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Current State of Research on Mathematical Beliefs VI

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edited by
Günter Törner

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Editor's Statement

We are proudly looking forward to presenting a first collection of papers prepared by the German-Finnish research group MAVI (MAthematical VIews on Beliefs and Mathematical Education) to you.

These papers contain the abstracts of talks given at the workshop on "Current State of Research on Mathematical Beliefs", which took place at the University of Duisburg on October 4-5, 1995. The aim of this research group, being the initiative of my colleague Erkki Pehkonen and myself, is to study and examine the mathematical-didactic questions that arise through research on mathematical beliefs and mathematics-education. There is a vivid movement around research on mathematical beliefs in Europe. At the last Conference on Mathematics-Education in Kassel (Germany) in March 1995, we established a working group of interested scientists called 'Mathematical World View' in order to build up a network of belief-researchers in Germany. The European Research Conference on Mathematics-Education took place in Osnabrück (Germany) a couple of days before our conference in Duisburg. In Osnabrück, we enlargened the network of belief-researchers into European dimensions.

Now, the initiators would like to encourage all interested colleagues to join our network and to participate in our activities.

Duisburg, October 1995 *Günter Törner*

Contents

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Astrid Brinkmann

Aspects of networks by mathematics instruction

1. Introduction

Nowadays we are living in a time, in which technique and science gain more and more importance and thus the influence that we take on our nature increases. The thinking in complex systems, in which every change of one component determines the change of many others is more necessary than ever.
This kind of thinking can be trained by learning and doing mathematics, as this subject represents itself as a very complex net of its elements, so for example concepts, theorems, definitions, algorithms, rules and theories. But mathematics is also manifold connected with our real world.
The Third International Mathematics and Science Study ([1], [2]) revealed that German students show a great failure in thinking in complex nets.
According to this background the aim of my actual research work is to find out some aspects according to the mathematical network composed by instruction.

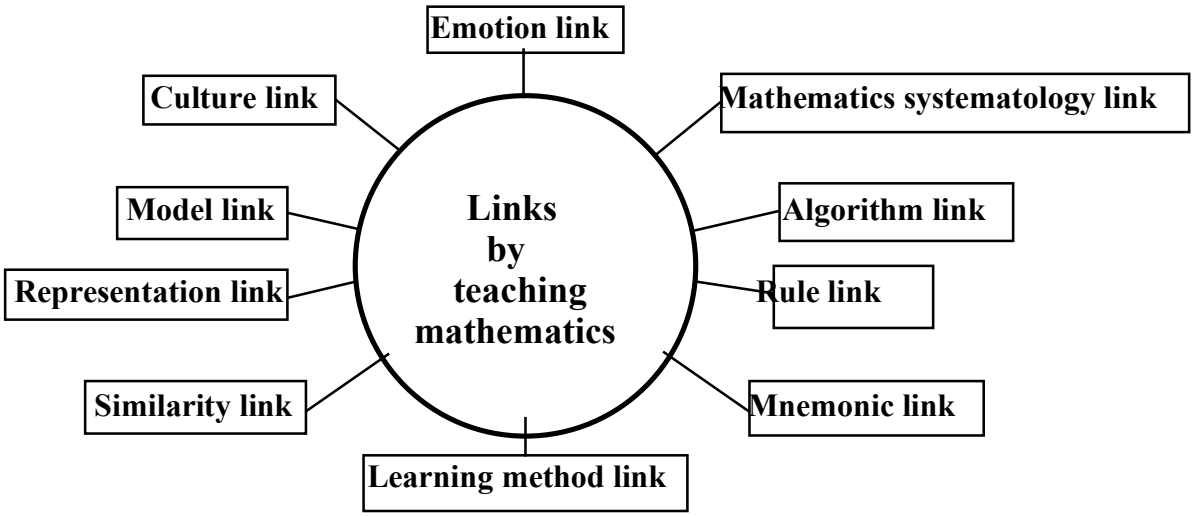
2. Categories of links

As a net consists of many single connections I first dealt with the task to categorise these in respect of a better possibility of further analysis.
The *categories of links* we have to consider are not only those referring to connections between mathematical contents or connections between mathematics and the non-mathematical reality, but also categories of links due to the instructional process: the emotional loading of every content (emotion link) or the connection of mathematical components with the way these were learned (learning method link).
Thus we receive the following categories of links.

1. *Mathematics systematology link*: link according to mathematics systematology
2. *Representation link*: link between different representations of a mathematical content
3. *Model link*: link between a problem and a model suitable for its solution
4. *Culture link*: link with non-mathematical culture
5. *Algorithm link*: link between a problem and an algorithm suitable for its solution
6. *Rule link*: link between a problem and a rule suitable for its solution
7. *Similarity link*: link of a mathematical content with the same or a similar content already known
8. *Mnemonic link*: link with a mark that supports remembrance
9. *Learning method link*: link between a mathematical content and the way it was learned

10. *Emotion link*: link with emotions

Figure 1: Categories of links



I further gathered and analysed some *characteristics of these connections* with respect to the *way they are built up and stored in the brain and how they are used*. For the sake of brevity I here omit the presentation of my investigations.
However it can be remarked that this is a very wide field and can only be treated by considering perceptions of psychology as well as neural physiology. Thus it is only natural that a first representation cannot have the pretension of completeness, but may only present an introduction in this topic.
The links of the different categories don't appear isolated. So I also tried to find out in which respect they *play together or coincide partially*.

3. Presentation of nets

If you deal with nets the question arises how these can be presented.
Written texts have a linear form and spoken words are ordered linearly according to the progressing time. But networks haven't got a linear order, they represent somewhat like a picture of many nods arranged in a two- or three-dimensional field and connected one with each other by many coexisting links.
Any pictorial representation may show a complete net of a mathematical topic because of its complexity. So every representation is a reduction of the net according to the consideration of special aspects and with respect to its later scope.

Some of the *pictorial representations* are:

- tables,
- representations in co-ordinate systems,
- logical graphs,
- flow charts,
- mind maps,
- concept maps or
- critical path diagramms.

Logical graphs suit to map up mathematical proofs ([9]).

Concept maps and *mind maps* are hierarchically ordered maps that show the connections between concepts related to a special theme ([10], [3], [4], [8], [13]). The method of concept mapping puts out the links between the considered concepts in a more detailed and precise way than mind mapping, whereas a representation by mind maps involves an artistic component in order to combine the thinking of both parts of our brain.

A *critical path diagram* shows possible sequences of aims of instruction that are assumptions for the teaching of a special content ([5]). Thus it may represent a plan of instruction according to the mathematical systematology.

The *maps* mentioned above can be *used for different purposes*, such as

- *the research purpose*: to reveal the structure of the net in a pupil's brain ([6], [7]),
- *the instructional purpose*: as diverse means of instruction, in order
    - to plan instruction,
    - to show relationships between concepts,
    - to support the building of nets in students' heads,
    - to repeat and put together contents that are learned.

Depending on the actual situation one has to decide which kind of map to use advantageously.
Whereas we are familiar with using tables, representations in co-ordinate systems and sometimes also flow charts in mathematics instruction, the use of logical graphs, mind maps, concept maps and the critical path method have only recently begun to play a role in mathematics research and instruction. In this domain a lot of work is still to be done.

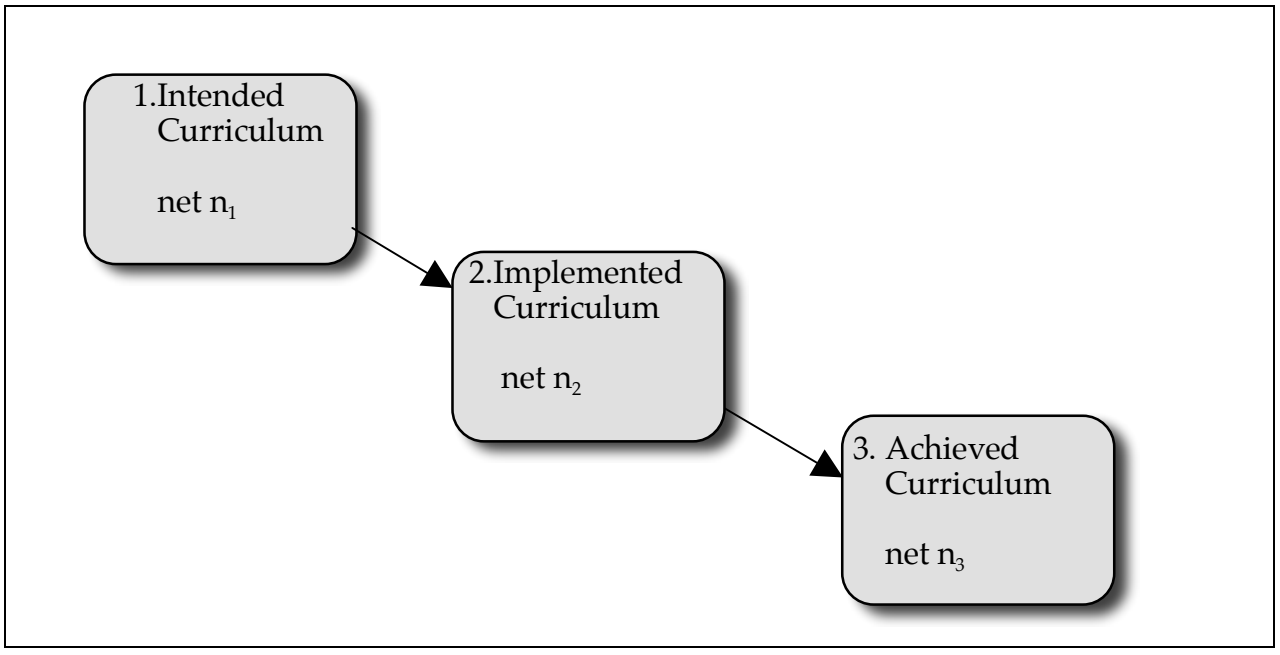
4. Evaluation of networks by mathematics instruction

If you want to make something better, so as the thinking in mathematical nets, you have to evaluate the status quo.
It isn't enough to map up the interesting parts of the students' minds, you also have to evaluate the frameworks for the formation of these mental constructions.

Following the example of the Third International Mathematics and Science Study ([2]) we may use - as it is done here - as frames the intended curriculum, the implemented curriculum and the achieved curriculum (see also [10],[12]) and look upon the *nets* and their *transfer from one frame on the other*.
TIMSS gives us already some very global information concerning this task.

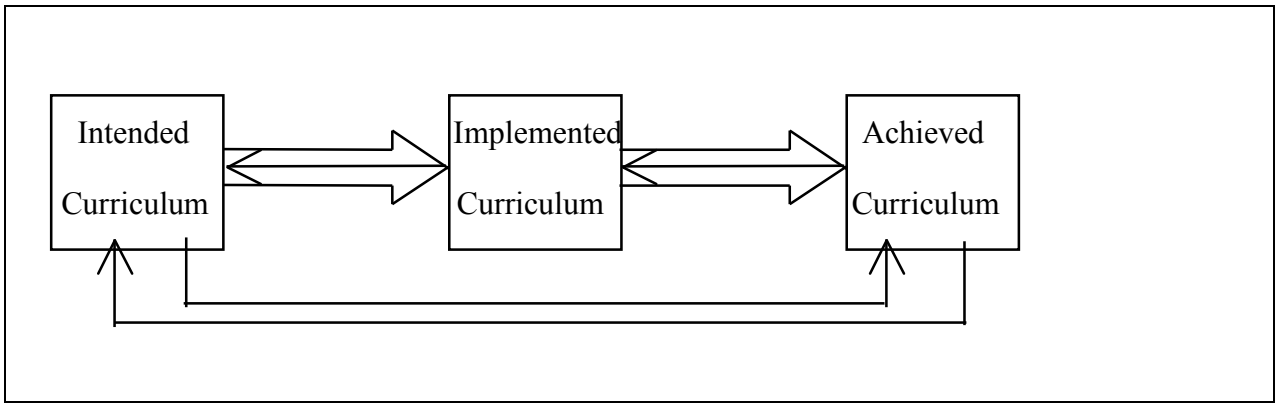
As a net built up by mathematics instruction doesn't only consist of linked mathematical contents but overlaps the emotional net, so that the single mathematical objects have an emotional loading ([14]) and considering further that every mathematical content can be connected with the way it was learned, we have to use the notion *curriculum* in a very wide meaning, i.e. by *involving emotional components and instruction methods*.

Figure 2:



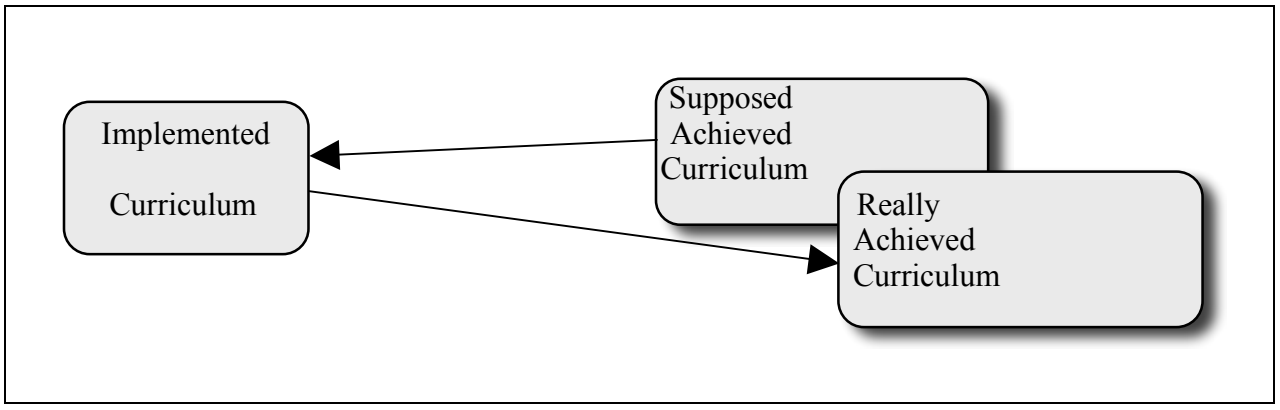
Of course the representation in figure 2 is a very simplified one, as the implemented curriculum influences also backwards the intended curriculum and likewise the achieved curriculum influences the implemented curriculum. And there exists also a direct interrelation between the intended and the achieved curriculum.

Figure 3:



In effect you also have to distinguish between the achieved curriculum in students' heads and that what the instructor supposes as achieved.
The interrelation between the implemented curriculum, the really achieved and the supposed achieved curriculum is shown by figure 4.

Figure 4:



In order to get more detailed information about the transfer of a net from one curriculum frame to another you may choose a special topic and considerate it in the different frames. This requires also special research methods, such as *teacher interviews* on the chosen topic and *student tests*.

I intend to show a way of possible research by focusing on a particular theme and I hope that the results of the investigation will allow some conclusions for an improvement of building up resistant branchy mathematical networks in students' minds.

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Beliefs of elder students about mathematics education

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In october 1997 I gave to students of a seminar about general education and beliefs a questionaire about mathematics education which based on pupils questuionaire used from E. Pehkonen et. al. in earlier times . Though only a few persons visited the seminar I got out some interesting aspects which led me to some further questions. Thus I decide to talk about my results here.

The population contained 19 Persons (11 female and 9 male). Their age ranged between 21 years and 31 years and they were in the 4th to 9th semester of study. The questions No 1 to No 32 have been the same like those from E. Pehkonen & B. Zimmermann mentiened in several other papers of MAVI. I added 8 questions about mathematical contents and general mathematical goals like "understanding proofs" and "logical thinking". Moreover I added 10 questions about objectives of mathematics education with respect to the theme of the seminar. By answering the students had to choose between +2 (full agreement) and -2 (full disagreement) once for the Is-state they remembered and secondly for the Shall-state in their opinion.
I will give now results of some items which I collected for later conclusions.

Item 1: *mentally calculations*

Is / Shall	**-2**	**-1**	**0**	**+1**	**+2**	***no a.***	**Sum Is**
-2	-	-	-	-	-	-	0
-1	-	1	1	0	1	-	3
0	-	-	-	-	3	-	3
+1	-	-	2	-	8	-	10
+2	-	-	-	1	2	-	3
no a.	-	-	-	-	-	-	*0*
Sum Shall	0	1	3	1	14	*0*	19

The means are: IS-state + 0,68 and SHALL-state + 1,47 . We see high agreement for both.

Now we want look at those items which concern applications of mathematics.

Item 9: *word problems*

Is / Shall	-2	-1	0	+1	+2	*no a.*	Sum Is
-2	-	-	-	-	-	-	0
-1	-	-	-	2	1	-	3
0	-	-	-	2	2	*1*	5
+1	-	1	-	3	5	-	9
+2	-	-	-	-	2	-	2
no a.	-	-	-	-	-	-	*0*
Sum Shall	0	1	0	7	10	*1*	19

Item 19: *mathematics for practical benefits*

Is / Shall	-2	-1	0	+1	+2	*no a.*	Sum Is
-2	-	-	-	-	1	-	1
-1	-	-	-	5	4	-	9
0	-	-	-	2	3	-	5
+1	-	-	-	2	2	-	2
+2	-	-	-	-	-	-	0
no a.	-	-	-	-	-	-	*0*
Sum Shall	0	0	0	9	10	*0*	19

The means are: IS-state - 0,47 and SHALL-state + 1,53 . Noticeable is the high agreement for the Shall-state and the light negative mean for the Is-State. The difference (SHALL - IS) > 1 appears in 68% of all cases while the difference (IS - SHALL) > 1 never appeared.

Item 59: *better understanding of every day world with mathematics*

Is / Shall	-2	-1	0	+1	+2	*no a.*	Sum Is
-2	-	-	2	2	2	-	6
-1	-	-	-	7	-	-	7
0	-	-	1	4	1	-	6
+1	-	-	-	-	-	-	0
+2	-	-	-	-	-	-	0
no a.	-	-	-	-	-	-	*0*
Sum Shall	0	0	3	13	3	*0*	19

The means are: IS-state - 1,00 and SHALL-state + 1,00 . We see the nearly total agreement for the Shall-state und in opposite the disagreement for the Is-state. The difference (SHALL - IS) therefore is > 1 in 74 % of all cases.

Looking at all three items it is very clear that the students wish more applications or connections with every day life in mathematics education.

Now we want to point at items which concern the way of teaching mathematics.

Item 4: *sometimes make guesses and use trial and error*

Is / Shall	**-2**	**-1**	**0**	**+1**	**+2**	***no a.***	**Sum Is**
-2	-	-	-	-	-	-	0
-1	1	1	2	2	1	-	7
0	-	-	1	6	-	-	7
+1	-	-	1	1	1	-	3
+2	-	-	-	1	-	-	1
no a.	-	-	*1*	-	-	-	*1*
Sum Shall	1	1	5	10	2	*0*	19

The means are: IS-state - 0,11 and SHALL-state + 0,58 . This item stands nearly in opposite to item 2. Notiecable is also the small rate of differences $|SHALL - IS| > 1$.

Item 11: *all pupils understand*

Is / Shall	**-2**	**-1**	**0**	**+1**	**+2**	***no a.***	**Sum Is**
-2	1	-	-	1	4	-	6
-1	1	-	-	3	4	-	8
0	-	-	1	2	2	-	5
+1	-	-	-	-	-	-	0
+2	-	-	-	-	-	-	0
no a.	-	-	-	-	-	-	*0*
Sum Shall	2	0	1	6	10	*0*	19

The means are: IS-state - 1,05 and SHALL-state + 1,16 . Noticeable is the difference be-tween diagreement with Is-state and agreement with shall-state which you also can see by the rate of 68% for (SHALL - IS) >1. This says that most explanations are not so good as wanted.

Item 26: *every stage is explained exactly by the teacher*

Is / Shall	**-2**	**-1**	**0**	**+1**	**+2**	***no a.***	**Sum Is**
-2	-	-	-	1	1	-	2
-1	-	-	1	3	-	-	4
0	-	1	2	5	2	-	10
+1	-	1	-	2	-	-	3
+2	-	-	-	-	-	-	0
no a.	-	-	-	-	-	-	*0*

Sum Shall	0	2	3	11	3	*0*	19

The means are: IS-state – 0,26 and SHALL-state + 0,79. This item shows the same as the item before but not so extremely.

Item 27: *pupils are led to solve problems on their own*

Is / Shall	**-2**	**-1**	**0**	**+1**	**+2**	***no a.***	**Sum Is**
-2	-	-	-	-	1	-	1
-1	-	-	1	4	1	-	6
0	-	-	-	5	1	-	6
+1	-	-	-	2	3	-	5
+2	-	-	-	1	-	-	1
no a.	-	-	-	-	-	-	*0*
Sum Shall	0	0	1	12	6		19

The means are: IS-state – 0,05 and SHALL-state + 1,26 . We see the high agreement in SHALL-state but undecidedness in IS-state. You could think item 27 is the opposite of item 26 but the result does not show this. The shall-differences between item 26 and item 27 with the same answerer can be seen by the following statistic showing the combinations (item26 / item27) by the same answerer: (-1/1) ||, (0/1) |, (0/2) ||, (1/0) |, (1/1) |||||, (1/2) |||||, (2/1) ||, (2/2) |.

I like to complete the view on teaching mathematics with the item about the often discussed role of proofs in mathematics education.

Item 55: *able to offer proofs*

Is / Shall	**-2**	**-1**	**0**	**+1**	**+2**	***no a.***	**Sum Is**
-2	-	-	3	2	-	-	5
-1	-	2	-	-	1	-	3
0	-	1	2	2	-	-	5
+1	-	-	1	5	-	-	6
+2	-	-	-	-	-	-	0
no a.	-	-	-	-	-	-	*0*
Sum Shall	0	3	6	9	1	*0*	19

The means are: IS-state – 0,37 and SHALL-state + 0,42 . We see the difference between the means but also we must see the relatively strong distribution so that we can not point out a special trend (exept that there is no +2 for the Shall-state and no –2 for the Is-state).

If we summarize these results and add results of some other items (I can not show here) as well as some results of earlier questionings (see e.g. MAVI 3 to 5) it comes out that we should look especially at the following five aspects:

1. The aspect of "*application / environment concern*" (see e.g. item 9, 19, 54) is represented as a big wish of all students but it is still paid to less attention to it in school reallity.
2. "*Clearness / good explanations / vividness of mathematics instructions*" (see e.g. item 11, 26) is also one aspect represented as a big wish of all students while the reality is not so good.
3. "W*orking on problems / seeing different ways for solution / no strict scheme*" (see e.g. item 4, 27) is another aspect represented as a wish of the students. Teacher should give more possibilities for tackling mathematics by the pupils and for learning kreativity, heuristics and other ways to handle with problems.
4. "*Proofs*" are seen as typical for mathematics by many students but als many of them don't like them. Not only formalistic proofs should be discussed in class.
5. An aspect of *typical items for mathematics teaching* which has agreement in Is-state as well as in Shall-state (see e.g. item 1) consists of "mentally calculations", "computations with paper and pencil" and "discipline".

The results of the open questions about good and bad experiences in school as well as wishes for future mathematics education underline these aspects.

Therefore I think in future we should look more on these aspects in detail, i.e. we should develop detailed questionaires which focus only on one or two of these aspects extend this by interviews of pupils as well as their teacher and ask for changes of mathematics teaching and the possible changes of beliefs by this.

Stefan Grigutsch

Beliefs and behavior
On theory and research on attitudes

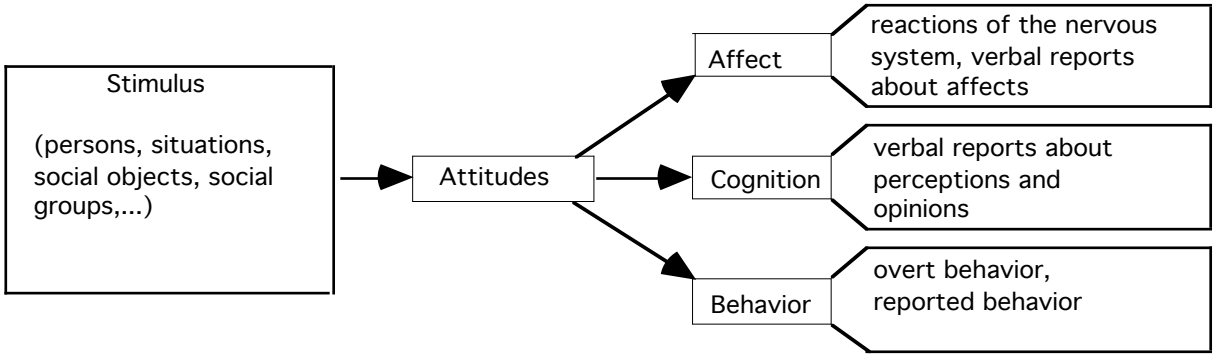
Criticism towards the concept of attitude and recent ideas and approaches

The concept "attitude" is a classical concept and one of the major concepts in social psychology, and it is used very often as a theoretical basis or framework for empirical investigations. We use this concept as a theoretical framework, too (Grigutsch, Raatz and Törner 1995 or 1998; Grigutsch 1996).
The concept of attitude is criticized as often as it is used. Especially the thesis that cognitions and affections have a relation to behavior (relevance for action) is - based on numerous investigations - called in question.
Therefore, I want to present the following:

1. Reasons, why I think that the concept of attitude is an appropriate theoretical basis or framework.
2. Ideas and approaches from recent social-psychological literature showing, how the theoretical concept and the practical research can be improved to raise the relevance that cognitions and affections have for action.
   (This is more a change in the attitude-behavior-model than in the attitude-model.)

Basic ideas

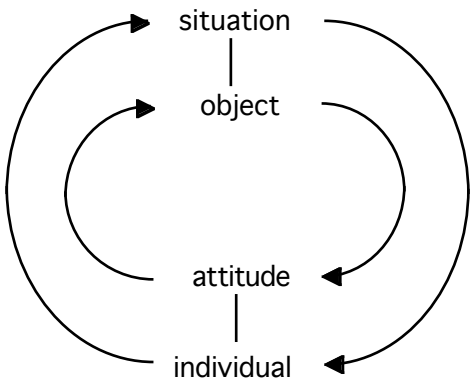
Since the investigation of Thomas and Znaniecki about "The polish peasant in Europe and America" (1918), the concept "attitude" has the following meaning: "Attitude" denotes a lasting, permanent and stable orientation and readiness (or intention) to action of an individual towards a social object. In modern, cognitivistic terms, "orientation" means a consistency in perception, cognitive representation and affective valuation.
Another classical approach in the theory of attitudes is the 3-components-approach: Attitudes are a system of cognition, affection and conation (readiness to act), which principally tend to correspond.



Now I would like to give reasons, why I think that the concept "attitude" is an appropriate theoretical framework.

Thesis 1: The 3-components-approach is very suitable as an ideal starting point for research because he expresses the entanglement and the ties between cognition and affection and between the individual and his social environment.

The concept of attitude follows the idea that the human being and his social environment are entangled and bound. And it follows the idea to catch this entanglement in the way that although attitude is a property of an individual, this individual is not seen as an isolated creature. On the contrary, the human being is confronted with environment, and in this confrontation, human beings create attitudes in social processes. Further, in this confrontation human beings react to social objects, and attitudes contribute to the planning of action.



Surely, the relationship between attitude or belief and behavior is no 1:1-relationship. This is not the thesis of the concept of attitude or of the 3-components-approach. They just express the idea that there are relationships.

The relationships between cognition and affection are known since years. There is a thesis grounded by empirical results: Affections (and the structures that they build) are substantially involved in structuring the memory, in the funcions of the memory (memorizing, remembering, combining) and in thinking processes, socalled intuitiv-holistic thinking processes.
There are significant relationships between cognitions and affections. The concept of attitude and the 3-components-approach take these relationships into consideration. Again, these relationships are no 1:1-relationships. But this is not the thesis of the concept of attitude. It just expresses that there are such relationships.

The concept of attitude expresses the entanglement and the ties between cognition and affection and between human beings and social environment. Due to this idea, I think that "attitude" is an appropriate theoretical framework for research.

1. Even if the relationships are not 1:1 or weak, there are many results and hints that there exist such relationships. And if there exist relationships, it is important that the theoretical framewort takes this into account.
   (There is no great difference if one uses the concept of belief and takes into consideration that there might be relationships to affection and behavior, or if one uses the concept of attitude and takes into account that the relationships may often be weak.)

2. A theoretical framework has, in my opinion, the task to support and not to narrow the view and the research. Therefore, a theoretical framework can be an ideal or a thesis, too.
   The framework "attitude" supports to investigate beliefs, but also supports to investigate in how far affections and action-guiding schemes are connected to these beliefs (or could be connected with them).
   The theoretical framework of attitude gives a wide viewpoint, it prevents to neglect affections and behavior.

Thesis 2: A close connection between attitudes and behavior could not have been proved in empirical investigations.
This means: A simple attitude-behavior-model does not fit to reality.

The relationship between attitudes or beliefs and behavior has been investigated in many empirical works.
Wicker summarizes in 1969: "[…] it is considerably more likely that attitudes will be unrelated or only slightly related to overt behaviors than that attitudes will be closely related to actions. Product-moment correlation coeffizients relating the two kinds of responses are rarely above .30, and often are near zero. Only rarely can as much as 10 % of the variance in overt behavioral measures be accounted for by attitudinal data." (Wicker 1969, quoted from Mummendey, p. 134)
Many studies have in total confirmed Wicker's statement.

There are various reasons that might explain the weakness of the relationship between attitude and behavior:

1. methodological lacks
   - lacks in the measurement of attitudes
   - lacks in the measurement of behavior
2. conceptual lacks
   - lacks in the concept of attitude, especially the thesis "there is a close relationship between beliefs and behavior"
   - lacks in the concept of behavior
   - lacks in the attitude-behavior-model, especially the thesis "there is a simple relationship between attitudes and behavior"

We can derive alternative proposals:

proposal 1: The concept of attitude should be given up and be replaces by a theory of behavior. *

proposal 2: The concept of attitude should be improved in many ways.

In my opinion, the criticism towards the concept of attitude should lead to immanent improvements. This should make it possible to measure cognitions or attitudes which are relevant for action.

Thesis 3: It has been the implicite aim of the research to decribe and register cognitions and affections which do have a relevance for action, e.g. which are (closely) related to action.
Collecting cognitions and affections which are not related to action is of minor interest.
In my opinion, the attitude-behavior-model (and not the concept of attitude) has to be improved to model the entanglement between individual and social environment better than before.

There are two reasons stressing that it is very important to measure cognitions and affections which are relevant for action:

1. Attitudes as explanations for behavior.

* Due to the weakness of the relationship between cognitions or affections and behavior, many researchers give up the 3-components-approach. They only regard the cognitive component (Rokeach) or only the affective component (Fishbein & Ajzen) or a combination of the cognitive and the affective component (Rosenberg; Pehkonen).

The starting point for our research in attitudes or beliefs are questions refered to the learning-behavior of pupils in mathematics education. For example "Why have pupils difficulties in problem-solving ?" Attitudes or beliefs are considered in order to understand the learning-behavior of the pupils. According to this example: A pupil is decribed as algorithm-orientated or used to traditional teaching, and thus the researcher can explain his behavior and his difficulties in learning.
Attitudes or beliefs are used as predictors of behavior, even more, they are used as explanations for behavior (difficulties, problems).
This setting for didactical research is based on the assumption that there exists a causal relationship between attitude and behavior.

2. Research into attitude change or belief change.
The aim of research into attitude change (or belief change) is to change the attitudes of individuals with the intention to change their behavior. For example, the attitudes of teachers (or pre-service teachers) should be changed in order to change their teaching-behavior. This setting for research is based on the assumption that attitudes and behavior are closely related.

In both cases it is important for mathematic-didactical research that cognitions and affections are related to behavior. If a researcher only intends to describe psychological processes which have no relation to action, then his investigations are of minor interest.

Thesis 4: There are a lot of approaches, on one hand in the theory or in the model and on the other hand in the methods of research, which can contribute to raise the relevance that attitudes (or beliefs) have for action.

Thesis 4.1.: The methods used in research should realize the (classical) concept of attitude better than before:
Consistency and Independence from situation.

One part of the (classical) concept of attitude is the demand that attitudes are expressed in consistency (of perception, valuation and readiness to action).
But in practical research, cognitions and affections are often "measured" in a single moment. Many of the data that is gathered are no consistent cognitions or even attitudes which are relevant for the individual, but spontaneous, unique and thus rather unimportant cognitions. Probably, such cognitions do not say much about patterns of perception, valuation and action. Therefore the relationship between cognition and behavior is often weak.
If the researcher is interested in patterns (and not in single, isolated cognitions), he must look for patterns. In order to gather cognitions or attitudes which are relevant for action, the researcher has to observe over a longer period of time in various situations and look for consistent patterns.

A second part of the (classical) concept of attitude is the independence from situation. Attitudes are expressed in a consistent perception, valuation and readiness for action towards a whole class of similar situations. Attitudes are generalized schemes (in the language of Piaget).
But in practical research, attitudes are often measured in a specific situation (or in a laboratory situation without specific circumstancas), and the results are generalized to any kind of situation. No wonder that they have no relevance for action in any kind of situation.
If we want to obtain attitudes which are relevant for action, we have two possibilities:

(i) We have to investigate attitudes or beliefs in a few situations and prove whether they are independent from situation. If we have found situation-independent cognitions and affections, there is a higher probability that a person uses these cognitions and affections in further, similar situations.
(ii) We have to decribe the situation, in which an attitude is "measured", very carefully and exactly.

If the theory we use makes the assumption that attitudes are situation-independent, then I suggest to define only a small class of situations because I am convinced that attitudes are highly differenciated to situations.

Thesis 4.2.: The attitude-behavior-model which assumes simple consistency, that means a simple relation between attitudes and behavior, has to be replaced by a model of conditional consistency.
This model of conditional consistency has to be realized in research practice.

The model of simple consistency assumes that behavior in a certain situation is determined by one attitude, e.g. that behavior corresponds to the attitude which is "active" in a situation.
There are three alternatives for the model of simple consistency: the model of conditional consistency, the model of Fishbein and Ajzen, and path-analysis.
I prefer the model of conditional consistency because it preserves the 3-components-approach and because it has relations to Piaget's theory of action.

Conditional consistency means:
The action or the behavior in a situation does not only depend on one attitude - this would be the thesis of simple concistency which failed to fit to reality.
Behavior or action in a situation depends on

- the interaction of various attitudes,
- personal factors,
- situational factors (or the perceptions of situation).

All these variables or factors might have a direct influence on the planning of action. In addition to that, they are related to another, so that each variable can have an intervene effect on the influence that another variable takes on behavior.

Personal factors.
Important personal factors are "self-monitoring" (german: "Selbstüberwachung") and "self-awareness" (german "Selbstaufmerksamkeit"), which in most cases have an intervene effect on the attitude-behavior-relation.
Individuals are differenciated according to their ability to self-monitoring and self-awareness, and the prediction of the behavior is only made for part of the population, not for every person.

"Self-monitoring" (german: "Selbstüberwachung").
Human beings often hide their real opinions, attitudes or intentions (of action), or they don't act according to them. Instead, they can steer and control their behavior in order to influence the picture that others receive. A person who is able to steer and control her behavior is called to have a high "self-monitoring". (The concept, the questionnaire and the scale has been developed by Snyder.)
Persons with high self-monitoring tend to behave more according to situational conditions and stimuli, persons with low self-monitoring tend to behave more according to their attitudes.
If we want to obtain relevant attitudes or beliefs, we should only ask persons with low self-monitoring.
(Schiefele, p. 75 ff.)

"Self-awareness" (german: "Selbstaufmerksamkeit").
Persons concentrate their attention in a certain moment either mainly to the self or mainly to external events. The situation in which a person puts herself into the focus of attention or awareness is called "self-awareness". (The concept, the questionnaire and the scale has been developed by Duval and Wicklund.)
Persons with self-awareness tend to act more according to their attitudes.
(Schiefele, p. 115 ff.)

Differentiations of attitudes.
Further personal factors are individual differentiations of attitudes. There are a lot of criteria to differenciate attitudes. The thesis is: Not every attitude is suitable for the prediction of behavior, only attitudes with additional qualities.

1. Level of centrality / relevance, extremity and intensity.
Attitudes can be differenciated according to the level of their centrality (or relevance or salience), extremity and intensity. According to this differentiation, attitudes have a different relevance for action.
One example: Centrality. An attitude can be more or less central or peripheral for an individual. Research has shown that attitudes which are very central change seldom or only very slightly. Thus we can assume that centrality of an attitude is an important intervene variable for the

relationship between attitude and behavior. The more central the attitude is, the more relevant is this attitude for action.

2. Attitude structures.
If we don't look only on a single attitude but on many attitudes that are held by a person, we can analyze the relationships and connections between these attitudes and we obtain an attitude structure.
Especially we can look at
- the level of complexity and differentiation,
- the level of internal consistency.

If an attitude is complex and highly differenciated, for example according to situations and circumstances, then the prediction of behavior must be as differenciated as the attitude. An undifferenciated prediction of the attitude-behavior-relation would not fit to reality.
It is also important whether a complex attitude is consistent or contradictory. Contradictory elements in a complex attitude can lead to contradictory behavior.

3. The consciousness of the attitude.
If an attitude is conscious then on the one hand it might have more influence on the process of planning the action. On the other hand, if an attitude is unconscious then it can't be measured but can influence the behavior.

4. Level of generality and universitaly.
General and universal attitudes are more relevant in comparison to attitudes which are especially fixed to certain situations.

Situational factors.

1. Natural situations.
The most researchers share the assumption that attitudes are learned dispositions, that means that they are developed as reactions of an individual towards a certain situation; later they are generalized as dispositions towards a class of similar situations. (In the words of Piaget: Attitudes are developed as an adaption of the individual to demands of the environment, and specific schemes are generalized later on.) In my opinion, it is of greatest importance to investigate these natural situations in which attitudes are developed and generalized because attitudes might be tied to these situations.
One may raise the objection that in the theory, attitudes are developed as dispositions which are independent from special situations and circumstancas. That means that they are general schemes (of perception, representation, valuation and action), and this generality is important for their function and value for the individual: In a broad class of similar situations, a person can act without developing new action schemes. Due to this independence from a certain situation, one might neglect the situation in research.

In my opinion, the following is true: attitudes are generalized schemes. But we have to prove to what extend these schemes are generalized.
First, these schemes might never be such general so that they fit to every situation. A generalized scheme is bounded to the class of situations in which this scheme was developed. So a researcher has to investigate these situations in which the schemes (attitudes or beliefs) are developed. These situations are natural situations and not laboratory situations.
Second, a scheme becomes generalized only if a person makes a lot of experience. But if there is a lot of experience, the general scheme might be highly differenciated. So a researcher must decribe the situation differenciated and carefully, too.

So my proposal is the following: The researcher should investigate the natural situations in which attitudes are developed. A class of similar and consistent cognitions, affections or actions towards a class of similar situations indicates an attitude. This attitude should be described together with the situational circumstancas and conditions. By this way, the probability for exact predictions of action increases, that means that the data gathered is more relevant for action.

2. Reference-groups or -persons (german "Bezugspersonen").
When attitudes are developed in natural situations, there are reference-persons present. If we observe attitudes or behavior in laboratory situations, then there are no reference-persons. It is known that in such laboratory situations without reference-persons, the level of the relationship between attitude and behavior is over-estimated. Reference-groups have an decreasing effect, sometimes an increasing effect on the attitude-behavior-relation.
Attitudes should be measured in situations in which reference-groups are present, for example natural situations. In laboratory situations, one should try to estimate the influence of reference-groups. For example, we can ask a pupil: "What do you think would your teacher say to your opinion ? What about your camerades ?"

3. Routine situations and norms.
It is well known that the level of the attitude-behavior-relation is under-estimated if a deviation from the routine is necessary in order to act conform with the attitude. In other words: In a routine situation, attitudes are more relevant for action.
In research, one should investigate a lot of situations to identify routine situations and routine behavior, and one should identify cognitions and affections corresponding to this routine behavior. If attitudes (cognitions, affections) correspond to habits, routines or norms, they are probably more relevant for action.

These improvements in the theory and in the practical research might (partially) refute the criticism towards the lacking relevance for action. Besides, there is a global criticism that the concept of attitude would be based on a "wrong" (unsuitable) model of human action.

Thesis 5: **Attitude theories and interactionistic theories describe models of the human being and human behavior that are (seem to be) different and alternative models.**
But in the light of Piaget's theory of learning and action, these two theories join into a senseful coexistence and a uniform model:
attitude theories decribe (more) assimilative processes, interactionistic theories describe (more) accomodative (and less assimilative) processes. Therefore, both classes of theory are justified, and they complete one another to describe human behavior and action.

Interactionism (example Blumer) draws the following picture of human beings:
An individual is an active actor, who posesses the ability to creative action that deviates from routine and norm. He does not act due to the influence of outer or inner factors, but he acts on the basis of definitions of the situation, e.g. that the individual ascribes meanings to a situation - the individual himself and in this special situation. Theses definitions of the situation are changeable, and they can be uncertain and even contradictory (inconsistent).

Attitude theories (model of simple consistence) draw the following picture of human beings: An individual reacts to situations: There is a perception according to (well known) schemes, an assimilation (cognitive representation) und a valuation into known schemes, and then an action according to known schemes of action; in total there is an action following norm and routine. The action is to a large extend influenced by an outer factor "situation" and an inner factor "routine" (scheme, attitude). These routines are rather fixed and certain, and they are proved good in former situations, and contradictions and conflicts between two schemes (dissonance) are tried to solved.

	interactionistic theories (Blumer)	attitude theories (simple consistence)
human being	• active actor • ability to action that deviates from routine and norm	• reacts to situation • action according to norm and routine
action or behavior	• according to a meaning that is ascribed in the special situation	• highly influenced by outer and inner factors: situation and routine
definition of the situation	• changeable, uncertain, contradictory	• routine: fixed, certain, proved good, no conflicts

As we can see, these two classes of theory seem to be contradictory and alternative paradigms; there seems to be a deep gap between them, and there are indeed researchers who stand on the one or the other side of this gap.

In Piaget's theory of learning and action, there is a vis-a-vis of assimilation and accomodation. Both processes complete one another.
In my opinion, in attitude theory the situation is the same. In some cases, situations or objects are assimilated; there are schemes applied which are proved good, and attitudes belong to these schemes. In this cases, the consistency between attitudes and behavior is rather high.
In other cases, situations and objects are not assimilated but schemes are accomodated, e.g. "new" schemes are developed or constructed. All the factors mentioned obove (schemes, situation, person, attitudes, norms, ...) join to the construction of new schemes and actions. Attitudes are one aspect among others, and the consistency between attitudes and behavior is rather low.

In some situations, the individual holds fast to routines and patterns, there cannot be a permanent change. Only because there exist such lasting, durable and rather general patterns as attitudes, orientation, stability and continuity is possible in society.
In other situations, there is an ascription of new sense and new meaning to this situation, and a construction of a new scheme; there cannot always be the same routine. Only because such specific and changeable processes take place, a new orientation, learning, progress, creativity and change are possible in society.

There cannot only be continuity, and there cannot only be change. Therefore there are situations in which for example attitude theories are a suitable modelling, and there are situations in which interpretative theories are suitable.

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"That was really stupid. You don't need such in life."

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Abstract

This article is a case study of the dynamics of attitudes of one seventh-grader. First I present her negative attitude towards mathematics ("stupid", "You don't need math in life"), then how a negative emotion develops in a problem solving situation. From insecure beginning it changes through frustration to rejection. She works in a group and this emotional process is connected with a social process. I suggest that "I don't need this" is her defence strategy, and that similar emotional experiences are a reason for her negative attitude.

Previous research and theoretical background

Mathematics is a school subject that many pupils have quite emotional relation with. Satisfaction and joy often accompany a successful solution of a problem (McLeod, 1988). The dynamics of attitudes are not well understood. Attitudes tend to become more negative as pupils move from elementary school to secondary school. Efforts to promote desired affects of students have usually induced only slight changes and sometimes even contrary to expectations. (McLeod, 1994)

McLeod (1988) sketched a theory of affective issues in a problem solving situation. He suggested the following aspects to be studied: the magnitude, direction, duration, level of awareness and level of control of the emotion. The short term emotions that usually are intense are called local affects. The relatively stable, but less intense attitudes and beliefs are called global affects.

Goldin (1988) presented "affective pathways" as a structure for the dynamics of affective domain in mathematics. These pathways are established sequences of states of feelings that interact with cognition and suggest strategies during a problem solving process. Affects are not 'noise' of human behaviour in problem solving, but a representational system parallel to and crucial for cognitive processing. Later (DeBellis and Goldin, 1997) this affective representational system has been refined into a model, where four components interact on individual level: emotional states, attitudes, beliefs, and values/morals/ethics. Interaction with environment is also included in the model.

Research project and the focus of this report

The research project explores changing beliefs about and attitudes towards mathematics through the grades 7 to 9. A type of action research is fitted to the project, in which the author acts as a teacher-researcher. A description of the

project was given at MAVI-3 (Hannula, Malmivuori & Pehkonen, 1996). This report focuses on the changing affects of one pupil.

Methodology and data

I work with enactivist methodology (Hannula, 1998), where the two key features are "the importance of working from and with multiple perspectives, and the creation of models and theories which are good enough *for*, not definitively *of* " (Reid, 1996, p. 207). Qualitative approach was chosen to understand the dynamics on individual level.

Large variety of data on Rita was available for me as her teacher and form master. I have tried to reach the multiplicity of perspectives through discussions with Rita's other teachers and fellow researchers. I have reviewed the material several times and discussed my interpretations with Rita.

This paper relies mainly on one interview (December 1996). Some episodes that were recorded in my diary and a few lines from a third interview (December 1997) will be used too.

The code-key for the transcription:

(x.y); (.)	pauses: x.y seconds; less than 0.5 seconds
(-); (--); (*text*)	unclear speech: one word; several words; plausible words
wo(h)rd	word has been spoken laughing
[*text1*]; [*text2*]	texts 1 and 2 spoken simultaneously
=	talking continues immediately after the other speaker
{text}; {...}	editorial comments: about context or tone of voice; text omitted

"You don't need math there in life"

Rita's primary school teacher gave a description of Rita, that I agree with:

> *She was all the time hustling and talking. {...} But Rita was easy in that sense that when you spoke with her, she always did try. She was a bit like 'Pippi Långstrumpa'. A good hart, but somewhat of an anarchist. But I think she was a nice pupil. Loads of good properties. I do myself prefer these lively that need tempering down compared to those that never raise their hand. From Rita you always got it. Sometimes correct sometimes false.*

At the time of the first interview I had been teaching the class for four months. I evaluated her success satisfactory. In this interview there were also two other girls from the same class. Rita's comments were not too flattering for me as her teacher.

[13] Rita: [*Mathematics*] *was nicer in elementary school than in secondary school.*{...} *I don't remember anything but at least we had so, that* {...} *And alike. I don't remember anything, it was so stupid.*
[38] Markku: *What has been most boring?*
[39] Rita: *The, thethethethe story problems.* {...} *I don't understand them ever.* {...} *You don't need math in life. I think. Because I do know enough math to manage when I go to buy a shirt or need to know the time or such.* {...} *I can't explain, but in a way like* (1.0) *now when we have really strange things in math. All that we have had at elementary school, all fractions and such, and these you do need, but not these* (1.1) *these other things.* (1.6) *These, I can't explain, the things that come for example on ninth and at high school. You don't much need those there in life.*

In Rita's last sentence the word '*there*' reveals how she sees school life alienated from real life. In the real life out there, she doesn't need the mathematics that is taught in school. She already masters what she needs.

"I don't like this t(h)ask at all"

In the first interview I gave the group of girls three tasks to solve together. I gave the written tasks one at a time and recorded their solving process. Here I shall summarise the process and concentrate on Rita's contribution.

Task 1. Salla is working on an abstract painting. She has divided an area with straight lines into parts. She wants to paint the picture with as few colours as possible. Parts that are side by side, may not be of same colour, but those touching only in corners may. How many colours will Salla need. (Below the text was a picture that could be coloured with three colours.)

Maria and Lisa start solving the task together, trying to find out one possible colouring. Rita's comments are few, and she gets no response. From the discussion I extracted here all Rita's statements and the answers when she got any. The running time is shown on the right side.

	Time
[277] {Beginning the task}	{0.00}
[293] Rita?: [(*-*yellow*)]	{0.43}
[308] Rita: *I don't like this t(h)ask at all.*	{1.15}
[325] Rita: *Yhm. Yes*	{2.00}
[327] Rita: *Is this then yellow, 'cos that* (-).	{2.03}
[332] Rita: *How come it's blue then?*	{2.18}
[333] Lisa [to Maria]: *Yes, probably it would go with three colours.*	
[334] Maria [to Lisa]: *Three colours.*	

[335] Rita: *Hey! Because that one is yellow.* (3.0) *Mm?* {2.20}
[336] Maria {to Rita, puzzled}: *What did you say?*

Rita seemed to have difficulties in the beginning. I had to tell the girls to move closer to another so that Rita could read the task. At the beginning of the solving process she got very close to frustration. After two minutes there was the first sign of understanding. When she tried to break in, the other two seemed to ignore her first, but she was persistent and was taken in.

In another interview she described how '*pissed off*' she had felt because she had been '*thrown outsider*'. Although a full year had passed, the feelings were still intense enough to alter the tone of her voice to faint and sad.

The second task was an estimation of the number of letters in a given book. Rita understood the task at once and was an active contributor in the solving process.

Task 3. Addition, subtraction, multiplication and division are operations. Let's define a new operation # in a following way:
When a and b are numbers, then a#b = (a+b)*(a-b).
An example: 2#3 = (2+3)*(2-3)= 5*(-1)=-5
a) Do the following calculations:
2# (-3) =; (-2)#3 =; (-2)#(-3) =
b) In addition you may change the order. For example 2+3=3+2.
May you change the order in the defined operation #?

This was a difficult task and there was 16 seconds of silence after they had read the task. Then there was a period of feeble attempts, where Rita was actively involved. As soon as the others grasped the idea of this task, Rita was left as an outsider.

[486] {Rita's last effort to contribute} {0.00}
[500] Rita {tired voice}: (*That is*) *a nice* (*task*).
[508] Rita {half yawning}: *What would have been the right answer?* {1.15}
[513] Rita {offers chewing gum}: *You want some?* {1.24}
[515] Rita {checks if she has more}: *Let me see*.
[518] Rita: *I don't understand a piffle of what they are even trying to do the(h)re*. {1.42}
[528] Rita {parodizing Lisa and Maria}: *Minusminus five minus minus six hundred.* (1.7) *Look, you don't need this for example in your life*. {2.40}
[530] I: *Yhm*.
[531] Rita: *These are exactly the kind* (*I mean*).

Here Rita claims that this kind of mathematics is not needed in life. However, that does not seem to be the reason for her giving up work. **First** she tells, that she

doesn't understand. **Next** she taunts the task. **Finally** she tells, that this kind of mathematics is not needed. After they had done the tasks I asked how they liked the tasks. Rita thought that the two first were OK, but the third...

[558] Rita: *That was really stupid. You don't need such in life.*
[559] Maria: *I liked to do that one especially.*
[560] Rita: *You will certainly become some philosopher* (-) *when you grow up.*

Discussion
This report illustrates how emotions in a problem solving situation shape beliefs and values of Rita. Emotions (frustration) awaken a belief in mathematics ("*you don't need this for example in your life*"). There is also a link to values: this mathematics has no value.

The social process of the group had a strong influence on Rita's emotions that guide her process of problem solving. Rita is willing to work with her peers. With the first task she was persistent and was able to break into the discussion. In the third task she remained outside and finally took a resistance-position.

Could it be, that telling that some kind of mathematics is not needed actually is a defence strategy of the self? Being left outside is not easy for Rita. As she found out that she couldn't follow, she tried other approaches. She started to make ironical comments on the task. As the others ignored that too and even seemed to enjoy the task, she probably felt more and more rejected. So she made a counter-attack to reject the task.

How far can we generalise this? On more general level she disvalued mathematics. I am tempted to believe the reason is repeated lack of understanding? Often pupils in the class ask "what we need this for?" The teacher should take it as a warning sign. Maybe they do not understand what has been taught?

Another episode from the next fall (Diary, 24.9.1997) reveals how Rita's defence strategy is linked with understanding. A friend argues that she doesn't need powers and Rita replies - not claiming the need - but the easiness.

> Pia: *What we need these powers for? {...} I don't need these.*
> Rita: *These powers are really easy.*

An epilogue: "I think that now mathematics is quite nice sort of"
Half a year later Rita said: "*I think that now mathematics is quite nice sort of. In elementary school I didn’t like it at all.”* The reasons Rita gives for this change seem to be in circles: It's *more fun* because she has *been understanding more,* because *mathematics is quite nice* because she *has learned more*. For Rita liking

mathematics is almost equal to understanding it. When she didn't understand a new topic she complained that *"now mathematics is becoming stupid again"* and wished that *"We don't need these anywhere, do we?"*

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