secondary school or grammar school), some of the elder ones attend a ›Gesamtschule‹ (comprehensive school). The coach is 24 years old, quite experienced in teaching but he has not become a qualified teacher. He has passed his intermediate examination for the diploma in mathematics and computer science. The analysis of the interview deals with three questions:

1. What are the main problems in learning mathematics according to this coach?
2. What are important elements of his concept of coaching mathematics?
3. What factors is this concept of coaching mathematics influenced by?

1. What are the main problems in learning maths according to this coach?

Some quotations (translated by the author):

a) *The coach's assessments of pupils' attitudes and behaviour*

- pupil is untalented;
- pupil is lazy;
- pupil is not attentive enough;
- pupil does not work carefully enough;

- pupil is not interested (»Mathematics is boring, too theoretical, ... is no fun.«);
- pupil is not able to get enthusiastic about mathematics;
- pupil is convinced of being unable to do mathematics successfully;
- pupil is quite disheartened and lacks self-confidence.

b) *Aspects on learning mathematics*

- they show many gaps with respect to several mathematical subjects (basic algebra, calculus, geometry);
- they do not understand mathematics in principle.

c) *Other aspects*

- pupil has changed school; teacher has changed;
- teacher's methodological concept is wrong or his behaviour is not acceptable;
- there are social problems in school situation or in classroom situation (»If they do not get along with the teacher, mathematics is no fun.«);
- there are private problems at home or with girl- respectively boyfriends.

There are quite many and diverse factors the coach connects with the bad results pupils have. Most of them are due to the pupils' situation, to their disposition, their attitudes and their self-confidence. Other aspects can be attributed to several social problems while the mathematical aspects are quite global. But it is very interesting to have a another look at the last aspect of list b), because this coach has quite an interesting view of the intensity of understanding. I will come back to this aspect.

Result: The general in-classroom-aspects this coach refers to are subtly differentiated whereas the problems concerning doing mathematics are seen more superficially.

2. What are important elements of his concept of coaching mathematics?

a) *Quotations referring to his aims of coaching*

»I want to show that mathematics can be fun, provided that you have a good command of mathematics.«

»I want to help them to catch up on learning mathematics.«

»I want to help them learn successfully without coaching.«

»I want them to have more fun than before.«

The quite general aim to help pupils in doing mathematics is obviously shaped by aims belonging to the affective dimension. ›Fun‹ might be a significant part of his own concept of doing mathematics and he even wants to transfer ›fun‹ into the concept of pupils' way of doing mathematics. In this case one can observe a relationship between his view of pupils' problems, see above, and his own aims.

b) *methods*

His methods might be influenced by his own experiences. At school he enjoyed mathematics, but in grade 8 he was ill for a long time. So he got some difficulties and there was no fun, he said. He himself was coached privately and simultaneously he got a highly qualified teacher at school, and he soon learned successfully. He concluded, that one lesson per week is not enough to start with coaching.

At the beginning of coaching	teachers comments	later
Pupils are very insecure.	All of them have some talents even in mathematics.	Pupils are more sure. Pupils have more courage and self-confidence.
All questions will be explained very often. Realisation of knowledge by coach whether • facts are only learned by heart • facts are understood • procedures of problem solving are understood. afterwards: • further explanation by pupil • more exercises	I give them new heart. I give them a logic (system) ... ... because knowledge has to become understanding.	Pupils have to learn methods of learning: • learn to look up formulas on their own • learn to look up exercises in their books. • learn to take notes. • learn why notes must be done carefully. • learn why comments are necessary in their exercise books.

Table 1. Teacher's predominant method in connection with his estimations

Result: The simple help in doing mathematics implies instructions how pupils should organize their way of learning. This progression goes together with the pupils' growing self-confidence.

Summary of comments to the exercises in quotation: It is important that ...

- poor pupils do their exercises correctly;
- exercises should gradually become more difficult;
- pupil is able to do 5 to 6 or more out of ten exercises on their own;
- pupils feel that they are able to work successfully;
- pupils make positive experiences.

	comments	reasons	comments
Conversation (until 1/2 hour)	... when the rela-	Pupils will work	I attach impor-

• about private affairs • about some general problems at school	tionship becomes closer	more naturally. Coach's influence is more intensive.	tance to this practice.
Start of learning • asking questions • solving problems • repetition	... ask questions to help think in the right way	Pupils feel that they have done it themselves. That increases their understanding and pupils will enjoy mathematics.	... very important. It may take up all the time.
Exercises • from school lessons • analogous exercises • exercises with mixed problems	with increasing difficulty	... to comprehend step by step ... to transfer step by step	

Table 2. Procedures of a singular private lesson.

Results: There are three important aspects in his assessment of teaching private lessons successfully:

1. establishing a positive private relationship between coach and pupil;
2. providing pupil with positive impressions and thus making sure that mathematics is fun;
3. no fixed schedule; taking time as necessary (mostly without any payment).

It is very interesting that ways of demonstration – one of the most important aspects in our didactics – do not seem equally important to this coach.

3. What factors is this concept of coaching mathematics influenced by?

Table 3 shows how the coach differentiates between various kinds of understanding mathematics and how he characterizes them. In addition to this one can observe a certain progression in the coach's understanding of the process of teaching mathematics: it must necessarily be combined with the experience of fun.

knowledge (›Wissen‹)	understanding (›Verstehen‹)	comprehension (›Begreifen‹)

• pupils only learn by heart • logic is missing	• pupils only make use of formulas • connections are not clear • pupils are unsure • mathematics is no fun yet • pupils only remember for a short time • pupils feel that they have done it themselves; that increases understanding and pupils enjoy mathematics.	• pupils develop connections quickly • pupils deal with mathematics more easily • fun is an important factor
Influences by his own experiences at university: »Understanding at university often has come very late, ... one year later sometimes.« »Something I have not understood until now. I think something I have not even comprehended yet.« Consequence: Coach wants to give up studying mathematics although he passed his intermediate examination.		

Table 3. Various kinds of understanding.

I think one key to the basis of his concept is the sensation of fun and pleasure (in German: ›Spaß‹, ›Lust‹). In the collection of all sentences of the interview in which one of these two words ›Spaß‹ (fun) and ›Lust‹ (pleasure) are part of, this affective aspect applies to several dimensions:

- to the aims of coaching: fun belongs to the aims of the coach and to the aims of his coaching. It is even the reason why the coach teaches mathematics;
- to his opinion of pupils problems in learning mathematics: lack of fun is regarded as a cause of problems;
- to his idea of the intensity of understanding (see Table 3)
- to the characteristics of teaching.

During the interview he repeatedly touched this aspect:

- Quotation: »First pupils say that mathematics is dry and there is no fun.«
- A later Quotation: »Then, if they are successful in doing mathematics they will have fun again.«
- Another quotation: »Fun and enthusiasm is absolutely necessary in mathematics lessons, but before they have to recognize that it makes sense to do mathematics.«

In his opinion fun and success in mathematics depend on pupils' insight into the logic of mathematics.

4. Conclusion

The coach's conceptions of teaching mathematics are mainly influenced by two factors, his own *experiences of learning mathematics* and *emotions (fun)*. Both aspects are part of his intensive self-reflection and are important factors that influence his procedure in private lessons. The *science and the didactics of mathematics* do not seem to be of considerable importance.

Jukka Ottelin

Views of Finnish Twelfth-Grades about Mathematics Teaching

1. On the study

1.1. The aim of the study

The aim of this survey is to clarify the views and conceptions of Finnish 12th-graders concerning mathematics learning/teaching. The following research problems can be derived from the aim of the survey:

1. What are students' views of mathematics teaching ?
2. What kind of experiences (good and bad) and wishes do students have about mathematics teaching?
3. Are there any differences in these two questions between boys and girls and between shorter and longer courses of mathematics ?

1.2. Practical realization of the survey

The questionnaires were filled up in two upper-secondary schools near Helsinki in January 1994 by altogether 152 students, 75 girls and 77 boys. There were 70 longer course students (15 girls and 55 boys) and 82 shorter course students (60 girls and 22 boys), see Figure 1.

In Finland the upper-secondary school means 10-, 11- and 12-grades. The longer course in mathematics includes 11–16 courses (4–5 lessons a week) and the shorter one includes 6–9 courses (2–3 lessons a week). The length of the courses is 38 lessons.

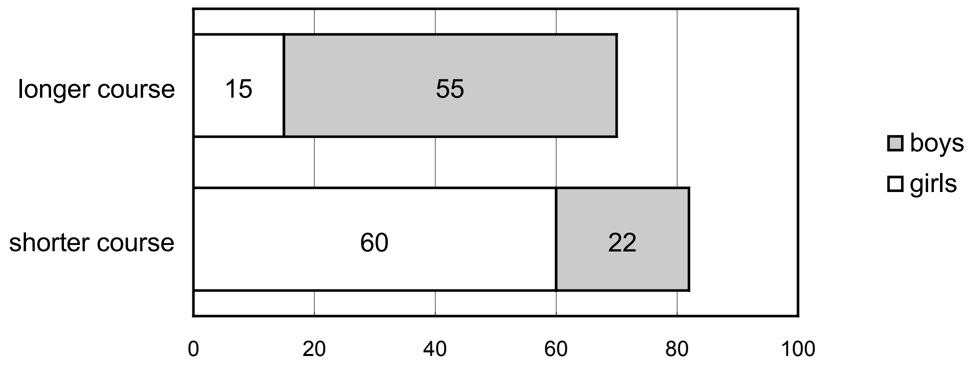

Figure 1. The structure of the students' groups.

2. Some results

2.1. ›Top ten‹-list of the statements

It seems that practical benefits are very important for all, especially for the shorter course students. About the mathematics lessons students often say : »Why are we studying this, where can we use this?«.

Statements	Mean	SD
19: The idea that studying mathematics has practical	1.49	0.80
30: All or as much as the pupil is capable of will be understood.	1.65	0.70
29: There will be as much practice as possible.	1.68	0.66
14: Use of calculator.	1.73	0.77
24: There is usually more than one way to solve the problems.	1.76	0.79

Table 1. The most agreed statements.

Statements	Mean	SD
20: Only the mathematically talented pupils can solve	4.27	0.93
7: One ought to always get the right answer very	3.97	0.83
12: Much will be learned by memorizing rules.	3.95	0.83
17: Different topics, such as calculation of percentages, separately.	3.70	1.06
10: There is always some procedure which one ought to	3.60	1.06

Table 2. The most disagreed statements.

I suppose that the top ten-list tells about the last-grade students' feelings. Their ›Abiturprüfung‹ is near and they are thinking of it a lot. They are concentrating on passing the test and it is very important for them to practise practical problems with a calculator and so that everything will be understood. In the last few years there have been more practical problems than before in ›Abiturprüfung‹.

2.2. Differences in means between the boys and the girls

Item with statistically very significant difference. There is a very significant difference between the boys and the girls when they make a choice between the longer and the shorter course. There were 15 girls and 55 boys in the longer course. Boys disagreed more than girls with the statement 17 (different topics will be taught separately). The boys saw the mathematics more holistically than the girls.

Items with statistically significant difference: items 4, 8, 18, 25 and 34. The girls wanted more practising and the strict discipline and the use of the games. The boys were more interested in mathematics and saw that it was important that students could sometimes make guesses and use trial and error.

Items with statistically almost significant difference: items 5, 11 and 20. The girls agreed more with the idea that everything ought always to be expressed as exactly as possible and that all students must understand. The girls disagreed more with the idea that only mathematically talented students can solve most of the problems.

2.3. Differences in means between the longer course and the shorter course

Item with statistically very significant difference: items 17 and 34. The students of the longer course considered that mathematics is holistic and they are more interested in mathematics. In the last years there has been discussion that the shorter course shouldn't be a shortened version of the longer course but it must have a clearly different holistic curriculum. The contents of the new curriculum are now almost ready.

Item with statistically significant difference: item 2. The students of the longer course agreed more with the idea that there are usually more than one way to solve problems.

Items with statistically almost significant difference: items 2,7,11,16,22,23,25 and 27. The students of the shorter course thought more that the process is more important than the quick answer. They wanted that all would understand and that everything would always be reasoned exactly. The students of the longer course thought that mathematics requires a lot of effort and it is important that they are led to solve problems on their own. They also thought that the good mathematics teaching included calculations of areas and volumes, but to the games they said weakly ›no‹.

2.4. Differences in means between the five groups

The five groups are:

1. The girls in the shorter course
2. The boys in the longer course
3. The girls in the longer course
4. The boys in the shorter course
5. The students who had moved from the longer course into the shorter one.

Items with statistically very significant difference: items 17 and 34. The boys in the longer course considered mathematics more holistic and were more interested in it than the girls in the shorter course.

Items with statistically significant difference: items 4,11,16,17 and 25. The differences between the boys in the longer course and the girls in the shorter course: The girls in the shorter course considered that everyone must understand and they agreed little more with the games.

The differences between the two groups of the girls: The girls in the longer course considered mathematics more holistic. It was a surprise that the girls in the shorter course preferred better that everything must be reasoned exactly.

The differences between the boys and the girls in the longer course: The boys agreed more with the idea that the student could sometimes make guesses and use trial and error.

Items with statistically almost significant difference: items 2,8 and 18. The girls in the shorter course disagreed more than the girls in the longer course with the idea that getting the right answer is always more important than the way of solving the problem.

Both the girl groups agreed more than the boys in the longer course with the idea that there would be as much practising as possible. The girls in the shorter course and those who had changed the course agreed more than the boys in the shorter course with the strict discipline.

2.5. Experiences

Survey: The majority answered the open questions telling about their experiences and wishes. There were 177 positive and 191 negative experiences. All the groups mentioned a little more of bad than good experiences.

The most frequently mentioned positive experiences:

1. good teachers, good tuition (28%)
2. it is great to learn (21%)
3. a good elementary knowledge at the Comprehensive School (10%)
4. the skill of thinking has developed (10%)
5. achieved mathematical skills (6%)

The most frequently mentioned negative experiences:

1. hurry (15%)
2. the teacher cannot explain (15%)
3. too theoretical (8%)
4. not understanding (7%)
5. too slow advancement at the upper level (7%)
(boys in the advanced group)
6. too big teaching groups (6%)

Positive experiences:

»I have always liked mathematics, so it has been quite nice.«

»I am really happy with myself if I can after trying hard solve a difficult problem.«

»It is useful to learn to do basic mathematical problems, like percentage calculation, geometry and solid geometry.«

»Accurate thinking is often connected with mathematics, which is inevitably shown in teaching. Surprisingly many teachers are willing to help in teaching (advanced) material beyond the schoolbook.«

»The math lessons at the lower level inspired me. At High School Maths develop human thinking: you have to try to understand the abstract in concrete terms.«

»At the lower level the teacher's concrete examples and lively tuition provided a good basic knowledge and aroused an interest in maths. At the upper level my math group was above average and we could go through some extra issues.«

Negative experiences:

»The groups are too big, the teacher hasn't got enough time for everybody.«

»Too seldom are problems taught again / revised.«

»At the upper level there were some discipline problems, at high school too many students in one group, too much separate knowledge, no clear unities.«

»The teacher at the upper level could never explain ›why‹, everything was self-evident.«

»If the students cannot solve a mathematical problem , the teacher writes the solution on the board at once and the students copy it. If you don't understand, the speed of teaching is too quick.«

»I have never been able to follow the teaching, because I don't understand maths and it irritates me that the groups are so heterogeneous that the good answer and the bad sleep during the lesson.«

» A bad teacher can make a simple thing difficult.«

»Too easy tasks at the lower and upper level, too demanding at high school.«

»Maths at high school is too theoretic. When I switched to the easier level I thought maths would be more practical. The difference was that we work less and the teacher does not explain where the formulas come from.«

2.6. Wishes

There were 140 wishes altogether. The most common wishes were:

1. more practical
2. smaller groups, personal tuition
3. slower speed
4. encouragement
5. more choices

»Smaller groups, more personal tuition, more interesting problems.«

»More practical problems, separate parts should be nearer each other.«

»Ability grouping might be good even nowadays, because then the student could choose the level that best corresponds to his level out of more than two levels.«

»Humour is recommended.«

»More possibilities to choose from.«

»The less talented students should be paid more attention to, not only the good students should be encouraged.«

»More time for learning or fewer problems to be solved, so that everybody could follow the teaching and understand it.«

3. Appendix

Student questionnaire about mathematics teaching
(University of Helsinki, Pehkonen & Zimmermann, 1990)

Consider the following statements concerning mathematics teaching.
Circle the point corresponding to your opinion. Use the scale:
1 = fully agree   2 = agree   3 = undecided   4 = disagree   5 = fully disagree

1. Good mathematics teaching includes ...

1 2 3 4 5 doing calculations mentally (1)
1 2 3 4 5 the idea that getting the right answer is always more important than the way of solving the problem (2)
1 2 3 4 5 mechanical calculations (3)
1 2 3 4 5 the idea that the student can sometimes make guesses and use trial and error (4)
1 2 3 4 5 the idea that everything ought to be expressed always as exactly as possible (5)
1 2 3 4 5 drawing figures (6)
1 2 3 4 5 the idea that one ought to get always the right answer very quickly (7)
1 2 3 4 5 strict discipline (8)
1 2 3 4 5 doing word problems (9)
1 2 3 4 5 the idea that there is always some procedure which one ought to follow exactly in order to get the result (10)
1 2 3 4 5 the idea that all students understand (11)
1 2 3 4 5 the idea that much will be taught learned by memorizing rules (12)
1 2 3 4 5 the idea that students can put forward their own questions and problems for the class to consider (13)
1 2 3 4 5 the use of calculator (14)
1 2 3 4 5 the idea that the teacher helps as soon as possible when there are difficulties (15)
1 2 3 4 5 the idea that everything will always be reasoned exactly (16)
1 2 3 4 5 the idea that different topics, such as calculation of percentages, geometry, algebra, will be taught and learned separately; they have nothing to do with each other (17)
1 2 3 4 5 the idea that there will be as much repetition as possible (18)
1 2 3 4 5 the idea that studying mathematics has practical benefits (19)
1 2 3 4 5 the idea that only mathematically talented students can solve most of the problems (20)
1 2 3 4 5 the idea that studying mathematics could not always be fun (21)
1 2 3 4 5 calculations of areas and volumes (22)
1 2 3 4 5 the idea that studying mathematics requires a lot of effort by students (23)
1 2 3 4 5 the idea that there is usually more than one way to solve problems (24)
1 2 3 4 5 the idea that games can be used to help students learn mathematics (25)
1 2 3 4 5 the idea that when solving problems, the teacher explains every stage exactly (26)
1 2 3 4 5 the idea that students are led to solve problems on their own without help from the teacher (27)
1 2 3 4 5 the constructing of different concrete objects and working with them (28)
1 2 3 4 5 the idea that there will be as much practice as possible (29)
1 2 3 4 5 the idea that all or as much as the student is capable of will be understood (30)
1 2 3 4 5 the idea that also sometimes students are working in small groups (31)
1 2 3 4 5 the idea that the teacher always tells the students exactly what they ought to do (32)

2. What kind of experiences have you had until today (from the elementary level up to now) about mathematics teaching? Explain.

good:

....
....
....
....

poor:

....
....
....
....

3. How would you like mathematics to be taught ?

....
....
....
....

4.

1 2 3 4 5	Mathematics is difficult subject compared to other subjects	(33)
1 2 3 4 5	I'm interested in studying mathematics	(34)
1 2 3 4 5	I'm satisfied with my results in mathematics	(35)

Christiane Römer

Discontinuities of the Mathematical Worldviews of Teachers During Probationary Period

Background

In up to date research about mathematical worldviews two things have become clear. On the one hand we are interested in the origin of worldviews and how they change. On the other hand the group of mathematics teachers and their mathematical worldviews seems to play an important role. When you take both these two points together into account, the question arises, how and when the mathematical worldview of mathematics teachers develops or changes. Some studies have already busied themselves with that problem, namely by questioning teachers, who have been teaching for at least ten years.

My investigation sets in a little bit earlier in the development of a mathematics teacher, namely in his or her probationary period (›Referendariat‹). The education of teachers in Germany is divided into two parts. First a theoretical education at an university, in the form of subject specific studies and also studies in the pedagogical science. This part is concluded with the first state examination (›Erstes Staatsexamen‹).

In addition there is the two year practical education, the Referendariat. During this time the future teacher (›Referendar‹) attends and teaches at a school. He also attends an advanced seminar (›Hauptseminar‹) for his pedagogical education as well as seminars with regard to his educational subject. This part of the education is concluded with the second state examination (›Zweites Staatsexamen‹).

The Referendariat surely is in some aspects a deciding part in the development of any teacher. It represents the change from learner to teacher, and is the first serious confrontation with the school as place of working. The role of mathematics also changes at the beginning of the Referendariat, where subject related knowledge is acquired to in the end be employed to teach the subject. In addition they experience a relationship with students and teachers, where the Referendar has to establish his own role. During this time of change I wish to assume, that the mathematical worldview would not be left unchanged.

1. The method of the study

For establishing a basis for the information, I have decided to make use of personal interviews with Referendars. This method is less representative as for example a questionnaire, because you are limited to less interview partners because of a lack of time. However this way of questioning results in gaining a lot of indirect information that you would not obtain with a questionnaire. During a video-interview not only the answers but also the gestures and mimics, as well as eventual pauses may say something about the interviewed person.

The direct contact between the interviewer and the person being interviewed makes it possible to simultaneously understand the actual questions, and also to clear aspects that have not been attended to in the beginning and which seem to be important to the person being interviewed. To create a quite relaxed atmosphere, the interviews are conducted in the interview partners' homes.

The investigation consists of 5 videotaped interviews with Referendars, who have attended the Referendariat at Oberhausen together from the 15. December 1995, although they studied at different universities. Each Referendar will be interviewed twice, and in addition he will have to fill in a questionnaire. This should help me to establish my investigation into the context of previously conducted research studies. These interviews should be conducted with few open-formulated questions as to establish a possible uninfluenced view from the person. The use of two different questioning devices bring about an abundance of information as well as mutual control.

The first interviews (see Table 1) have already been conducted in February 1996, thus the Referendars had next to nothing experience of teaching.

(1) Report about your experiences with mathematics during
(a) your time at school
(b) your time at university.

(2) Why do you want to become a mathematics teacher?

(3) How did you experience the transition from school to university with regard to mathematics?

(4) Do you think, that you can prepare your pupils better? Or is it the task of university to help the first year students at the beginning of their studies?

(5) How do you experience the transition from university to the Referendariat?

(6) How would your ideal of the education of a mathematics teacher look like?

(7) How do you imagine your own mathematics teaching?

(8) What rank has formalism in your view on mathematics and mathematics teaching?

(9) Which terms do you associate with mathematics?

Table 1. Questions of the first set of interviews.

The second set of interviews is planned for the end of June just before the summer holidays' start. This is the latest possible date with regard to organisational aspects. During the first interview I firstly tried to establish previous experience in mathematics and secondly their present view on mathematics and mathematical education. Supporting this interview the interview partners fill in a questionnaire on mathematical education developed by S. Grigutsch.

The second interview busies itself with possible changes in the Referendars' mathematical worldviews and the reasons for this changes. For this reason the interview partner should answer freely with regard to his experiences with mathematics and mathematical teaching up to now. Furthermore I would like to ask them about factors by which they felt influenced. Corresponding to the second interview I plan to draw up a further questionnaire, to recognize eventual changes to the first one. On the basis of these interviews and questionnaires I would like to do an interpretative assessment. This investigation would not want to give the impression of being representative because of the way and extent to which it has been conducted.

Through this investigation based study I will try to analyse the results with the assistance of other investigations and corresponding subject matter. In this way it could be possible to indicate directions in which intensive research would be interesting.

2. Some results based on the first set of interviews

For this report I could only analyse four interviews, because the last two interviews have been postponed because of health reasons. The Referendars have more or less the same amount of experiences in school and university. All four liked mathematical education in school, with the remark by Anke been characteristic, namely »or else I wouldn't have studied it.« [AS]

As motivation all of them give their own mathematical education as a reason. Three explicitly mention one of the own mathematics teachers as an example on which their future career is based. With regard to own motivation for becoming a mathematics teacher, Michaela totally differs from the rest. Whereas the others have started their studies with the goal of becoming a teacher, she has started with the ›Diplomstudiengang Technomathematik‹.

After a year she found the study very ›trocken‹ (dry) [MK]. Just because of her interest in subject ›Technik‹, which is only offered as a subject of teaching profession, she became a teacher. This access through the mathematical science forms a clean contrast to the motivation of the other Referendars. Her direct counterpart is Anja, who already at an early stage wanted to become a teacher but had difficulties on deciding on the subject of teaching. In this area of tension between inspiration for the subject and the wish to become a teacher the given motivation for becoming a mathematics teacher are to be found.

Now it should be investigated, if this different approaches have an influence on the self-realisation of mathematics teachers, mathematics teaching and the mathematical worldview.

Figure 2. Transitions.

A further interesting aspect of the interview is the question of transition (3 & 5). The development of a mathematics teacher can be represented as shown in Figure 2. Transitions I and II are interesting for my investigation.

The first series of interviews established that transition I has been experienced as a shock by three out of four interview partners. They feel deserted (left alone [MK]) at the beginning of their studies as well as been dumbfounded (›erschlagen‹ [LK]) by the mass of study-material. They find university mathematics too theoretical and sophisticated (›abgehoben‹) [LK].

In general all interview partners feel definite distance with regard to university mathematics. They do, however, agree that mathematics teachers have to know more than only the school subject, but the distance between mathematics in school and the contents of the main part of their studies is too vast for them. In addition they miss a connection to the practical side of their future job. The practical training period (›Blockpraktikum‹) is not enough for this practical part.

Transition II is experienced by the Referendars as a logical continuation of their education. But this seemed to be based on their knowledge about the course of their education and not on its natural structure. Only one of the interview partners answers to question 6 that he wants to continue this two parted system of teacher's education. All the others would like a parallel course of theoretical and practical education at least after the fourth semester.

At this moment, I am at the very beginning of my investigation. I cannot at this stage say much about the real subject, the discontinuities in the mathematical worldviews of teachers during the Referendariat. Establishing sound and deeper results will require a comparison of the two sets of interviews and questionnaires.

Günter Törner

›Basic Concepts of Mathematical Contents‹ and ›Mathematical Worldviews‹ (Mathematical Beliefs) as Didactical Conceptualizations

Introduction

Mathematical didactics views itself as a science in its own right and must therefore elaborate on some specific terminology and approaches (Wittmann 1992) within its research paradigm with regard to the context of mathematical teaching and learning processes. The fact that the corresponding research concepts will relate to very different disciplines at the same time (see graphic – Wittmann, 1992), does not necessarily decrease its science theoretical independence, but initiates a constant interdisciplinary dialogue.

A justification for the existence of the science didactics of mathematics could for instance be explained by the fact that it would, with regard to all related sciences or disciplines, be impossible to gather all the relevant multi-faceted relations and then come to a satisfactory analysis from the research questions at hand, or ideally, to a solution. Mathematics, being the original relevant science, of course plays an important role. As a matter of fact, they simultaneously have a direct or indirect relationship: direct because of the relatively objective subject matter, and indirectly because of subjective judgements. Vollrath (1994) feels that mathematics teaching inevitably has to do with the forming of concepts.

> What does it mean to understand a concept? ... to know the definition ... to give examples ... to give counterexamples ... to test examples ... to know properties ... to know relationships between concepts ... to apply knowledge about the concept. But it is more difficult to describe what we mean by ›having images of a concept‹, ›to appreciate a concept‹ or ›knowing the importance of a concept‹.

In other words, it could not be excluded that information – to use a metaphor – is not merely experienced as black or white, but always in colour. No teaching or learning process could be described as being free of subjectivity (see Ernest 1994). Here the philosophy of mathematics gains a role as ›giving colour‹, which, however, has not been valued highly enough with regard to the consequences it has for mathematics, or by implication for university mathematics. It comes as no surprise that this philosophy is often ignored when it comes to mathematics and it plays a peripheral role because it merely becomes relevant with regard to inherent basic mathematical concepts. The meta-mathematical discussion is being conducted mainly by mathematicians who feel obliged to do actual research work. This absence of a philosophical position with regard to, for example a teacher, could not be seen as indifference, but rather as the expression of a basic philosophical outlook (see Ernest, 1994).

A discussion of such aspects, of course, forms part of mathematical didactics, although it would at first rather be the high school students who would ask ›why‹ than university students. A recent study by vom Hofe (1995) seeks to prove the above mentioned approach as being correct, which has tried for a long time to draw detailed attention to a familiar concept within the discussion around the mathematical learning processes, namely focusing on basic concepts. Within this background, vom Hofe (1995) makes a valuable contribution to the discussion on absent conceptualization within a theoretical dialogue. A detailed discussion of previous contributions, especially by Bender (1991), would not be possible here.

When, according to vom Hofe, there is a link between the theoretical construction of the basic concepts in mathematical didactics and pedagogical and psychological theories, the following remarks should not be regarded as questioning the above mentioned approach. However, it is important to the author that a parameter not mentioned by vom Hofe (1995) and which could not be ignored with regard to this concept-idea, should be explicitly explored, namely: the fundamental beliefs or worldviews on mathematics and the teaching or learning of mathematics. It should be understood as a relative or subject-partial revelation of basic concepts.

Our approach towards the matter could be explained by our involvement with the role of so-called beliefs. The work on mathematical worldview came into being as a result of a study conducted by the author and Grigutsch (Grigutsch et al. 1995) and could be seen as an attempt to verify and classify a term which has been vaguely used in mathematical didactical literature, namely ›beliefs‹ and its relation with the worldview on mathematics. Thereby, the term ›mathematical worldview‹ is psychologically linked to the theory of attitudes and at the same time it becomes a theme within mathematical didactics. Extensive field-research by Grigutsch (1996) conducted with more than 1 600 pupils, proved it possible to describe and explain reality by means of this specific theory construction.

The result of this paper could be the proof that basic concepts are not neutral with regard to beliefs or worldviews, but metaphorically spoken – reveals ›colour‹. Thus, Thom's thesis (Thom 1973) is again proved correct*: mathematical didactics, whether one likes it or not, is based on a mathematical philosophy.* Basic concepts should never be understood as being ambivalent extracts from monolithical mathematical theories, but they reveal strong subjective relations. With regard to this, the word ›basic‹ in the terminology of vom Hofe should be used more or less relatively, according to the specific circumstance. A co-existence of basic concepts should at least be assumed within the same subject-context. Through this conscious analysis of the term basic concepts – the process of making the explicit implicit – an enrichment of the discussion takes place with regard to the question of which worldview is the result of which possible basic concept.

1. The notion of ›basic concept of mathematical contents‹

The conceptual approach ›basic concepts of mathematical contents‹ refers to classical theories of learning-psychology. It involves the general ›concept‹-term of the contents, as far as being relevant for mathematics. As vom Hofe's book shows, there are numerous didactical contributions (Pestalozzi, Herbart, Diesterweg, Hentschel, Kühnel, Breidenbach, Öhl, Griesel), which have a lot of similarities with the present point of view. It is, however, not easy to give a precise definition (see vom Hofe 1995, pp. 97).

The basic concept-idea describes the relationship between mathematical contents and the phenomenon of an individual terminology. Through their different forming they each characterize different aspects, three of those being important with regard to this phenomenon:

- giving meaning to a term by linking it with familiar matters or aspects, or concepts of action.
- creating corresponding visual representations or ›internalizing‹ – which makes it possible to operate on the level of concepts.
- the ability to use a term in correspondence with reality through the recognition of a corresponding structure in the context or by modelling the problem with the assistance of mathematical structure.

This discussion has of course also taken place within Anglo-American didactics. Vom Hofe partially mentions it, although he refuses to base his understanding on a previous suggestion. Here, of course, Bruner's (1976) much quoted *fundamental ideas* could have been rendered more precisely. Maybe the term ›concept image‹ found in the English literature (von Vinner & Dreyfus, 1990) could be closely related to the idea of vom Hofe:

> All mathematical concepts except the primitive ones have formal definitions. Many of these definitions are introduced to high school or college students at one time or another. The student, on the other hand, does not necessarily use the definition when deciding whether a given mathematical object is an example or nonexample of the concept. In most cases, he or she decides on the basis of a concept image, that is, the set of all mental pictures associated in the student's mind with the concept name, together with all the properties characterizing them... The student's image is a result of his or her experience with examples and nonexamples of the concept. Hence, the concept is not necessarily the same as the set of mathematical objects determined by the definition. If these two sets are not the same, the student's behaviour may differ from what the teacher expects. To improve communication, we need to understand why it fails.

2. The concept of mathematical worldviews

Here we only shortly want to refer to the concept of mathematical beliefs, and focus the attention on the forthcoming publications in ZDM (Pehkonen & Törner 1996) as well as on previous papers (see Törner & Grigutsch 1994, Grigutsch et al. 1995, Grigutsch 1996).

›Mathematical worldviews‹ are systems of norms, convictions, background theories, attitudes and guiding concepts about mathematics. They develop through mathematical practice and to a great extent influence the way in which mathematics is approached. It could be accepted that they represent attitudes. The aim of didactics is to identify and to ›trace‹ mathematical worldviews. There are two mathematics-didactical motivations: (i) individual attitudes produces the context with regard to the way in which pupils react to mathematical tasks and problems. (ii) the attitudes of the pupils being expressed through their mathematical worldview precisely reflects reality with regard to teaching. Therefore the teaching chain *university – mathematics – teacher training – worldviews of teachers – worldviews of pupils* should gain a lot of attention.

3. Mutual concept elements of ›mathematical worldview‹ resp. ›basic concepts of mathematical contents‹

3.1. Constructional character of both conceptualizations

It should be emphasized that both terms have constructions which claim their own explanatory relation. They do not, a priori, hold empirical value, but rather heuristic meaning. Both terms associate cognitive or affective conceptualizations: the term ›basic concepts‹ refers to a (psychological) concept, and the term ›mathematical worldview‹ approaches the (psychological) theory of attitudes. It must be noted that theoretical approaches could merely be established separately from fundamental theories. With regard to belief concepts, the possibility that basic modifications within attitude theories could have an effect on the conceptualization of the mathematical worldview, could not be excluded. Vom Hofe is more rigid here (1995):

> On the psychological level there are many different and partially contradictory concepts – on the didactical level however, the possibility is excluded that a continuity of formed concept emerges from elements, which are to a great extent independent of psychological description models.

This position should rather have been critically valued, as it could not be said that didactical concepts are without further consideration compatible with arbitrary psychological concepts.

Finally it should be noted that, as being emphasized by vom Hofe and referring to Wittmann (1992), that basic concepts are terms *that should be understood as a* genuine mathematical didactical category *in the central meaning of this discipline.* (see vom Hofe 1995, pp. 98). We are however of the opinion that the fundamental conceptual pattern, thus basic concepts corresponding to the subject-contents, could also be valuable within other scientific didactics (see the discussion in Jäckel 1994). The same applies to beliefs.

3.2. Meta-character of both conceptualizations

Both terms partially involve information on information, information on emotions and affections. Here their conscious or also unconscious management function might be added.

> ... another way to look at this aspect of metacognition is to think of it as a management issue ... (Schoenfeld, p. 190)

The constituent characteristics of mathematical beliefs resp. mathematical worldviews are discussed in Grigutsch et al. (1995):

> 1. Individual attitudes towards mathematics and the teaching of mathematics are essential influential factors with regard to the processes of teaching of teaching and learning mathematics. They describe, even unconsciously, the context in which pupils practice and regard mathematics.

Characteristic 1 corresponds with the description in vom Hofe (1995, pp. 98), namely:

> The term basic concept characterizes fundamental mathematical terms or methods and their meaning in real situations. He describes the relation between mathematical structures, individual psychological processes and existing coherence of matter, or in short: the relationship between mathematics, the individual and reality.

Also the second characteristic of beliefs corresponds with the idea of concepts:

> 2. The worldview on mathematics and the teaching of mathematics reflected by pupils' attitudes, is a very precise reflection of the actual mathematics teaching, because attitudes are formed during the learning process and reflect the circumstances.

It becomes even more clear when the term ›attitude‹ in the following citation from Grigutsch et al. (1995) is replaced by the term ›basic concept‹:

Because attitudes are acquired during the learning process within actual (social) circumstances, you could also formulate the thesis that the attitude of the teacher would to a great extent influence the attitudes of the pupils – on the one hand with regard to the direct communication and interaction in the classroom, and on the other hand indirectly through the concrete contents (choice of matter and methods , system of evaluation) of mathematics education.

3.3. Functional character of both conceptualizations

In Pehkonen & Törner (1996) it is pointed out that *mathematical worldviews or beliefs* show the following characteristics:

... *as a regulating system:* The latter is easily understood when we remember that an individual's mathematical beliefs, his view of mathematics, form a regulating system for his knowledge structure.

... *as an indicator:* Since the view of mathematics transmitted through beliefs, expressed by an individual, gives a good estimation of his experiences within mathematics learning and teaching, we will thus have a method to indirectly evaluate the instruction he has received or has given. So, it seems quite obvious that beliefs of pupils as well of teachers act as a tachograph. They present condensed information on personal experienced ›meetings‹ with cognitive elements in the past: In the case of a teacher, the view of mathematics may act as an indicator (1) of teachers' university studies, (2) of teachers' professional views, (3) of teachers' in-service training. In the case of pupils and students, the view of mathematics could function as an indicator, (4) of how students experience teaching (in schools and universities). Generally, one may consider the view on mathematics as an indicator (5) of the functioning of the whole school system.

... *as an inertia force:* If we aim to develop mathematics teaching in schools, we are compelled to take into account teachers' beliefs (their view on mathematics) – and also pupils' beliefs. Usually, the concern is about experienced teachers' rigid attitudes and their steady teaching styles, which will act as an inertia force for change. Experienced teachers know through their long practice what kind of mathematics teaching is (according to them) good, and this subjective knowledge (beliefs) is usually deeply rooted.

... *have prognostic character:* As a consequence of the mentioned arguments, one should stress that mathematical belief systems also have a prognostic aspect.

It is surprising that all these features also fit to basic concepts of mathematical contents, you only have to replace the word ›beliefs‹ by ›basic concepts‹.

4. The ›beliefs-colour‹ of basic concepts of mathematical contents

Or: Basic concepts of mathematical contents are not neutral with respect to mathematical worldviews; some examples

Clearly, understanding does not take place in some or other sterile environment. This is also postulated by vom Hofe for the development of basic concepts; he distinguishes several levels through which information permeates before it emerges as, for example, a certain basic concept in the head of a author of a text book (see Figure 1; vom Hofe 1996, p. 124). To summarize: Basic concepts are coloured already during their genetic processing.

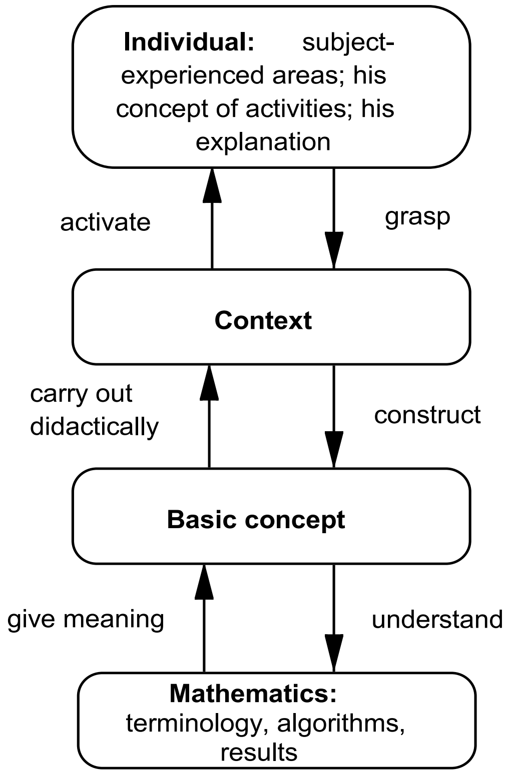

Figure 1. Genesis of basic concepts.

According to what has been said so far, it becomes clear that basic concepts are positioned on the border line, or, as Griesel points out, play the role of a mediator, between the world of notions and the world of objects. We must conclude that attitudes play a role here, that beliefs are involved when, for example, basic concepts are taught.

4.1. ›Basic concepts in elementary arithmetic‹ according to Griesel (1973)

It was New Math which led to a detailed analysis of concepts in elementary arithmetic. The period of New Math coincided with a rise of algebraic concepts under the dominance of Bourbakism. So it is not surprising that analysis during this period seemed to carry the flavour of the colour of Bourbakism. We quote some work on the basic concept of binary operations published by Griesel (1973), see vom Hofe (1996, p. 89):

›Operation‹	Appropriate basic concept
Addition of two elements	Combination (union) of (quasi disjoint) representatives
Subtraction of two elements	Separation of one representative from

	another
Multiplication of an element with the number *n*	Combination (union) of *n* equivalent (mutually disjoint) representatives
Division of an element by a natural number *n*	Decomposition of a representative into *n* equivalent mutually disjoint representatives
Division of an element *a* by an element *b*	Combination (or decomposition) of a representative of size *a* from (or into) representatives of size *b*; (repeated subtraction of a representation of size *b* from a representation for *a*)

Table 2. Basic concept of binary operations.

In order not to be misinterpreted: the description in the right column is mathematically correct using corresponding set-related models of representations. However, the question remains whether these explanations are the basic concepts of adequate representation models for elementary operations addressing students.

4.2. ›Basic concepts‹ of derivatives

In their monograph ›Didaktik der Analysis‹ (Blum & Törner 1983) the authors discuss, that any realisation of the derivative in a high school course on calculus can be associated with at least two different, however equivalent concepts: derivatives as *rates of changes* with respect to some functional relationship; alternatively derivatives can be understood as defining the slopes for linear approximation of (differentiable) functions. Obviously, any of these points of views can be elaborated to detailed basic concept in the sense of vom Hofe. However, any decision for one of the two concepts involves some philosophy on calculus with the curriculum in the background. Whereas the change-rate-approach seems to be more application-orientated, the approximation-concept is open for generalization to function with more than one variable. This shows again in which way the basic concepts may differ by their colours.

4.3. ›Basic concepts‹ of space

Whereas the last example addresses calculus, it is clear that the fundamental concept of space can be handled differently, thus there is more than one ›basic concept‹. Here we cite two tables of contents from text books for Linear Algebra. The first example (see Table 3) is from *Themenhefte Mathematik. Lineare Algebra und Analytische Geometrie 1. Lambacher-Schweizer. Stuttgart: Klett. 1975.*

	1 Gaussian Elimination 1.1 Introduction 1.2 An example of Gaussian Elimination

Preface
1 Vector Spaces
1.1 Repetition: Numbers
1.2 Properties of Number Domains
1.3 Groups
1.4 Examples of Groups
1.5 Subgroups
1.6 Multiplication of Group Elements by Integers
1.7 Generating Systems in Groups
1.8 Vector Spaces
1.9 Examples of Vector Spaces
1.10 First Consequences Using Axioms of Vector Spaces

Table 3. *Themenhefte*.

1.3 Matrix Notation and Matrix Multiplication
1.4 Gaussian Elimination = Triangular Factorization
1.5 Row Exchanges, Inverses, and Roundoff Errors
1.6 Band Matrices, Symmetric Matrices, and Applications
2 The Theory of Simultaneous Linear Equations
2.1 Vector Spaces and Subspaces
2.2 The Solution of *m* Equations in *n* Unknowns
2.3 Linear Independence, Basis, and Dimension
2.4 The Four Fundamental Subspaces

Table 4. *G. Strang. Linear Algebra*.

Obviously, any realisation of mentioned concept would enlighten the concept of space by means of a module theoretical approach: space is defined through axioms within algebra as a highly-elaborated theoretical frame.

In contrast to this ›basic concept‹ we cite from the textbook of *G. Strang. Linear Algebra and Its Applications. 1976* (see Table 4) which was written in the same year as the Themenheft from above. It should be mentioned that Themenheft contains a course on calculus for grade 12, whereas Strang's book is used at the universities.

It is clear that there are numerous topics in school math which evoke different ›basic concepts of ideas‹. Often these concepts may be equivalent under mathematical terms, however they differ by the underlying worldview on mathematics.

References

Bender, P. 1991. Ausbildung von Grundvorstellungen und Grundverständnissen. Ein tragendes didaktisches Konzept für den Mathematikunterricht – erläutert an Beispielen aus den Sekundarstufen. In: Postel, H. (Ed.): Mathematik lehren und lernen. Festschrift für Heinz Griesel (p. 48–60). Hannover: Schroedel.

Blum, W. & Törner, G. 1983. Didaktik der Analysis. Göttingen: Vandenhoeck & Ruprecht.

Bruner, J.S. 1972. Der Prozeß der Erziehung. Berlin: Schwann.

Cooney, T.J. et al. 1989. An Agenda for Research on Teaching Mathematics. In: Grouws, D.A. (Ed.): Perspectives on Research on Effective Mathematics Teaching (p. 253–261). Volume 1. National Council of Teachers of Mathematics. Hillsdale NJ: Lawrence Erlbaum Associates. 1989.

Dreyfus, T. 1990. Advanced mathematical Thinking. In: Nesher, P.: Mathematics and Cognition: a research synthesis / by the International Group of Psychology of Mathematics Education (p. 113–134). Cambridge: ICMI study series. Cambridge University Press. 1990.

Ernest, P. (Ed.) 1994. Mathematics, Education and Philosophy: An International Perspective. London: The Falmer Press.

Fischer, R. 1976. Fundamentale Ideen bei den reellen Funktionen. Zentralblatt Didaktik der Mathematik 8, 185–192

Green, T.E. 1971. The activities of teaching. New York: McGraw-Hill Book Company.

Grigutsch, S. & Raatz, U. & Törner, G. 1995. Mathematische Weltbilder bei Lehrern; Schriftenreihe des Fachbereichs Mathematik der Gerhard-Mercator-Universität Duisburg, Nr. 296. Duisburg.

Grigutsch, S. 1996. Mathematische Weltbilder von Schülern. Struktur, Entwicklung, Einflußfaktoren. Fachbereich Mathematik. Gerhard-Mercator-Universität Duisburg. Dissertation.

Hersh, R. Some proposals for reviving the philosophy of mathematics. In: Tymoczko, T. (Ed.): New directions in the philosophy of mathematics (pp. 9–28). Boston: Birkhäuser.

vom Hofe, R. 1995. Grundvorstellungen mathematischer Inhalte. Heidelberg: Akademischer Verlag.

Jäkel, L. (Ed.) 1994. Der Wandel im Lehren und Lernen von Mathematik und Naturwissenschaften. Band II: Naturwissenschaften. Symposion 1994. Pädagogische Hochschule Heidelberg. 4. bis 7. Oktober 1994. Weinheim: Deutscher Studien Verlag.

Pehkonen, E. & Törner, G. 1996. Mathematical Beliefs and their Meaning for the Teaching and Learning of Mathematics. To appear in ZDM 1996.

Schoenfeld, A.H. 1987. What's all the fuss about metacognition? In: Schoenfeld, A.H.: Cognitive Science and Mathematics Education (p. 189–215). Hillsdale, NJ: Lawrence Erlbaum Associates. 1987.

Thom, R. 1973. Moderne Mathematik: Gibt es das? Didaktik Math. 1 (4), 251–263.

Törner; G. & Grigutsch, S. 1994. ›Mathematische Weltbilder‹ bei Studienanfängern – eine Erhebung. Journal für Mathematik-Didaktik 15 (3/4), 211–251.

Vollrath, H.-J. 1994. Reflections on mathematical concepts as starting points for didactical thinking. In: Biehler, R. et al. (Eds.): Didactics of Mathematics as a Scientific Discipline. Mathematics Education Library. Dordrecht: Kluwer Academic Publishers. pp. 61–72.

Wittmann, E.Chr. 1992. Mathematikdidaktik als ›design science‹. Journal Mathematikdidaktik 13 (1), 55–70.