Research on Mathematical Beliefs
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edited by

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Editor's Statement

The book consists of the abstracts of the presentations at MAVI-9 workshop. The conference language was English. There were no plenary talks, but every presentation had a time slot of 30 minutes with a follow-up discussion of another 30 minutes. The concept ‚beliefs' was seen in a wide meaning and presentations in this workshop dealt also with conceptions, images, views, and attitudes.

There is no prereviewing process on MAVI-workshops. One of the aims is to promote belief research by younger scientists on a European level. So, each contributor is responsible for his/her own publication; the content as well as the presentation.

The MAVI-9 event took place in VIENNA, Austria, one of Europe's most attractive cultural cities. This, in addition with the interesting MAVI research subject, effected in numerous international participants of this workshop. We were pleased to welcome also some colleagues who enjoyed MAVI for the first time. Even two scientists from the far city of Hong-Kong presented their results.

Last but not least, we like to thank the colleagues at VIENNA, Prof. Dr. Reichel and Dr. Goetz, for organising a remarkable scientific conference and combining this event with a nonforgettable impression of VIENNA.

Finally, my thanks address Mrs. Astrid Brinkmann for compiling the different files of the contributors and making this report possible.

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Pupil's Beliefs in Vienna and Lower Austria

Markus Berger

Abstract

In 1998 and 1999 I visited different schools in Vienna and Lower Austria for my research about mathematical Beliefs as I had to write my diploma. I took a questionnaire from E. Pehkonen and St. Grigutsch and worked it out on those criterions:

- *The questionnaire was evaluated on five different groups of pupils' beliefs.*
- *Working out differences between 13 and 17 year old pupils*
- *Working out differences between pupils in Vienna and Lower Austria*
- *Working out differences between boys and girls.*

Additionally, there were three more questions on the questionnaire about good and bad experiences and desires for maths. Lots of surprising answers came up.

1. Introduction

Last year I wrote my diploma with help from Dr. H. - Chr. Reichel and I was concerned with mathematical conceptions of Austrian pupils. Of this purpose I went to seven schools in Vienna and Lower Austria in order to hand out 740 questionnaires to 13- and 17- year old pupils. I handed them out during the mathematical lessons and their mathematics teacher was also in the class. I told the pupils that their teacher will not be allowed to see the questionnaires. The questionnaire was taken from E. Pehkonen and St. Grigutsch who made some examinations in Germany, Hungary, Estonia and other European countries. It's worth pointing out that there were 32 questions which pupils had to answer. There was a Likert typical scale, 1 means total agreement, 3 indecision and 5 total rejection. Moreover there were three more questions about good and bad experiences in and wishes for mathematics. At home I calculated the mean value and in addition some more statistical values. In conclusion I classified the mean value in five groups:

Total agreement	mean value less than 2,00
agreement	2,00 – 2,75
indecision	2,75 – 3,25
rejection	3,25 – 4,00
total rejection	mean value more than 4,00

All in all I handed out 740 questionnaires, half I gave to schools in Vienna and half in Lower Austria. Some more girls were interviewed than boys, about two-thirds were 13 – year olds. The aim of the report was to show the most interesting results of this diploma.

For one thing, there were five groups of conceptions on the questionnaire, but then, they were totally mixed up. This five groups are:

Conceptions about: mathematical contents
mathematical methods
learning maths
part of pupils
part of teachers

2. Results of the five groups of mathematical conceptions

To come to the point, at first I want to give a report about the most interesting results of these five groups of conceptions. First of all I want to say, that there are only a few questions which got rejection.

The following I want to say is based on the category of mathematical contents. Firstly it's quite surprising that there is a fairly strong agreement to use the pocket calculator. That's why some will draw the conclusion that it is rather unimportant to do mental arithmetic, but this is not so. In spite of that, I think pupils should be confronted with flashover mental arithmetic from time to time. If one compares that with other European countries, one notices that there is a remarkable big difference in using the pocket calculator.

All the questions in this group get agreement just for the one: It looks as if it is quite unusual to produce and work with real things, for instance cubes, pyramids and so on. You can recognise that this question got rejection. This is interesting because most of the pupils learned geometry in the lessons before I handed out the questionnaires. By the way, my opinion is that teachers are able to use such objects in many different parts of mathematics, for example in geometry.

Secondly I want to give a general view of conceptions in mathematical methods. To start with, all questions which are stated on the questionnaire are formed with agreement, just for three: The question if only the result of the examples is important became a mean value of four. For one thing, this is a very pleasant result for me but then there is also agreement in this question that there is always a specified way to get the result. In my opinion you can draw the conclusion that therefore this result is not so pleasant as it looks at first. The other question which gets strong rejection is that doing mathematics is only possible for talented pupils. As far as I know from some personal interviews in Lower Austria in Vienna often parents and grandparents give pupils an insight that mathematics is only a concern for talented persons.

Also you can recognise that pupils think that it is very important for maths to be very exactly and in addition you have to give exact reasons if you say something.

Now I want to give a general view of conceptions in learning mathematics. There is only one question in this part which get rejection. This is the question if it is essential to maths to learn by heart. In general there is a strong agreement that pupils want to do some exercising in the lessons and that they want to be able to understand what the teacher is talking about. All the other questions in this group got agreement and that was to expect.

Also in the fourth group which deals with the part of the pupils is no surprising result. I think it is worth to point out that there is only a little agreement in the question if it is possible to guess or try when you are solving mathematical problems. This questions got a lot of answers with total agreement and lots of total rejection. This is the reasons that the mean value shows only a little agreement.

In the last group, the conceptions in the part of the teacher, comes up, that the teacher shall help as soon as possible and anytime there come up some problems. In my opinion this is not a very surprising result, but it is rather impossible for every teacher to do this every time the pupils want to.

You can see from these facts that the mathematical lessons in Austria are not in a complete other way as in other European countries. To draw a conclusion, you can get the impression that the lessons in Austria are maybe a little more traditional as in some other countries, the results are nearly the same as in Germany. It seems that the mathematical contents and calculating are in the foreground. But in conclusion I want to point out that the differences between most of the countries are not very noticeable.

3. Differences between boys and girls

Now I will be concerned with the differences between boys and girls. In general there are no big differences just for two: For girls it is a bit more important to do a lot of practice in the lessons as for boys. Nearly the same is the difference in the question about the discipline in the mathematical lessons. So you can see that the differences are not very big, if someone wants to he can interpret the girls have some more traditional conceptions of the mathematics than boys.

4. Differences between Vienna and Lower Austria

Coming to the next point it is not so surprising that there are no big differences between pupils in Vienna and Lower Austria. Only the two following questions got a little difference: On the one hand the pocket calculator but on the other hand also mental arithmetic got a little bit more agreement in Lower Austria than in Vienna. Nearly the same result came up on the question if discipline is important for maths, also in this question is a little more agreement in Lower Austria. Maybe these results depend on this point that the schools in Lower Austria are located on the country and not in such a big town like Vienna is.

5. Differences between 13- and 17 year old pupils

There is certainly no doubt that there are more differences between 13- and 17 year old pupils than before. For the younger pupils it is more important to do mental arithmetic as for the older ones but for me this is not a very surprising result because most of the schools I went to allow the pocket calculator the first time when the pupils are thirteen years old, before they have to do mental arithmetic. For this reason the pocket calculator gets more important when you get older.

Another result is that for 17 year old pupils practising in the maths lessons is much more important than for the younger ones. Contrary to this result it is for 13 year old pupils more interesting to do something with concrete things. But I think this is also not very unexpectable because most of this group of pupils learned geometry in the days before I handed out the questionnaire.

In conclusion there are lots of more differences between 13- and 17- year old pupils but the mean value is always in the same group.

6. Good experiences with maths

Finally I want to give a general view about the three more questions on the questionnaire. For me it was not very surprising that there are more bad experiences with maths as bad ones, but it was very pleasant that there were a lot of wishes for maths lessons in future.

To come to the good experiences it is worth pointing out that lots of pupils have the opinion that the practising lessons for preparing to a test are very sensefull and also helpful. Also many pupils told me that they think that mathematics is very important to train their logical thinking.

A lot of 13 year old pupils mentioned a mathematical content as good experience, that is geometry.

7. Bad experiences

The next point I want to give a general view about are the bad experiences with maths. 16% of the interviewed pupils are thinking that the tests are too heavy. Most nearly the same amount told me that the teachers are too fast in dealing with a mathematical problem. As a consequence from this bad experience it is not surprising that lots of the pupils critize that there is too less practising in the maths lessons because it looks like that some teachers are solving only one or two examples in a maths lesson and the pupils have no time to come to the result alone. It was also a wish from a lot of pupils that there should be some time in school that they can solve problems alone and the teachers is only a helping person if there are any problems.

The next bad experience is that many of the interviewed persons have the opinion that there are to many different contents in maths in school. More expectable was the following answer, this is that some pupils critize that the teachers are too strong in teaching maths and in judging the tests.

Nearly 15% told me that they get too much examples to do at home because for a few of them it takes sometimes more than two hours to do only the exercise in maths they have to do at home.

Before I went to the schools I thought that most of the bad experiences are the names of some teachers and I was very pleased that a lot of answers I got were not in this way.

8. Desires for maths lessons

The last question is about the wishes for the mathematical lessons in future. Almost 20% of the pupils who have given answers to the questionnaire hope that the explanations from the teachers get better.

Also nearly the same amount thinks that it would be better for teaching if some lessons are organised with the principle "Offenes Lernen". Most of them who told me that did some work in this way before. Some teachers told me that the pupils get a lot of motivation and are very interested in such lessons but the time for preparing these lessons is enormous.

16% of the interviewed persons hope that there is more working in groups in future or practising greater projects maybe together with other subjects in school. So everybody can see that pupils know that there are also in mathematics some alternatives to the talking of teachers in the front of the class. I think it will be very important for the fame of mathematics to make a use of this chance and to do some work in this way.

19% of the pupils wish that there will be more reference to reality in the examples which are solved in maths. Maybe this answer is a consequence of one of the questions on the questionnaire.

More than every tenth of the kids think that it will be extremely more motivation to learn if the math lessons are sometimes a little bit humorous. Also some pupils have the opinion that it will be more pleasant if some units are explained more often and in some different ways, because this will maybe help to understand some units a little bit better. Another wish is the great hope for more practising in math lessons.

Almost every twenth pupil asks for more working with the Personal Computer in the mathematical lessons. There were two schools where they have done something in mathematics on the Computer before and nearly everyone asks me for more doing in this way.

These were most of the wishes which more than twenty pupils told me. It is worth pointing out that there were no ideas on the questionnaire for these three questions. This is the reason why I think these are wishes which are not unessential.

This was a general view of the results of my diploma. The greatest problem for me is that I asked on the questionnaire for their conceptions of mathematics but I think that most of the pupils told me who their math lessons are in reality.

Beliefs about mathematical contents

	Mean value	σ^2
Mental arithmetic	2,03	1,07
Calculating	1,87	0,85
Drawing of figures	1,78	0,95
Textual examples	2,22	1,23
Pocket calculator	1,52	0,82
Learning different themes separately	2,13	1,29
Examples from reality	1,72	0,91
Surface planitation	1,92	0,97
Concrete things	2,98	1,67

Beliefs about mathematical methods

	Mean value	σ^2
Only the result is important	4,01	1,22
Speaking exactly	2,69	1,55
Only one way to the right result	2,67	1,44
To give exact reasons	2,28	1,14
Only for talented pupils	4,16	1,16
Not every time humorous	2,40	1,44
Demands pupils very hard	2,91	1,42
More possibilities to the right result	1,71	0,78

Beliefs about learning maths

	Mean value	σ^2
Is clear for everyone	1,33	0,67
To learn often by heart	3,90	1,07
Repeat a lot	2,17	1,24
Practice a lot	1,70	0,71
Everything has to be clear	1,26	0,38

Beliefs about the part of pupils

	Mean value	σ^2
It is also possible to guess or try	2,60	1,87
Only the result is important	3,29	1,44
Discipline	2,40	1,61
Own questions or examples are allowed	2,56	1,57
Alone calculating	2,00	0,94
Working in groups	2,34	1,82

Beliefs about the part of teachers

	Mean value	σ^2
Teachers have to help very fast	1,41	0,55
Mathematical games	2,33	1,58
Explain exactly every step	1,36	0,59
Exact instructions	2,56	1,72

Multiple Realities and the Making of Worlds

A Multi-perspective Approach to Mathematical Belief Systems

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Abstract

After a period of rich empirical outcomes, in the field of mathematical belief research a certain 'lack of theory' becomes visible. It more and more emerges that the complexity of individual as well as cultural belief systems cannot be understood solely from the perspectives of a single discipline such as (social) psychology. This paper is aimed at giving a short outline of a theoretical model for the scientific construct 'belief system' which extends the classical psychological perspective by aspects of current sociology, ethnography, and cognitive science, especially of neuro science and cognitive linguistics.

Background

Over the past two decades, numerous empirical studies have shown up the decisive impact of mathematical beliefs on math teaching and learning processes. In their bibliography, Törner & Pehkonen (1996) list more than 800 relevant items. As a first approach, an individual's mathematical belief system can be understood as "the compound of his subjective (experience based) implicit knowledge (and feelings) concerning mathematics and its teaching/learning" (Pehkonen & Törner 1996). Most authors agree in conceptualizing mathematical beliefs as mental structures located in the borderland between cognition and affect, gaining considerable relevance as they function as individual director systems guiding personal perception, assessment, and behavior in the context of mathematics.

Not least in view of the rich and manifold empirical results that belief researchers have been coming up with by now, the research interest at present increasingly focuses on the question of how to theoretically substantiate the central scientific concept of *belief system*. The traditional reference to the concept of *attitude* – and thereby the settlement of belief research within the domain of social psychology which may be mainly due to pragmatic reasons (attaining the benefits of an established theory) – more and more emerges to be a limitation of both research perspectives and methodological approaches. For a considerable time, even social psychologists are skeptical about the scientific advantage of the attitude concept. Today many of them prefer a more complex model including social forms of knowledge such as group beliefs, subjective theories, social representations etc.

The transformative circle of knowledge

There is much indication, that an adequate understanding of the emergence, representation, and mechanisms of mathematical and other science-related belief systems will not be possible without considering the different forms of knowledge as social phenomena. In a 'transformative circle' (Flick 1995), those forms of knowledge interact on two levels –

analogues to the two realms of the *sacred* and the *profane* in ancient religion-based cultures: the first one being the collective, acknowledged, and official level of the mythical-religious, ideological, or scientific knowledge, and the second one being the private level of (pre- and post-scientific) everyday life with its forms of common sense, social representations, and everyday knowledge, mirroring the 'higher' forms of knowledge (cf. Fig. 1).
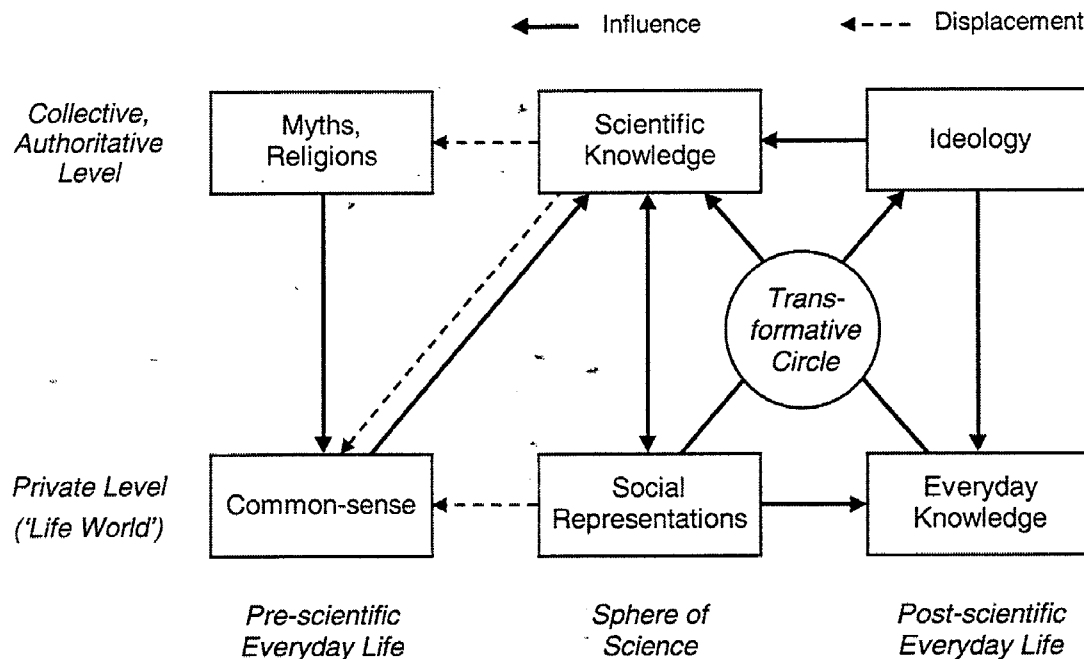

Figure 1: Transformative circle of knowledge forms (after Flick 1995)

A constructive process: sense making as reducing complexity

Mathematical belief research must not lag behind the current findings of scientific disciplines involved, such as sociology, ethnography, cognitive linguistics, and neuro science. Without continuous efforts of keeping a high level and extensive range of theoretical perspectives, our research would risk a considerable loss of scientific relevance. One of those perspectives is that of neuro science which has substantially changed our understanding of the brain.

To the *traditional view*, the brain is more or less a *mirror of reality*. Among the guiding assumptions of this traditional concept is that reality is sending a stream of sensory information to the brain and that the brain is answering by producing a sort of 'photograph of reality'. Consequently, there are objective (extra-personal) quality criteria for brainwork, namely adequacy, congruence, identity. This traditional view, however, in recent years turned out to be a too naïve one. Wolf Singer, Director at the Max Planck Institute for Brain Research, Frankfurt, summarizes some findings of modern neuro science which are in remarkable contrast to the traditional concept.

> Actually, there is much evidence that our cognitive system is behaving very selectively. ... It is interested only in very small partial aspects of reality. ... Its processing of sensory signals is based on prejudices in plenty. ... Our perceptions highly depend on the brain's guesswork about the world. ... The brain continuously produces concepts, adjusting them to the sensory information available. (Singer 2000)

Thus, sensory information does not play the fundamental role of being the source and aim of all knowledge, it is just used as a means of adjustment. Contrary to the widely held opinion,

only roughly 10% of the synaptic connections within the cerebrum (resp. the cerebral cortex) is processing sensory information, while the remaining part of around 90% is "devoting itself to an interior monologue" (Singer) creating concepts and 'making sense' (cf. Fig. 2). Actually, perception is not to be thought of in terms of 'getting information', but rather in terms of 'verifying hypotheses'. Man is mainly not a reality perceiving but a reality constructing species, not an observing, but rather a sense making animal.
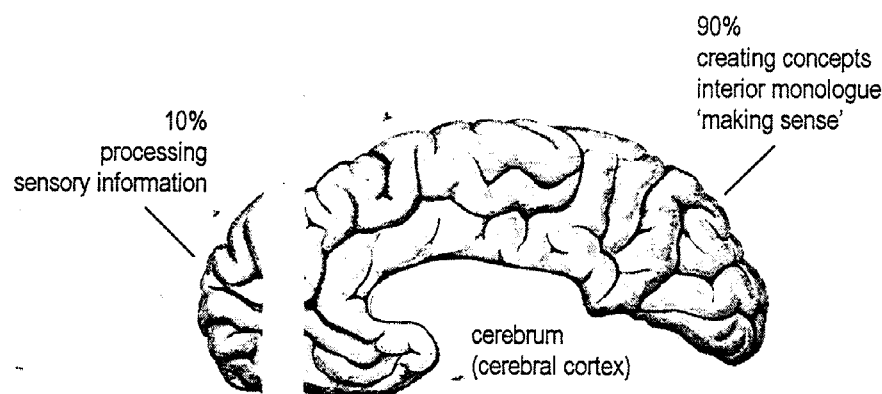
Figure 2: Man is a 'sense making animal'

To the modern view, the brain is constructing 'its own reality'. In the form of a continuous interior monologue, it is producing a stream of concepts, conjectures, and hypotheses about reality. Those hypotheses are continuously but selectively adjusted by means of sensory information. Consequently, there are only subjective (intra-personal) quality criteria for brainwork, such as viability and practicability. Fig. 3 depicts the two contrary models.
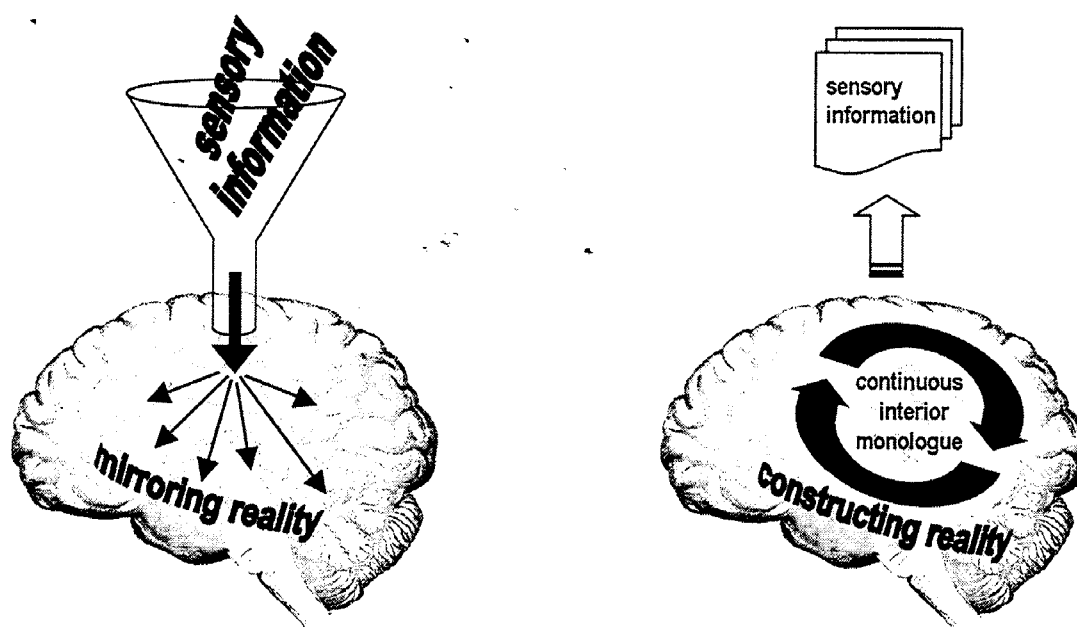
Figure 3: Traditional (naïve) versus modern view of the brain

The neuro scientific understanding of man as a sense making animal has a general biological interpretation, as we may recognize 'sense making' as the fundamental process of reducing the complexity of the world, due to evolution theory being a general adaptive behavior of most species. In a socio-cultural context that also applies to individuals, groups, and cultures.

Societies are built up by processes of commonly making sense, where sense is represented in the 'durable form' of culture. Culture is preserved by language (narratives, myths), actions (rituals), and symbols (totems).

Multiple realities: worldmaking as making of worlds

In early advanced cultures, sense making by reducing complexity had been relatively simple and led to the religion-based societies with simple, homogeneous, and uniform structures. The shift from ancient to modern societies, however, came along with what we may call a *cultural catastrophe*, attended by a loss of certainty, of simplicity, homogeneity, and unity. In modern advanced cultures 'the world' is disintegrated into several 'partial worlds', while 'the person' is disintegrated into different 'social roles'. In modern societies, thus, reducing complexity means reducing the complexity of partial worlds. Social reality in modern cultures actually takes the form of a conglomerate of multiple realities, a conceptual landscape of socio-cultural frames (worlds).

As it was the case with ancient cultures, even in modern cultures social worlds are not set up by physical objects, but rather by narrative elements, by themes and stories. It is not the machines that constitute the 'world of computers', but the stories about it. The 'world of politics' does not consist of politicians or events, but of stories about politicians and events. Those worlds are not made in factories, they are made in the media. They are relatively autonomous, occasionally linked by common themes – as the world of sports, of politics, and of medicine are linked by the common (but separately conceptualized) theme of doping. What often may appear as a contradiction within a person's thinking, is mostly nothing more than coming from different worlds. The only form of (social, cultural) consistency achievable in modern cultures is the reduced form of a 'partial consistency' within a specific partial world.

We may summarize some central *research assumptions*:
- An individual perceives reality in a complex, dynamic, active, subjective, and constructive process of deconstructing 'the world' and reconstructing it into 'multiple realities (worlds)'.
- Those multiple realities are preserved by the individual in the form of 'personal constructs' about the world(s).
- Personal constructs are not 'photographs of reality', but 'adaptive maps'.
- Personal constructs (and even the construction system itself) are subject to continuous adaptive processes which make them more stabile; they develop an inertia force against change.
- Personal constructs gain stronger stability largely not as a result of verification processes (truth), but as a result of their viability (practical benefit in a certain context) and by socio-cultural reinforcement processes.
- The ordinary conceptual system of human beings, in terms of which we think and act, is fundamentally metaphorical. There are personal as well as interpersonal, socio-cultural metaphors. There are metaphorized actions (rituals) and metaphorized objects (totems).

A multi-perspective framework for belief research

Based on a series of empirical studies on teacher conceptions in the context of computer science, mathematics, and computer culture (cf. Berger 1998, 1999), the author worked out a theoretical model for the scientific construct of *belief system.*
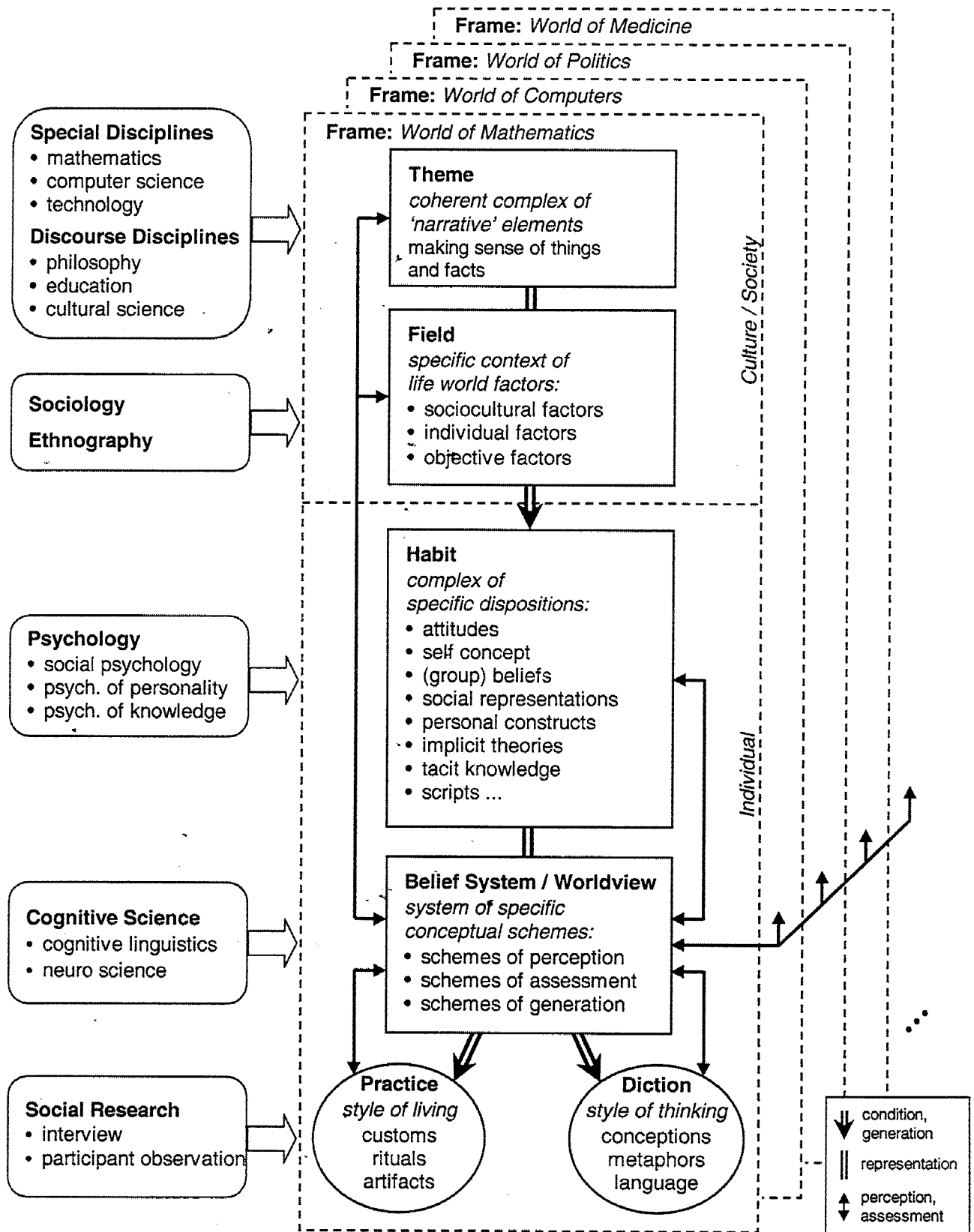
Figure 4: Multiple realities: social reality as a complex of socio-cultural frames (worlds)

The model (cf. Fig. 4) extends the classical psychological perspective by aspects of current sociology, ethnography, and cognitive science, especially of neuro science and cognitive linguistics (Berger 2000). In short terms, the model performs a 'focusing climb-down' with the stages *society/culture* (sociology/ethnography), *individual* (psychology), and *brain/mind* (neuro science/cognitive linguistics). It conceptualizes the multiple realities of individuals as specific socio-cultural frames ('worlds': world of mathematics, world of computers, world of medicine, world of politics etc.).

Belief systems, then, can be understood as habitualized conceptualizations of those worlds ('world views/belief systems') which are cognitive representations of frame-specific personal dispositions ('habit': attitudes, self concept, implicit theories, tacit knowledge etc.) forming specific schemes of perception, assessment, and generation (cf. Bourdieu 1998, 1999). Those world views emerge from a specific interpersonal context, i.e. a coherent complex of narrative elements ('theme'), which induces a specific context of life world factors ('field': socio-cultural, individual, and objective factors). World views result in specific styles of behavior ('practice': customs, rituals, artifacts) and thinking ('diction': conceptions, metaphors, language). World views, in a fundamental way, are metaphorically organized. Thus, linguistic interview analysis – within a qualitative methodological framework – provides an appropriate approach to the study of belief systems.

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The mathematical world view of pre-service teachers: first results

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Abstract

This paper presents first results of an investigation on the mathematical world view of pre-service teachers. First there are some information about the structure of the investigation and particularly the questionnaire. Here, especially the results of the quantitative methods are given. The factor analysis of some sections of the questionnaire give a first impression of possible dimensions and structures in the mathematical world view of pre-service teachers.

Introduction

The pre-service education is in some aspects surely a crucial part in the development of any teacher. It represents the change from learner to teacher, and is the first serious confrontation with school as workplace. in this initial period the role of mathematics changes from pure subject related knowledge to the ability of employing it for the purpose of teaching. In addition, the pre-service teacher establishes his own position between teachers and students. This leads to the assumption that the mathematical world view would not left to be unchanged during this time of change.

The structure of the investigation

The investigation focuses on the mathematical world view of pre-service teachers and possible changes in it. I deal with a mix of quantitative and qualitative methods by using questionnaire and interview. However, the main emphasis will be on the questionnaire and therefore on the quantitative methods.

The investigation is divided into four parts. First the preparation including the design of the questionnaire and preliminary investigation. The next two parts consist of two surveys followed by a final analysis. Next I want to present some results of the first survey.

The questionnaire

The questionnaire is divided into 7 sections. First there is a section with statistical questions about age, gender, second subject of teaching and so on. The sections 2 to 5 contain items with a five-graded Likert-scale; there are statements about teacher education, mathematics and teaching mathematics. In section 6 the pre-service teachers are asked about factors which influence their mathematics teaching. The probands are asked to evaluate shall "weigh" 11 given factors from 1 to 10, where 1 means low influence and 10 very high influence. Besides they have the chance to name own factors and evaluate them as well. At the end, in section 7,

they have some space to write down some remarks about "teacher education", "mathematics" and general impressions. The following table shows the structure of the questionnaire.

Tab.1: Structure of the questionnaire

Section 1	Statistical questions
Section 2	Education of mathematics teachers
Section 3	My view on mathematics as a school subject
Section 4	Mathematics and reality
Section 5	Teaching mathematics
Section 6	Influencing factors
Section 7	Remarks

The items of the sections 3 to 5 are based on a questionnaire of GRIGUTSCH/ TÖRNER which was given to university students and to teachers.

The section on teacher's education was developed with the support of two groups of pre-service teachers and by using some interviews with other prospective teachers. The same applies to section 6 "Influence factors".   .

First results

From the first survey started in 1999, I received about 65% of the questionnaires with valid data. Therefore, I have 130 probes at my disposal. Here are some results of the first data analysis.[1]

Factor analysis of section 2

Section 2 contains 32 statements about the education of mathematics instructors and the situation of pre-service teachers. The factor analysis provides 10 factors with an eigen value larger than one.   .
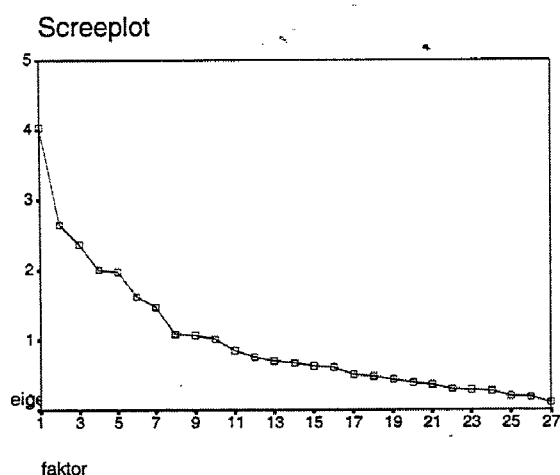

Fig.1: Screeplot of the factor analysis of section 2

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[1] I work with *SPSS 8.0 for Windows*.

I decided to work with the first seven factors, which explain about 60 % of the variance. The screeplot shows a clear break between factor 7 and factor 8. The seven factors have an eigen value larger than 1,4.

The following table contain some data about the seven-factor-solution. The number of items with the highest loading on the single factor, the number N of cases, Cronbachs α and the mean value of the factor on a scale from 0 to 50.

Tab. 2: Seven-factor-solution for section 2

	Factor	Number of items	N	Cronbachs α	Mean value
F_1	Problems with the situation	6	128	.6828	18,20
F_2	Mathematical self-assurance- university study	5	129	.7145	26,24
F_3	Hard work	3	129	.7969	32,69
F_4	Importance of the education in the "Seminar"	4	126	.6296	26,66
F_5	Negative judgement of teacher education-	4	117	.5981	37,21
F_6	Impact of the own time as a pupil	3	130	.7160	22,34
F_7	Impact of the mentor	2	129	.3395	35,22

As example, I present the items with the highest loading in factor 2, I call *Mathematical self-assurance and university study*.

Tab. 3: Factor 2 of the seven-factor-solution

Item	Loading
If I have problems with the subject, I use my material from university.	.429
The studies at university prepared me sufficiently for my job at school.	.440
While teaching mathematics, I discovered some problems with the material.	-.791
I have no problems with the material for my mathematics lessons.	.799
The university study provided a good foundation in mathematics.	.745

Apart from the single factors, the correlation between the factors is interesting. The following table shows the correlation between the factors of the seven-factor-solution.

Tab. 4: Correlation between the factors of the seven-factor-solution

	F_1	F_2	F_3	F_4	F_5	F_6	F_7
F_1							
F_2	-,223*						
F_3	,241**	-,124					
F_4	-,053	,125	,080				
F_5	,195*	-,465**	,209*	-,057			
F_6	,014	-,090	,028	,051	-,038		
F_7	-,116	,003	,180*	,289*	-,026	,094	

* the correlation is significant on the level 0,05
** the correlation is significant on the level 0,01

The table shows some significant correlation between the factors. This becomes more obvious in a figure.
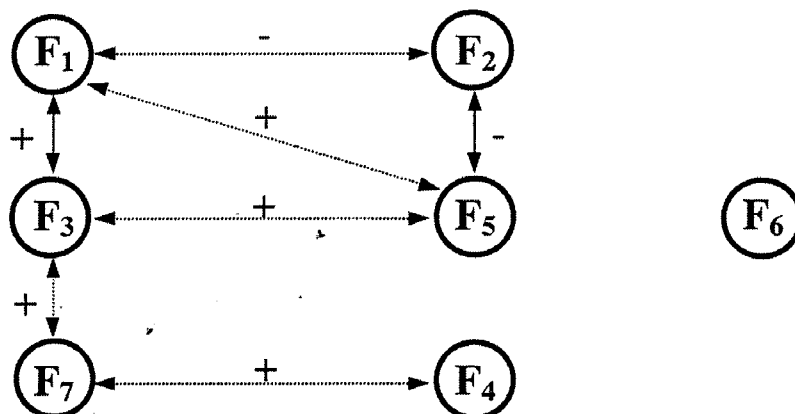
Fig.2: Correlation between the factors of the seven-factor-solution

For example there is a strong positive correlation between factor 1 *Problems with the situation* and factor 3 *Hard work*. And there is also a positive correlation between these two factors and factor 5 *Negative judgement of teacher education*. Whereas the factor 6 *Impact of the own time as a pupil* is absolutely isolated from the other factors.

Factor analysis of section 3 and 4

The statements of sections three and four were taken from earlier investigations on mathematical world views by GRIGUTSCH & TÖRNER. Therefore, these sections were analysed in conjunction. Here I have the possibility to compare my results directly with other data.

The factor analysis shows nine factors with an eigen value larger than one. But there are four factors with an eigen value larger than two, explaining 44% of the variance. The screeplot shows also an obvious turn between factor 4 and factor 5. Therefore, I decided to work with the four-factor-solution.
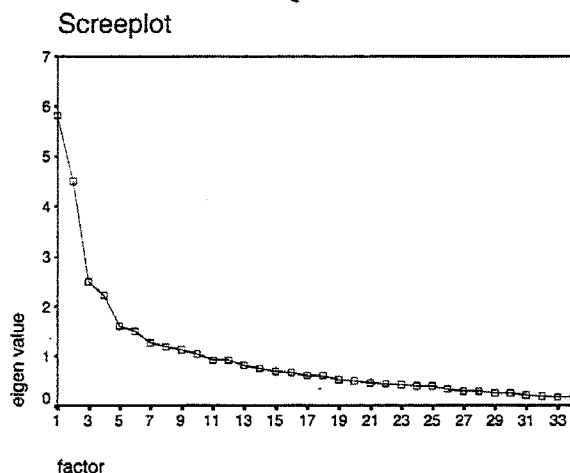
Fig.3: Screeplot of the factor analysis of section 3 and 4

Tab. 5: Four-factor-solution for section 3 and 4

	Factor	Number of items	N	Cronbachs α	Mean value
F_1	Formalism	9	127	.8430	28,23
F_2	Process	9	121	.7879	34,69
F_3	Application	8	127	.7789	22,69
F_4	Scheme	8	124	.7737	37,66

Dealing with these four factors you can also find some significant correlation.

Tab. 6: Correlation

	F_1	F_2	F_3	F_4
F_1				
F_2	,152			
F_3	,385**	,180*		
F_4	,190*	,347**	,087	

* the correlation is significant on the level 0,05
** the correlation is significant on the level 0,01

Here also a figure makes the correlation more obvious:
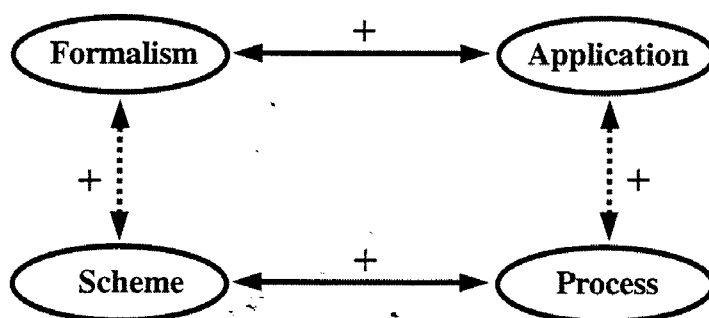
Fig.4: Correlation between the factors of the four-factor-solution

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Aesthetics - Complexity - Pragmatic Information

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Abstract

It is often stated that the aesthetic component of mathematics has a great importance in scientific research as a heuristic leitmotiv for putting forward a theory. The appreciation of the beauty of mathematics is one of the wellsprings of this subject, not only in research but also in school education. This has implications for the teaching of mathematics. However the beauty making elements are not very well analysed. Nevertheless it seems that there is some correlation between aesthetics and complexity.

When teaching mathematics, the degree of the complexity of the presented subject has to consider, above all, the effectiveness of the transported messages upon the students, the pragmatic information.

The relationship between complexity and aesthetics on the one hand and complexity and pragmatic information on the other hand in the realm of mathematics education is thus a subject of interest.

The paper deals with the dimensions aesthetics, complexity and pragmatic information, presenting some statements and a number of questions for further research.

1. The role of aesthetics for perception and education

In literature it is multiple reported about aesthetics as guideline when putting forward a scientific theory or selecting ideas for mathematical proofs.

The first who introduced simplicity and *mathematical beauty* as criteria for a physical theory was COPERNICUS [1, p. 30]. Since then these criteria have continued to play an extremely important role in developing scientific theories [1, p. 30; 2; 3]. DIRAC, for example, tells about SCHRÖDINGER and himself [6]:

> It was a sort of act of faith with us that any questions which describe fundamental laws of Nature must have great mathematical beauty in them. It was a very profitable religion to hold and can be considered as the basis of much of our success.

VAN DER WAERDEN [16] reports about POINCARÉ and HADAMARD [see also 12]:

> POINCARÉ und HADAMARD ... betonen besonders die Rolle des Schönheitsempfindens bei der Auswahl fruchtbarer Kombinationen.

> POINCARÉ stellt die Frage so: Wie soll das Unbewußte aus allen den vielen möglichen Kombinationen die richtigen, das heißt fruchtbaren, herausfinden? Seine Antwort lautet: durch den Schönheitssinn. Bevorzugt werden die Kombinationen, die uns gefallen.

A similar statement is given by HERMANN WEYL [8, p. 209]:

> My work has always tried to unit the true with the beautiful and when I had to choose one or the other I usually chose the beautiful.

Thus theories, that have been described as extremely beautiful, as for example the general theory of relativity, have been compared to a work of art [3]; PAUL FEYERABEND [9] even considers science as being a certain form of art.

Mathematics and mathematical thought are obviously directed towards beauty as one profound characteristic. (PAPERT and POINCARÉ [7, p. 2; 12] even believe that *aesthetics play the most central role* in the process of mathematical thinking.) The appreciation of mathematical beauty by students should thus be an integral component of mathematical education [7]. But DREYFUS & EISENBERG [7] remarked 1986, that developing an aesthetic appreciation for mathematics was not a major goal of school curricula [NCTM, 1980], and he expressed that "this is a tremendous mistake". In the new curricular guidelines of Northrhine Westfalia [13, p. 38], Germany, however the development of students' appreciation of mathematical beauty is explicitly demanded in the context of the fostering of long-life positive mathematical views. The importance of this demand may be stressed by the following statement given by DAVIS & HERSH [5, p. 169]:

> Blindness to the aesthetic element in mathematics is widespread and can account for a feeling that mathematics is dry as dust, as exciting as a telephone book, as remote as the laws of infangthief of fifteenth century Scotland. Contrariwise, appreciation of this element makes the subject live in a wonderful manner and burn as no other creation of the human mind seems to do.

2. Criteria of aesthetics

If we want to increase the students' appreciation of the beauty of mathematics, we have to ask what it is, that makes the beauty of the subject. What does it mean for example that a theorem, a proof, a problem, a solution of a problem (the process leading up to a solution, as well as the finished solution), a geometric figure, a geometric construction is beautiful?

Although assessments about beauty are very personal, there is a far-reaching agreement among scholars as to what arguments are beautiful [7]. Thus it makes a sense to search for factors contributing to aesthetic appeal. But HOFSTADTER [11, p. 555] for example believes, that it is impossible to define the aesthetics of a mathematical argument or structure in an inclusive or exclusive way:

> There exists no set of rules which delineates what it is that makes a peace beautiful, nor could there ever exist such a set of rules.

However we can find in literature several indications of criteria determining the aesthetic rating.

The Pythagorean took the view that beauty grows out of the mathematical *structure*, the mathematical *relations* that put together initially independent parts in the right way to a unit [10]. CHANDRASEKHAR [2] names as aesthetic criteria for theories their display of "*a proper conformity of the parts to one another and to the whole*" while still showing "some *strangeness* in their proportion". HERMANN WEYL [19] states that beauty is closely connected with *symmetry* and IAN STEWART [15, p. 91] points out that *imperfect symmetry* is often even more beautiful than exact mathematical symmetry, as our mind loves surprise. DAVIS & HERSH [5, p. 172] take the view that:

> A sense of strong personal aesthetic delight derives from the phenomenon that can be termed *order out of chaos*.

And they add:

> To some extent the whole object of mathematics is to create order where previously chaos seemed to reign, to extract structure and invariance from the midst of disarray and turmoil.

ALLEN WHITCOMBE [19] lists as aesthetic elements a number of vague concepts as: *structure, form, relations, visualisation, economy, simplicity, elegance, order*. DREYFUS & EISENBERG [7] state, according to a study they carried out, that *simplicity, conciseness* and *clarity* of an argument are the principle factors that contribute to the aesthetic value of mathematical thought. Further relevant aspects they name are: *structure, power, cleverness* and *surprise*. CUOCO, GOLDENBERG & MARK [4] take the view that:

> The beauty of mathematics lies largely in the *interrelatedness of its ideas*. ... If students can make these connections, will they also see beauty in mathematics? We think so ...

EBELING, FREUND & SCHWEITZER [8, p. 230] point out, that the beautiful is as a rule connected with *complexity*; complexity is necessary, even though not sufficient, for aesthetics:

> Ästhetische Kriterien sollten den Begriff Komplexität einschließen. Auch das Schöne ist in aller Regel komplex ... Allerdings ist das Vorhandensein komplexer Relationen in der Regel nur eine notwendige Bedingung; in keinem Falle ist Komplexität hinreichend für eine Bestimmung des Ästhetischen.

Complexity and simplicity are both named as principal factors for aesthetics: how does that fit together? If simplicity is named, it is mainly the simplicity of a solution of a complex problem, the simplicity of a proof to a theorem describing complex relationships, or the simplicity of representations of complex structures. Thus, also in this consideration *complexity* is involved.

3. Complex structures

A complex structure consists of many interrelated elements and is hierarchical ordered. Complex structures are situated between the extremes of maximum ordered (periodical) and maximum unordered (uncorrelated) structures; between perfect banality and chaos. As visualisation EBELING, FREUND & SCHWEITZER [8, p. 22] give the following example:

periodic structure   uncorrelated structure   complex structure
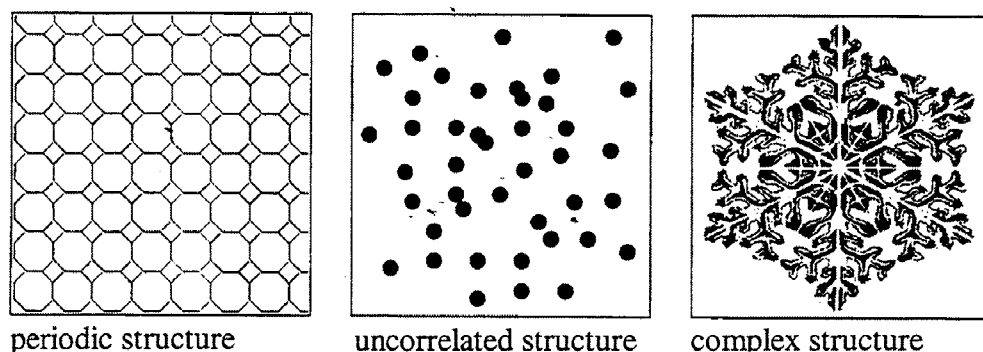
Figure 1: Geometric example for a periodic structure, an uncorrelated structure and a complex structure

The degree of the complexity of a structure is reflected by the number of equal respective unequal elements, the number of equal respective unequal relations and the number of hierarchies. In respect to mathematics for example simple geometric figures are situated at the beginning of the complexity scale, fractals are complex structures; problem solutions that only use simple rules or algorithms have a low complexity rating whereas open ended problems are characterised by high complexity.

Thus, if we want to promote the students' aesthetic feelings for mathematics it might be profitable to treat mathematical structures (figures, problems, geometric constructions, ...) that are characterised by a great number of elements connected by several relations and by a great number of hierarchies.

But complexity alone is, as already told above, not sufficient for aesthetics. Complex structured mathematical objects and procedures (theorems, proofs, problems, solutions of problems (the way and the result), geometric figures, geometric constructions, the context to a theme including all its relations/linkages, ...) should be represented in a simple, elegant, concise and clear way. Here is involved that aesthetic feeling is a binary relation between an object (for example a mathematical object) and a subject (for example a student), whereas complexity is a property only of the object. The complexity of the object must be in some way understood by the subject so that aesthetic feelings can arouse: decisive is the *information* that we get about the complexity of an object.

As learning of new contents begins with the reception of information, it is obvious that information, especially *pragmatic information*, plays a central role in the teaching and learning process. As we will see, the amount of pragmatic information is strongly connected with the concept of complexity, so that the complexity of a mathematical object or procedure might bring about both, aesthetic feelings and a good learning pre-condition.

4. Pragmatic information

Pragmatic information is defined as the effectiveness of a message upon the recipient. In mathematics education the recipient is the student and the messages that have to be transported are mathematical contents.

The effectiveness of an information is determined by the relation of confirmation and novelty, of the well-known and the new. The following figure proposed by ERNST VON WEIZSÄCKER [14; 17, p. 60; 18] may serve as a thinking and memory aid:
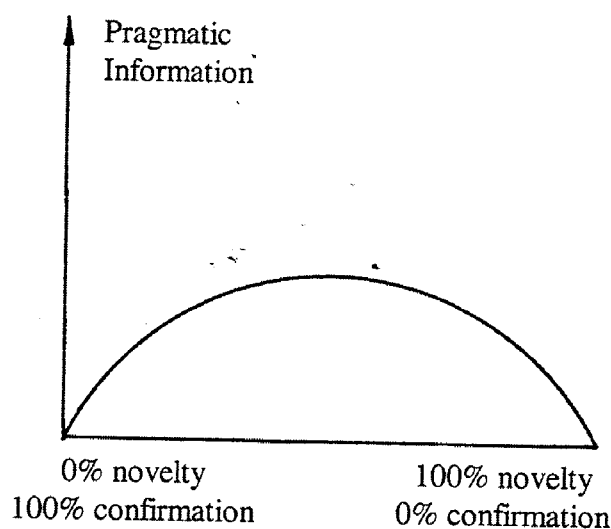
Figure 2: Schematic plot of pragmatic information in dependence on the degree of novelty and confirmation

Accordingly pragmatic information is minimal (equal zero) when the information is already well-known (100% confirmation, no novelty) or when the information doesn't refer at all to something known, that is when the information is completely new and thus not understandable (no confirmation, 100% novelty). Maximum pragmatic information, that is maximum communication efficiency, is provided by an optimal relation of the well-known and the new, of habit and surprise.

A more detailed representation of the existing relations is given by the following figure by ERNST VON & CHRISTINE VON WEIZSÄCKER [17, p. 202]:
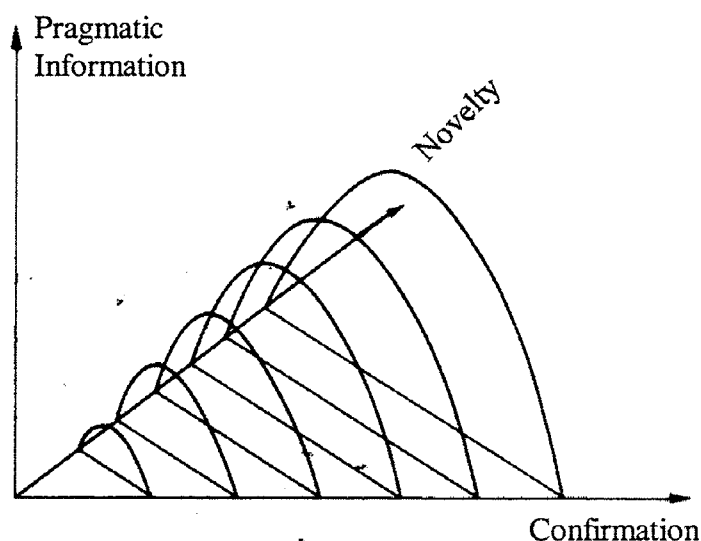
Figure 3: Pragmatic information in dependence on the degree of novelty and confirmation.

Objects of higher complexity may thus provide more pragmatic information than objects of lower complexity.

5. Final remarks

In respect to mathematics education it would be advantageous to consider the aesthetic dimension of mathematics more than it has been done up to now. Mathematics should be presented to and experienced by the students in a way that yields the appreciation for the beauty of the subject. Therefore the beauty making elements have to be better analysed.

Likewise, in mathematics education, the communication efficiency expressed by the pragmatic information should reach a maximum. The optimal relations between that what is well-known and that what is new for the students must be found.

As both, aesthetics and pragmatic information, are closely connected to complexity it is a subject of interest to find, dependent on the knowledge basis of the learning individuals, complex structured mathematical objects and procedures that may provide a maximum of pragmatic information combined with aesthetic feelings for the subject of mathematics.

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Beliefs of Teacher-Students and Teachers about Problem Orientation in Mathematics Teaching

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Abstract

Problem orientation was one point which came out in earlier questionaires of mine as a point for more profound reseaech. Thus in autumn 1999 in Bielefeld a questioning with teacher-students as well as teachers who are in practicewas done. The main trend which came out was: All questioned persons have a strict wish about more problem orientation in mathematic lessons while in the view of the students the reality differs from this wish.

1. Introduction (Outline of the history of problem orientation at school)

The importance of learning by doing was already mentioned by SOCRATES. In the modern times especially ROSSEAU and FRÖBEL first put emphasis on this method of learning. As a systematic principle for teaching it was used first about hundred years ago from reform padagogues like DEWEY, MASON, MACMILLAN and MOTESSORI. For Germany we should mention GAUDIG who promoted the support of activities of askings in school in the 30th of the 20th century. Also in the holistic didactic conception of J. WITTMANN which was published first in 1929 the method of learning by doing was to be found. A problem orientated approach with learning by discovery also was supported through the Gestalt psychology (see e.g. KÖHLER, DUNCKER, WERTHEIMER). The first study about learning by discovery came from McCONNELL in 1934.

An intensive and broad dicussion with many publications about learning by discovery, its pros and cons and empirical research then took place since the end of the 40th until the middle of the 70th. The most famous names in this discussion are BRUNER and AUSUBEL. In Germany for the first time we should mention COPEI and WAGENSCHEIN as well as WITTENBERG for the didactics of mathematics. In the end of the 60th and the 70th in Germany there can be named several padagogues and didacticans who discussed the method of learning by discovery or learning by doing and the conception of orientation on action.

Since the middle of the 80th stimulated by the cognitive psychologists and their constructive theory of learning a new discussion about problem orientation and open approach started first in the USA and a little bit later also in Europe and Asia. For didactics of mathematics the initiation came from POLYA in the 60th and then from SCHOENFELD with his book "Mathematical Problem Solving" 1985.

One aspect Schoenfeld focussed on was the influence of beliefs for the success of teaching mathematical problem solving. Already in 1983 LERMAN showed the importance of the factor "beliefs" for the process of learning. In this context my present research on beliefs - I will show one little part of it now - should be seen.

2. Basic conditions of the questioning

Together with one of my students I created a questioning focussiong on problem orientation in mathematics teaching in general and geometry teaching in special. For each part twelve items were put down. At the end there was left a space for remarks. For all items it was used the scale   −2 (total disagreement), −1 (disagreement), 0 (both...and/ don't know), +1 (agreement), +2 (total agreement) and k (no answer). For all items this scale was used twice: Once in respect to mathematics education they know from school ("IS-state") and secondly in respekt to their wishes ("SHALL-state"). The items have been the following ones:

For my belief it is part of mathematics teaching that (1) you have to espress everything very exactly, (2) you also work in groups on problems, (3) the right solution will be found as quickly as possible, (4) you have to reflect on problems/tasks and find ideas and solutions, (5) pupils should develop·and present own problems sometimes, (6) pupils should work on problems alone more frequent, (7) you may guess, puzzle, suppose sometimes, (8) the way of solution is secondarily, (9) you must learn rules, expressions and procedures by heart, (10) there are normally more ways to solve problems/tasks, (11) there are given schemes to get the result, (12) you always get told exactly what to do.

For my belief it is part of geometry teaching that (13) you can understand reality good with geometry, (14) calculation is to do only a little, (15) aesthetic and architectural aspects are important, (16) there are often different levels of solution (vivid, constructive, formal, etc.), (17) clarifying of concepts is important, (18) you often draw and make things with your hand, (19) you have to proof a lot, (20) bringing out and applying formulas for length, area and volume is in the foreground, (21) you often have to try or attempt and solve problems, (22) you often have to deal with abstract constructs of ideas, (23) applications appear only in the lead-in-phase for better motivation, (24) the main focus is geometry of mappings.

The questioning was done in autumn 1999 in Bielefeld. The questionaire was given to teacher-students who want to become mathematics teacher of junior secondary school as well as to mathematics-teachers of all four types of junior secondary school (i. e. Gymnasium, Realschule, Hauptschule, Gesamtschule).

The **number of students** who gave back a filled up questionaire was **67** with 20 from them who were first year students (freshmen). The **number of teachers** who gave back a filled up questionaire was **25**.

3. Beliefs of teacher-students

In the following we will have a lock at the results of some selected items which refer directly to problem orientation. We start with the results of the students and the items 7, 4 and 5.

Already with these three items we can see a **general trend** which came out nearly by all answers of the students refering to problem orientation: The point of gravity lies between the two last columns or even only in the last column. This means that the students have serious wishes in respect to problem orientated mathematics teaching (SHALL-state more than +1). We also can see another trend: Differences between the IS-state and the SHALL-state whereas in item 5 the IS-state even is clear negativ. To develop own problems in mathematics therefore must be an aspect of mathematics teaching which has to be installed or intensified. Item 4 differs a little bit from this trend. Probably the sense of this item has to be cleared up because the term "reflect on" (in the German text: "nachdenken über etwas") has at least two meanings, in a very simple way on one hand and on the meta-level on the other hand.

Item 7 : you may guess, puzzle, suppose sometimes

Is / Shall	-2	-1	0	+1	+2	*no. a.*	Sum Is
-2	1	-	-	2	2	-	5
-1	-	1	1	4	4	1	11
0	-	-	3	13	7	-	23
+1	-	1	1	8	9	1	20
+2	-	-	-	1	5	-	6
no. a.	-	-	-	2	-	-	2
Sum Shall	1	2	5	28	29	2	67

Mean: IS-state +0.17   SHALL-state +1.26   Difference (Shall-Is) +1.09
           (Freshmen +0.33)   (Freshmen +1.00)

Item 4 : you have to reflect on problems (tasks) and find ideas/solutions

Is / Shall	-2	-1	0	+1	+2	*no. a.*	Sum Is
-2	-	-	-	-	-	-	0
-1	-	-	-	4	9	-	13
0	-	-	-	3	9	-	12
+1	1	-	-	6	22	1	30
+2	-	-	-	1	9	-	10
no. a.	-	-	-	-	2	-	2
Sum Shall	1	0	0	14	51	1	67

Mean: IS-state +0.57   SHALL-state +1.73   Difference (Shall-Is) +1.16
           (Freshmen +0.50)   (Freshmen +1.55)

Item 5 : pupils should develop and present own problems sometimes

Is / Shall	-2	-1	0	+1	+2	*no. a.*	Sum Is
-2	-	-	3	21	7	1	32
-1	-	-	1	6	8	1	16
0	-	-	-	5	1	-	6
+1	-	-	-	1	7	-	8
+2	1	-	-	1	-	-	2
no. a.	-	-	-	-	2	1	3
Sum Shall	1	0	4	30	29	3	67

Mean: IS-state −1.06   SHALL-state +1.34   Difference (Shall-Is) +2.40
           (Freshmen −1.22)   (Freshmen +1.00)

If we want to distingues between the first-year-students (freshmen) and the elder students we can say: From all items (with one exception also the other ones) it comes out that the means of the freshmen are lower than those of all students for Is-state as well as Shall-state. I don't kwow how to interprete this. It might be that freshmen tend to see everything more bad or that for elder students the past has become transfigured already a little bit.

Let us now have a look on two other items which refer to problem orientation in geometry teaching. The picture is very similar to that of the others.

Item 16 : there are often different levels of solution in geometry (vivid, constructive, formal, etc.)

Shall \ Is	-2	-1	0	+1	+2	*no. a.*	Sum Is
-2	-	-	-	2	1	-	3
-1	1	-	1	6	7	-	15
0	-	-	3	5	7	*1*	16
+1	-	-	-	13	8	-	21
+2	-	-	-	1	6	-	7
no. a.	-	-	-	2	*1*	2	5
Sum Shall	1	0	4	29	30	*3*	67

Mean: IS-state +0.32   SHALL-state +1.36   Difference (Shall-Is) +1.04
       (Freshmen +0.28)   (Freshmen +0.75)

Item 21 : you often have to try or attempt and solve problems in geometry

Shall \ Is	-2	-1	0	+1	+2	*no. a.*	Sum Is
-2	-	-	-	-	3	-	3
-1	-	-	1	10	3	-	14
0	-	-	6	7	5	-	18
+1	-	-	2	12	7	*1*	22
+2	-	1	-	3	4	-	8
no. a.	-	-	-	2	-	-	2
Sum Shall	0	1	9	34	22	*1*	67

Mean: IS-state +0.28   SHALL-state +1.17   Difference (Shall-Is) +0.89
       (Freshmen +0.28)   (Freshmen +1.15)

4. Beliefs of teachers of mathematics in secondary schools

We will complete the picture about beliefs of problem orientation within mathematics teaching by looking at the results of the teachers refering to the same items as before.

Item 7 : you may guess, puzzle, suppose sometimes

Is / Shall	-2	-1	0	+1	+2	*no. a.*	Sum Is
-2	-	-	-	-	-	-	0
-1	-	1	-	1	2	-	4
0	-	-	1	1	2	-	4
+1	-	-	-	8	3	-	11
+2	-	-	-	-	6	-	6
no. a.	-	-	-	-	-	-	0
Sum Shall	0	1	1	10	13	*0*	25

Mean: IS-state +0.76   SHALL-state +1.40   Difference (Shall-Is) +0.64

Item 4 : you have to reflect on problems (tasks) and find ideas/solutions

Is / Shall	-2	-1	0	+1	+2	no. a.	Sum Is
-2	-	-	-	-	2	-	2
-1	-	-	-	2	-	-	2
0	-	-	-	1	1	-	2
+1	-	-	1	5	12	-	18
+2	-	-	-	-	1	-	1
no. a.	-	-	-	-	-	-	0
Sum Shall	0	0	1	8	16	0	25

Mean: IS-state +0.56   SHALL-state +1.60   Difference (Shall-Is) +1.04

Item 5 : pupils should develop and present own problems sometimes

Is / Shall	-2	-1	0	+1	+2	no. a.	Sum Is
-2	-	-	2	3	1	-	6
-1	-	-	1	7	2	-	10
0	-	-	-	2	-	-	2
+1	-	-	-	3	3	-	6
+2	-	-	-	-	1	-	1
no. a.	-	-	-	-	-	-	0
Sum Shall	0	0	3	15	7	0	25

Mean: IS-state − 0.56   SHALL-state +1.16   Difference (Shall-Is) +0.60

Item 16 : there are often different levels of solution in geometry
(vivid, constructive, formal, etc.)

Is / Shall	-2	-1	0	+1	+2	no. a.	Sum Is
-2	-	-	-	-	-	-	0
-1	-	-	1	-	2	-	3
0	-	-	3	5	-	-	8
+1	-	-	-	5	5	-	10
+2	-	-	-	-	4	-	4
no. a.	-	-	-	-	-	-	0
Sum Shall	0	0	4	10	11	0	25

Mean: IS-state +0.60   SHALL-state +1.28   Difference (Shall-Is) +0.68

Item 21 : you often have to try or attempt and solve problems in geometry

Is / Shall	-2	-1	0	+1	+2	*no. a.*	Sum Is
-2	-	-	-	-	-	-	0
-1	-	2	1	4	-	-	7
0	-	-	2	3	1	-	6
+1	-	-	1	4	7	-	12
+2	-	-	-	-	-	-	0
no. a.	-	-	-	-	-	-	0
Sum Shall	0	2	4	11	8	0	25

Mean: IS-state +0.20   SHALL-state +1.00   Difference (Shall-Is) +0.80

All items except number 5 have their centre of gravity in the edge right down. This means that the teachers have a positive belief about problem orientation in mathematics teaching in respect to the SHALL-state (like the students) as well as in the IS-state. Their view of the reality therefore is different to those of the students. Until now I cannot say if the teachers' focus on problem orientation (with other topics we find also other pictures) is different to that of the students or if the teachers don't see their own teaching reality with critical eyes. This aspect must be examined more in future.

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Interactive Work Sheets for Teaching Geometry

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Abstract

This research project makes a contribution to the subject of computer-assisted learning.
It is based on a Class 7 teaching unit composed of standard topics from the teaching of geometry.
This article presents some initial experience with the use of work sheets produced by Cinderella Dynamic-Geometry Software in class.

1. Introduction

Previous curricula will undoubtedly change due to the increasing use of Dynamic-Geometry Software (DGS) in teaching. The question of the influence of the computer in the success of teaching mathematics needs to be examined closely. The conditions under which children learn have changed, but tried and tested teaching concepts are still lacking. Teaching units on the application of computers in lessons must be set up. Empirical studies of the teaching and learning processes in connection with DGS are few and far between.

2. Range and Goals of the Project

The research project being considered here is intended to make a contribution to the realm of computer-aided learning. The goal is to research the changed self-controlled learning process. Within this framework, we hope to retain the pupil's normal environment as far as possible whilst taking advantage of the computer as a medium.

Previous results from research into learning environments using DGS in the classroom are rare.

Those available, cf. Hölzl [1999, p. 290 ff, 301], lead to the following conclusions:
- New technical problems arise due to the new medium and the use of DGS
- Technical problems arising during work on exercises influence pupils' learning processes
- The large number of available tools alone puts high demands on pupils' concentration
- Possible result: lower learning levels due to a reduction in concentration
- The use of dynamic drawing areas is seen to be of particular advantage in cases where advanced experience in learning mathematics already exists.

These conclusions tend to indicate new problems rather than new possibilities for mathematics teaching. From my perspective, however, the changes in the learning environment and in the design of the research project will lead to new and different results.

An assertion made by Rainer Grießhammer points towards an important factor in the learning process when using a computer that has so far not been heeded: the computer does not replace paper!

"New technologies usually complement older ones: the book does not replace the spoken word, nor the telephone the letter, nor the television the radio, nor the computer the paper."

Apart from that, according to Knöß [1989, p.63 ff] and transferred from the teaching of computer science, the following fundamental activities belong to the teaching of Mathematics: modularising, structuring, presenting, realising, evaluating. These can be a guideline for determining the learning environment. In particular, the presentation and documentation of the results are for the pupil an important aid in his learning process.

In the framework of our project, this perception guided us to a different learning environment and research plan. Comparing the marginal factors, for example, in the work of Hölzl (1999, S.292), with our research results, the following picture appears:

Factors in the Hölzl study:	Marginal factors in the Class 7 research project:
• Selected test persons with supplementary training facilitate the teaching • Special teaching topics • A long introductory phase in the use of the tools	• Limitation of changes to a familiar learning environment. • Retention of the teacher • Standard topic • Text of the problem taken from school books • Different types of documentation

Fig. 1: Comparison of the marginal factors

Since changes in learning conditions can influence success, any changes in external factors of the learning process are minimised. The specific teacher is an important influencing factor and is retained as a constituent of the pupil's trusted learning environment. Standard problems from issued school textbooks represent the basis for the interactive work sheets we use.

Within the framework of a two-week teaching unit in geometry for Class 7, interactive work sheets are being planned to cover the obligatory topics 'Perpendicular Bisectors and Angle Bisectors' from the NRW curriculum (1993, p. 49). Standard problems from school textbooks will be transposed to interactive work sheets with the aid of the geometry software Cinderella[1] The basic tools to be used are: a pair of compasses, a ruler and moveable elements.

3. Work Sheets with Cinderella

The Geometry-Software Cinderella[2] makes it possible to produce so-called 'interactive' work sheets. They are produce by the teacher using DGS and can be worked on in a browser by the pupil. The teacher determines which tools are to be used and can, therefore, choose those most suitable for his goal.

The other tools are not visible to the pupil while he is working on the given work sheet. The interactive work sheets include accessible help files and an internal check on the offered solution is available through an automatic proof window (Cf. Kortenkamp 1999), even if the pupil should have changed the initial form of the problem.

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[2] Cinderella was conceived and written by Jürgen Richter-Gebert and Ulrich Kortenkamp. It was based on a non-commercial pilot version by Richter-Gebert and Henry Crapo. The programme is marketed by Springer-Publishing and as a school version by Klett.

Independent of his chosen solution or of changes to the initial problem, the pupil will receive a message of confirmation on finding a correct answer. The nature of this message can be determined arbitrarily by the teacher.
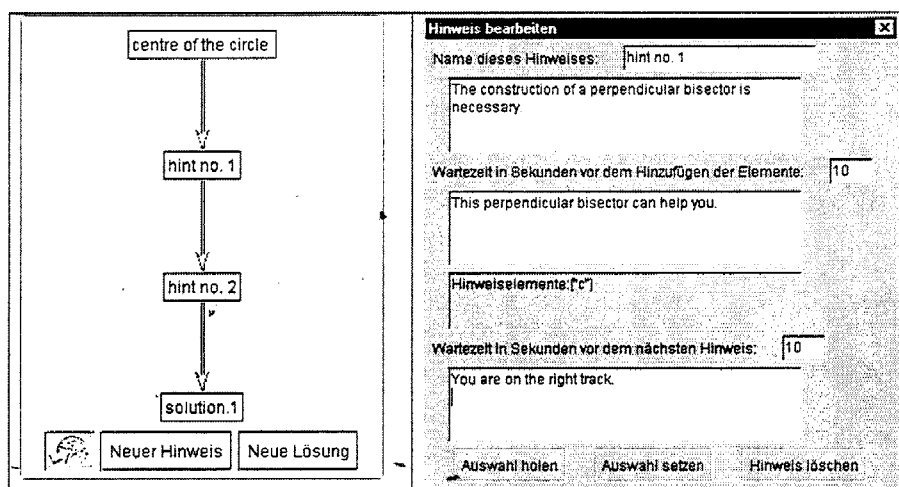

Fig. 2: Construction and Input Editor of the Hints

Teaching can only be really effective for the pupil if his learning process is not hindered or influenced by turnkey solutions. The teacher often unconsciously provides helpful but objective tips that frequently miss the pupil's own ideas completely and at best hinder or nip in the bud what could otherwise be excellent individual concepts.

Exercises should encourage pupils to accept the problem, leave open many paths to the solution and make possible various learning objectives. The needs of society and the needs of the individual to see an evaluation, demand a way of checking solutions.

Fig. 3: Parts of the Interactive Worksheets

4. Types of Documentation Used

In order to open up the different phases of a pupil's knowledge, various forms of documentation are necessary. Ruf/Gallin [1998] pointed out a diary-based method of achieving feedback that is based on the interpretation of teaching as an individual learning process. If

different functions are ascribed to each stage of the learning process, differing forms of documentation for learning ensue.

Stages of the Learning Process	Functions	Type of Documentation
Preview	Discovering the beginnings of a solution	Notebook
Path	Organising the Learning process Learning to control the learning process	Notebook Record-files
Product	Self-control General discussion of results Testing ideas	Notebook Record-files Copy
Review	Self-evaluation	Diary Entries

Fig. 4: Correlation of Types of Documentation and Stages in the Learning Process

4.1 Notebook

Parallel to the interactive work sheets, to provide a better overview and avoid scrolling in the windows, the pupil has a notebook that accepts the text of the current problem and provides space for personal notes. Initial ideas for a solution and also results can be stored here. Furthermore, it allows the pupil to work with the conventional tools of geometry.

4.2 Record-Files

During the work phase, all a pupil's on-screen activities are recorded in project record-files. The files provide information on the use of programme tools. Conclusions on changes in the use of tools can also be drawn.

4.3 Diary Records

The ideal situation would be represented by a place of learning where the pupil would not first search for a formula, a standard solution or other guidelines, but would allow himself to learn by testing his ideas, by rejecting some whilst recognising their value and trying again. He would be prepared to document his paths and give information on his conclusions to others. By helping a pupil to build such knowledge and capabilities, the teacher gains a new appreciation and evaluation of pupil products and recognition of a more advanced definition of achievement. The diary records that are used in this project, showing individual traces of the learning path, represent first steps in developing the concept of independent and individual learning.

Hölzl {1999} draws attention to the influence of technical difficulties during work on problems using the above-mentioned geometry software. The large number of available tools demands concentration from the pupils and could, therefore, limit improvements in the learning process. Such possible difficulties, rejected attempts at solutions and unexpected hurdles in the text of the problem are examined by using the diary records. This is the medium the pupil uses after each lesson to re-examine his work. It permits the pupil's self-evaluation of his learning process.

The diary records embrace the following levels of reflection:

- Assessment of the cognitive level
- Recording of difficulties with the work sheets
- Assessment of affective learning objectives
- Feedback towards co-operation with fellow-pupils.

5. First Results

The following results can already be recognised from the current study:

- Interaction between pupils changed in the course of a teaching unit. Increasing familiarity with the new learning situation caused the experimenting phases to expand. This is clearly shown by the increasing size of the record- files. In the 6th Learning Block, learning retention was tested which led to a reduction in time spent on interactive computer work.
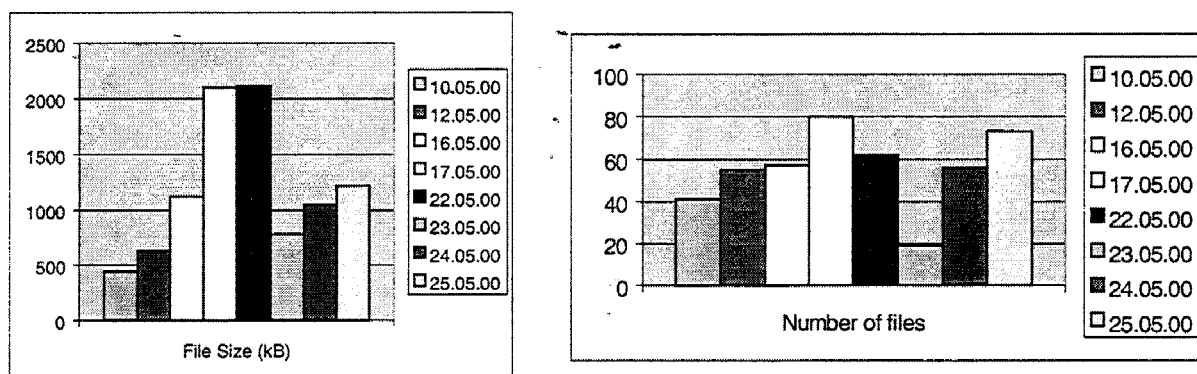

Fig. 5: Changes in File Size and Number of Record files

- The work sheets can be easily integrated into the various learning environments. No time-consuming introductory phases are necessary. However, a phase of free experimenting with the tools at the beginning of the teaching unit seems to be judicious. Technical problems with the tools were almost non-existent and had no effect on the pleasure of experimenting and learning.
- The prime idea of "Distance-Thinking" seems to be an acceptable concept for the application of movable elements.

6. Outlook

The evaluation of the research results centres on the relationship 'Computer-Pupil' and then concentrates on the following points:

- Influence of the interactive work sheets on the learning process
- Changes in the approach to problem solving
- Possibilities of transposing conventional problems to texts appropriate for computers
- New difficulties caused by using interactive work sheets
- Acceptance of diary records by the pupil
- Effects of written records on a pupil's learning achievement

It will be one of the tasks of mathematics teaching in the future to design programmes for teaching with the aid of the computer.

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Laddering - A skill to know more

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Abstract

Laddering is a simple method to hear what people are going to say behind their words. The questions are simple: Why? Which do you prefer the construct or its opposite and why? In an interview a girl, aged 16 is thinking of her learning in mathematics.

Do we know why we do things the way we do? Are we able to answer if somebody should ask 'How do you learn?' 'Why do you study?' Does a student know it? All people have their own implicit theories based on their. earlier experiences. They are either conscious or unconscious and they regulate people's behaviour. Kelly [5] has called these theories a construction system in his theory of personal constructs. It consists of a group of 'thoughts' through which a person looks at the world around him or her, makes observations of it, interprets and tests his or her theory of it and changes it if necessary. According to Kelly this system consists of bipolar constructs, i.e. a person uses perceived similarities and differences to interpret events (e.g. good vs. bad) (Fransella & Bannister [3])

Laddering

According to Kelly the construction system is hierarchical where some constructs, core constructs are central. They are important for the individual and difficult to alter. Some others are peripheral. Hinkle [4] used the term laddering to describe the skill to hear what people are saying behind their words (Fransella [1]). Laddering up simply means asking 'why?' The interviewee is asked which of the two poles of a construct he or she prefers and why. The interviewer continues and asks again 'why did he or she choose this alternative?'. After some questions, you usually come to the region of core constructs. It is possible to get to know something of the values of the interviewee. If you want to know more about the core constructs, you ladder down. Then you ask 'how?' or 'how are x and y different?'.

The theory of constructs shows that everybody has basic constructs to interpret the world around them. It is strange and traumatic to change these constructs. Good – bad is a core construct for many of us, it is the same with the religious constructs, too. If the interviewee can't explain anymore, he or she is near the core constructs. The interviewee only repeats the constructs and they frequently show up by accident. There are many reasons why one should be very careful when considering the core constructs:

1. *Self-deception.* People often have an image of themselves different from the reality. They may become disturbed if the grid interview reveals that their core constructs are different from their treasured self-image.

2. *Internal contradictions.* It is possible that people have mutually contradicting core constructs, which are kept apart with considerable psychological energy, mostly

unconsciously. It may result a difficult trauma if the contradictions are revealed in the grid interview.

3. *Need to know.* Is there really any need to know the core constructs? The interview should be a confidential discussion and the interviewees should feel that they can trust the interviewer. (Stepwart [6])

Interview

In my study I try to find out what the students themselves think of their learning in mathematics. The method used is a repertory grid interview. Different events in students minds when learning are the elements of this study. They have to group these elements to formulate constructs. My interviewee, Anna, aged 16, is girl on the ninth grade. According to her teacher she has succeeded in learning mathematics slightly above the average.

The interview begins when Anna is asked to think what features a good student in mathematics has. She is also asked how she understands the phrase 'to be poor in mathematics'. She answers the questions and chooses the ones which are relevant to her. They are the essential elements in this study. (The character plus means that the element applies to her the minus doesn't apply to her.)

What do you think a good student is like?

What is the way a good student studies?

What does it mean to be poor at mathematics?

She groups these features (elements) according to their common characteristics and they make up constructs. Then she is asked to select from a group from three elements the two that are the most similar and say in what ways they are similar and what the opposite of this similarity is.

An example of the laddering technique

The interview continues with laddering in order to get closer to the core constructs, the most important constructs. The following is an example of the constructs – a pair of opposites: Working together with other people (openness) – withdrawal

K: What do you mean by the withdrawal?

A: In the first place that ... working with other students is openness. You are able to express your own opinion, to give ideas. And withdrawal means that ... those ideas ... you are not able to share them.

PAUSE

K: Of the construct – opposite pair 'openness – withdrawal' you chose the first and said it's easy for you to get on with other people. Is that the reason, why you'd chosen the fist?

A: Well, I could add that it's easy to give ideas and it's also easy to utilize them.

K: If we continue this. You say that it's easy to receive ideas. What is the opposite?

A: Take. No. Mm.

PAUSE

K: What is the opposite?

PAUSE

A: I find ... take ideas. To utilize others' ideas.

K: Which of these two suits you better?

A: two.

K: This suits you better. (Mm) Why?

PAUSE

A: I realized it so many times ... or perhaps easier ... or don't know ... utilize ... I am a little afraid of giving my ideas, if somebody turns them down ... even though I shouldn't be afraid but ...

K: Is your fear connected somehow ... the fear of somebody rejecting your ideas?

A: Mm

K: Has it something to do with studying?

A: Yes.

PAUSE

What is 'the opposite?

PAUSE

A: Perhaps, ...that it doesn't matter to you if you are turned down, so ... you grow because of that turning down or perhaps you can develop that idea into a new one.

K: Which of the two is typical of you?

A: The first.

K: Why?

LONG PAUSE

A: No, perhaps therefore you have a feeling what kind of image they will get if ... I propose an idea and then ... if it's stupid, I don't think so, but the others, what kind of an image they get if I propose that a certain idea, that I have in my head, I proposed that idea, will they remember, what I am like?

K: Are you talking about revealing your self?

A: Somehow that proposal is connected to me.

K: Did you mean that you are afraid of revealing your self?

A: Well, yea to some people, they can get a wrong image, although your self is ok, and means well , they can get a wrong image of it.

K: What's the opposite?

PAUSE

A: Something like ... that you are open. You are able to express... all the things from your inside... you can be .. PAUSE .. you can reveal the ideas, it doesn't matter what kind of image the others get. If you yourself know what you are' like it doesn't matter so much to you. You yourself know what you are like, you don't bother with he others' opinions of you..

K: Which of them is more typical for you?

PAUSE

A: It depends on the person. You are able to tell familiar people everything because they know what you are like. You have not changed in any way. You have a fear of revealing yourself to strangers and you don't worry about people you know.

K: Both in principle. (Mm) Can you say why?

PAUSE

K: What is it about?

PAUSE

A: That fear has something to do with self-confidence. I'm not necessarily so self-confident ... that ... with the people you know you can be self-confident. You are afraid of revealing something of yourself to strange people. Although there is nothing to hide.

K: What's the opposite?

LONG PAUSE

A: It could be a fear of revealing your own self or something like it. PAUSE. Well, that is what I mean.

K: Which of them do you prefer?

A: The first.

K: Can you give reasons for it?

PAUSE

A: In all situations I trust on myself at least. I try to do my best. At least I have tried.

K: Does it show your self-confidence?

A: Yea, I trust myself. I try to do my best.

(13 min)

CORE CONSTRUCTS?

It is possible show this interview in a table:

```
Working together with other people (openness) – withdrawal
    I
(it's easy to manage with other people, it is easy to give and take ideas)
    I
It's easy to give ideas – to utilize others' ideas volunteered
        I
        A fear of turning down the ideas – redeveloping the ideas
        I
        A fear of revealing your own self – the image made of you by others
                                            doesn't bother
        I                       I
        A new person            a familiar person
        I
        Self-confidence – a fear of revealing your own self
        I
        A strive to do one's best
```

The last construct with the opposites was self-confidence - a fear of revealing your own self, where she wanted to say she was doing her best. This same construct was a result in another interview, too. The interview continued until Anna began to repeat her thoughts. Laddering gave the constructions: seeking for one's own limits, self-confidence - fear of the self and using the thing learned in real life.

Anna was a student of mine for one and a half years and so we knew each other. She volunteered when I asked my students for an interview. I think our relationship was confidential and she felt secure. In the interview she said many times 'I have never thought about these things' or 'it is a difficult question'. In the course of the interview she told me a lot about herself, about her feelings to school and some teachers, which I regarded as a sign of trust. Never did she say she would prefer not to talk about something.

This technique reveals the constructs which the interviewees don't tell in the grid interview. Purpose is to try to understand the interviewees and to get to know how they connect one construct to another. I agree with Fransella [2], when she says that laddering is a method that helps us to understand each other.

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Building Theory of Student's Own Mathematics: An Application of Grounded Theory Method

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Abstract

The principal objective of this paper is to reveal how the style of qualitative research known as the "grounded theory approach" was applied in my study (Huhtala 2000) which attempted to generate a theory of a student's own mathematics which leads to drug calculation errors.

1. Introduction

Grounded theory method (developed by Anselm Strauss and Barney Glaser in the early 1960s) has its roots in the symbolic interactionist tradition (Blumer 1986, Bogdan & Biklen 1992, Woods 1992) which is based on three premises:

Human beings act toward things based on the meanings that the things have for them. The meaning arises out of the social interaction that one has with one's fellows. [And] that these meanings are modified through an interpretative process used by the person in dealing with the thing he encounters. (Blumer 1986, 2)

The grounded theory method is qualitative and inductive, analysing data from the empirical world, from which categories and concepts are emerged. The grounded theory method stresses that theory must come from data and that the operations leading to theoretical conceptualisations must be revealed.

The Elements of Grounded Theory

Concepts are the basic units of analysis since it is from conceptualisation data that theory is developed.

A concept is a labelled phenomenon. It is an abstract representation of an event, object, or action / interaction that a researcher identifies as being significant in the data. The purpose behind naming phenomena is to enable researchers to group similar events, happenings, and objects under a common heading or classification. (Strauss & Corbin 1998, 103)

After naming phenomena a researcher must begin to group these concepts according to their properties under *categories* which form the second element of grounded theory.

...once concepts begin to accumulate, the analyst should begin the process of grouping them or categorising them under more abstract explanatory terms, that is categories. (Strauss & Corbin 1998, 114)

Categories are higher in level and more abstract than the concepts they represent. They are generated through the same analytic process of making comparisons to highlight similarities and differences that is used to produce lower level concepts. Categories are the "cornerstones" of developing theory. They provide the means by which the theory can be integrated. (Strauss & Corbin 1990, 7)

2. Backgrounds of my study and data collection

My study was based on a national test given to all the first qualifying practical nurses (November 1995) an on the results of the test. There was one drug calculation in the test and 53,6 % of the 3344 qualifying practical nurses were able to solve the calculation. The practical nurses are, however, required correctness in drug calculations as early as during their studies and in their future working life at the latest. The contradiction between the students´ poor ability and the aim of accuracy on the other hand started my study.

I wanted to know what mathematics means to practical nurse students. What do they think about mathematics, what kind of emotions do they have towards mathematics, how do they study mathematics. What is a student´s own mathematics which leads to drug calculation errors?

The student´s own mathematics here refers to the ways which the students themselves have developed to study, to understand and to use mathematics. The student´s own mathematics develops over a long period of time and mainly through school instruction. That is why it cannot be described as it is at present, but it is also essential to know how it has been formed.

The principal way to collect the data was to run a mathematics clinic in one Institute of Social and Health Care. I offered remedial mathematics instruction and small group instruction during the actual mathematics lessons. Nearly 70 students visited the clinic in two years´ time. All the clinic sessions were tape-recorded (total about 130 hours).

3. Data analysis

In my study data collection and data analysis were interrelated. Data analysis involved generating concepts through the process of coding. There are three types of coding: open coding, axial coding and selective coding.

Open coding refers to that part of analysis that deals with the labelling and categorising of phenomena as indicated by the data. (Pandit 1996)

Open coding is the analytic process through which concepts are identified. (Strauss & Corbin 1998, 101)

Whereas open coding fractures the data into concepts and categories, *axial coding* puts those data back together in new ways by making connections between a category and its subcategories. (Pandit 1996)

Axial coding is the process of relating categories to their subcategories. (Strauss & Corbin 1998, 123)

Selective coding is the process of integrating and refining the theory. (Strauss & Corbin 1998, 143)

Selective coding involves the integration of the categories that have been developed to form the initial theoretical framework. (Pandit 1996) A researcher must find a core category (or more) which tells about the central phenomenon of the study.

The core category must be the sun, standing in orderly systematic relationships to its planets. (Strauss & Corbin 1990, 124)

Other categories are related to this core category to develop a theory.

An excerpt from labelling my data (identifying concepts) follows. It is about one student trying to understand and calculate percents in the mathematics clinic.

Part of the data	*A concept (or a subcategory)*
Is the hundred percents that 2900? **Because I don´t see any other numbers...** **I can´t ever learn how to get that** **hundred percents.**	Difficulty to understand word problems I am not able to learn mathematics
I only wish I could understand...	Despair
I don´t have time when you are already calculating...	Slowness, feeling of inferiority
This is unfair that you only teach those others and you don´t show me the way which I can!	Getting nervous, attacking
I´ve had enough. **I can´t ever pass this.** **Oh hell, I don´t understand this!**	Getting depressed, despair, giving up Defending, attacking, aggression
2100 is that difference. **What difference? I subtracted here also, because that was done in the previous task. In these you can never use the same rule...**	Copying Superficial strategy Trying to find out a rule, a pattern
This is a very stupid question.	Defending, attacking, trying to hide her difficulties
What do you need that knowledge for anyway?	Not wanting to have anything to do with mathematics You don´t need mathematics Attacking

After naming, labelling the phenomena I had a lot of concepts. The concepts seemed to tell about the experiences the students have had in mathematics and about experiences of themselves as learners in mathematics in the past years, about their emotions toward mathematics and about the ways they are now trying to study mathematics, the strategies they are using to learn mathematics. So I could group the concepts under different categories like "mathematics has always been difficult for me", "I hate mathematics", "math anxiety", "mathematics makes me feel stupid", "trying to find out rules", "using minitheories", "using superficial strategies", "learning by heart", "avoiding mathematics", "escaping from mathematics", "guessing" etc. Categories were sorted out to subcategories and higher categories. This was followed by discovery of three core categories which form the story line of the theory of student´s own mathematics. These core categories are experiences, emotions and encounters.

4. Theory of Student´s Own Mathematics

My data shows that a student´s relationship with mathematics, or his or her own mathematics, starts to develop at the beginning of the lower level of comprehensive school. The learning experiences over the years form the basis of the relationship with mathematics. Different emotions towards mathematics in turn arise from the nature of these experiences. The experiences and emotions together have an influence on how mathematics presents itself to the students at present. Therefore, the student´s own mathematics is formed of the three core categories: experiences, emotions and encounters. (Figure 1.)
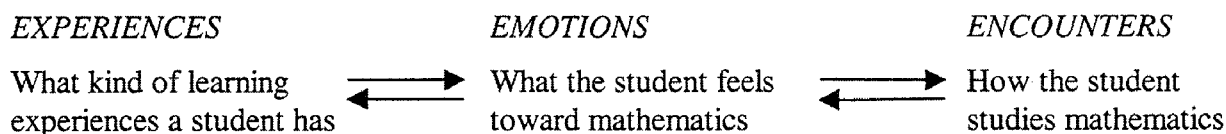
EXPERIENCES	EMOTIONS	ENCOUNTERS
What kind of learning experiences a student has	What the student feels toward mathematics	How the student studies mathematics

Figure 1: Student's Own Mathematics

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The study of the componential structure of Greek pupils' (aged 12-15) beliefs

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Abstract

This specific study attempts to scrutinize the componential structure of the Greek secondary-school pupils' beliefs per class as regards the teaching and learning of Mathematics. The survey took place in five (5) countries simultaneously: Cyprus, Finland, Russia, the USA and Greece. The results of the survey in Greece demonstrated that the created model is in accordance with the formed international model concerning four (4) factors (content, nature, learning and teacher) but it differs as regards the work on Mathematics factor.

Introduction

The significance of pupils' *beliefs* about Mathematics is well-known in mathematical education. These *beliefs* and attitudes have attracted researchers' interest in relevant studies. (Mc Leod, 1989, 1992; Schoenfeld, 1989; Barkatsas and Hunting, 1996; Leder, 1993). The emotional sector seems to contribute significantly to the learning of Mathematics no matter whether Mathematics is described as pure science or not. (Goldin,1988; Debellis & Goldin, 1997).

Cognitive theories had avoided the study of the emotional field for a long period of time. Several theories have recently dealt with this cognitive object (Hannula, 1998b; Goleman, 1995; Risnes 1998; 99).

The findings of the studies state that the cognitive emotions related to learning are direct and often intense. Moreover pupils' *beliefs* are organized in structured models which depend on the socio-cultural environment, behaviour and internal personal factors.

Pupils'*beliefs* about Mathematics also seem to be influenced by mathematical education and the character of mathematical learning in each country (Dematte, Eccher, Furinghetti,1999). This is so because the content of mathematical education is not universally acceptable whereas that of Mathematics is. Mathematicians face Mathematics mainly as a pure science which has its own reasons of existence independently of the real world and the social definition of Mathematics. On the contrary, pupils develop a social view about Mathematics and face it not only as a pure scientific object but also as a cultural fact, a part of the world outside school (Hannula & Malmivuori, 1999).

The question asked is whether the pupils' *beliefs* about the concepts of Mathematics and its teaching and learning are of a universal nature or whether they confine themselves to the education and culture of each country (Christou, Pehkonen, Philippou, 1999).

The objectives of this study are:

- The scrutiny of the componential structure of the pupils' (aged 12-15) *beliefs*. In Greece.
- The comparison of the study findings with the generated international model of Christou & Philippou, 1997, Pehkonen, 1998.
- The scrutiny of the stability of the structure from one class to another at secondary school.
- The study of the influence of the pupils' class and sex to the extent their *beliefs* about Mathematics agree to the modern teaching trends.
- To study the students' attitudes towards open questions.

Study Methodology

1399 secondary school pupils (aged 12-15) at 22 secondary schools in Athens and the provinces participated in the survey during the 1998-99 school term. The schools were selected by the process of random sampling. We took a random sample of a section from each class (A',B',C') at every secondary school.

Pupils' *beliefs* were gauged with the aid of a specially designed identical questionnaire for all five countries. Table 1 shows the distribution of pupils according to the class and sex.

A Likert questionnaire comprising 32 questions (Pehkonen 1998) with an answer range 1-5 (1=fully agree, 2=agree, 3= no opinion, 4= disagree, 5=fully disagree) was used to demonstrate secondary school pupils' *beliefs*. Older studies (Christou and Philippou, 1997; Pehkonen, 1998), showed that the pupils' belief system regarding Mathematics is a structure of five factors.

Results

We scrutinized the factorial structure of the secondary school puplis' beliefs with the aid of the Principal Component Analysis (PCA) and Varimax as a rotation method. We used 15 out of the 32 questions in this study. According to the model of Christou and Philippou, 1997; Pehkonen, 1998 each of the five components of pupils' *beliefs* is expressed by three questions.

At first we studied the *beliefs* questionnaire factorial structure for each class separately. In all three cases (A', B', C' class) we chose to extract five components according to the criterion of component dispersion (>1). The results are given in table 1, in which we have used the total number of pupils.

Then we went on to study the development of the pupils *beliefs* from one class to another as well as the differences between the two sexes and did a Multivariate analysis of Variance (Manova). We found significant differences between the two sexes {Wilks' $F_{(4,1360)}=5,544$; $p<0,001$}, and among the three classes {Wilks' $F_{(8,2720)}=9,74$; $p<0,001$} in the multivariate test (table 2 and table 3). The pupils' class seems to influence their *beliefs* as regards the learning, content and nature of Mathematics.

In the reply texts we looked for key words in order to classify the students' answers into categories. The answers to the question:

"What positive experiences have you had so far in relation to the teaching of Mathematics?" were classified into the following categories:

teaching, supportive attitude, interest in the subject, epistemological impediment, assessment, usefulness, psychosocial environment.

As regards their negative experiences we found the following categories: teaching, supportive attitude, interest in the subject, epistemological impediment, assessment, usefulness, psychosocial environment, anxiety, rejection of school.

The answers to the question: "How would you like the teaching of Mathematics to be?" were classified into the following categories:

teaching time, computing environment, teacher, supportive attitude, epistemological impediment, assessment, usefulness, psychosocial environment, curriculum range, interest in subject, rejection of school, textbook.

We analysed the frequency of the answers to the three questions according to sex and class level.

The results (table 4) showed that the positive experiences mainly focus on the teaching methodology followed by the teacher (31% to 63%). The positive experiences are linked to the interest in Mathematics (8% to 25%) as well as to its usefulness (21% to 30%). Assessment (3% to 13%) does not seem to contribute to the creation of positive experiences and the pupils' attitude is only slightly influenced by the environment of the classroom.

The cognitive -epistemological impediments (33% to 46%) seem to play the prominent role in the shaping of negative experiences. The negative experiences are to the same extent related to the teacher's attitude (21% to 32%) and to the teaching of the subject (19% to 35%). It is interesting to observe that the assessment does not significantly contribute (8% to 16%) to the shaping of negative experiences. The negative experiences are barely connected with anxiety and the classroom environment.

As regards the teaching of Mathematics the pupils consider the teacher's teaching practice most important (60% to 74%). Then comes the teacher's behaviour in the classroom (15% to 33%).

14% to 30% state their need for the incorporation of a computing environment into the teaching of Mathematics.

Table 1. Principal component analysis for the 15 questions of the Pehkonen model for the total sample (Secondary school classes A', B',C')

Items		Components				
		1	2	3	4	5
E9	The solution to problems	.66				
E22	Area and volume calculations. E.g. The area of a rectangle and the volume of a cube.	.65				
E6	The design of shapes . E.g. triangles	.65				
E19	The idea that Mathematics is valuable	.63				
E16	The idea that everything should always be precisely justified		.77			
E5	The idea that everything should always be expressed with utmost precision.		.71			
E32	The idea that the teacher should always tell			.67		

the pupils exactly what to do.

E26	The idea that the teacher should explain every stage of solving a problem precisely...	.58	
E15	The idea that the teacher should assist his/her pupils when they face difficulties.	.56	
E27	The idea that the pupils could solve problems by themselves without the teacher's help.	-.49	
E25	The idea that games could be used to help the pupils learn Mathematics.		.74
E31	The idea that sometimes the pupils work in small groups.		.67
E14	The use of computers / pocket calculators		.46

E7	The idea that we should always find the correct answer promptly.		.69
E1	The execution of operations from memory		.65

Table 2. Means and standard deviations of the *beliefs* according to sex.

	Male			Female		
	M	N	SD	M	N	SD
Learning	2.44[a]	692	.86	2.32[b]	678	.76
Content	1.92	692	.71	1.96	678	.63
Nature	2.21	692	.97	2.27	677	.96
Teacher's role	1.87[a]	692	.72	1.75[b]	678	.65

- The mean figures with a different exponent have a statistically significant difference.

Table 3. Means and standard deviations of the *beliefs* according to class

	A		B		C	
	M	SD	M	SD	M	SD
Learning	2.52[a]	.81	2.33[b]	.85	2.28[b]	.77
Content	1.80[a]	.63	1.97[b]	.66	2.07[b]	.69
Nature	2.10[a]	.91	2.37[b]	.99	2.26[b]	.98
Teacher's role	1.81[a]	.69	1.79[a]	.73	1.83[a]	.66

- The mean figures with a different exponent have a statistically significant difference.

Table 4. Distribution of student's experiences according to sex and class

	boy			girl		
	A	B	C	A	B	C
Positive experiences						
Teaching	35.0	42.2	30.9	46.7	63.2	52.9
Supportive attitude	10.0	6.7	9.9	12.1	21.1	15.3
Interest in subject	25.0	16.7	22.2	15.0	7.9	14.1
Epist. Impediment	15.0	10.0	7.4	13.1	11.8	9.4
Assessment	6.3	10.0	4.9	6.5	2.6	12.9
Usefulness	27.5	31.1	33.3	29.9	21.1	24.7
Class. environment	2.5	2.2	2.5	1.9	1.3	
Negative experiences						
Teaching	23.8	34.8	18.9	23.9	38.1	33.3
Supportive attitude	20.6	24.2	27.0	20.5	27.0	32.0
Interest in subject	12.7	6.1	17.6	8.0	9.5	9.3
Assessment	9.5	12.1	16.2	9.1	9.5	8.0
Epist. Impediment	46.0	43.9	37.8	45.5	38.1	33.3
Class. environment	3.2		2.7	2.3		2.7
Anxiety	3.2		1.4	8.0	1.6	4.0
Usefulness		1.5			3.2	1.3
Rejection of school		1.5				

Conclusions

The factor analysis of Greek students' *belie*fs offered us five factors: the *content, nature, teacher, learning* and *work* on Mathematics.

The results of analysis agree to the model of Christou and Philippou, 1997 – Pehkonen, 1998 as regards the first four factors. We observe a difference in the fifth factor regarding the work on Mathematics. The questions which create this component (calculations, usefulness of Mathematics, pupils can solve the problems by themselves) seem to disperse to the other components which were created by our analysis.

The pupils connect the usefulness of Mathematics with its *content* factor. Their prevalent opinion is that Mathematics is useful in a series of acts (problem solving, shape construction and so on) which are of use to our everyday needs.

The Greek pupils perhaps perceive the work on Mathematics as an execution of operations from memory and believe that they should find the correct answer promptly. However, these are activities which concern their own learning abilities.

As regards the classes, class A´ of high school seems to be different from classes B´ and C´ of high school concerning the *content* of Mathematics (problem solving, area and volume, calculation, shape design, Mathematics is useful). The younger pupils agree more than those in classes B´ and C´ to the items that determine the *content* of Mathematics. Our justification of this is that the younger pupils still display the primary school mentality. They face Mathematics as a series of acts.

As regards the *learning* of Mathematics (the games can be used to help the pupils learn Mathematics, the pupils work in small groups, the use of calculators/computers) the older pupils seem to have stronger *beliefs*. They seem to believe less in questions which determine the *nature* of Mathematics (everything must always be precisely justified, everything must be expressed in the utmost precision).

This states that the older pupils seek new learning environments, such as the use of new technologies, calculation environments, group cooperative teaching.

We could claim that there is a change in the direction of the pupils' *beliefs* in the more advanced classes towards new teaching models. The pupils of classes B´ and C´ seem to ponder exploratory teaching patterns. The fact that they seem to have weaker *beliefs* as regards the *nature* of Mathematics states that they seek teaching patterns in which severity and being absolute do not prevail. Therefore, the change that is observed in the more advanced classes allows us to consider that there is a suitable environment for the application of the new teaching models. Relevant research is in favor of the above –mentioned finding. Pupils' *beliefs* and the *belief* systems are developed through the below-mentioned high-school levels (12-15) (Hannula, 1996) and change as pupils grow up.

We do not observe any discrepancies between the 2 sexes. The girls seem to have more appropriate beliefs than the boys as regards learning, a fact that implies that they are more open to the new learning environments. However, they seem more conventional regarding the behavior they expect the teacher to have in class (the teacher should always tell the pupils what exactly they should do, he/she should explain every stage of the solution to a problem precisely, he/she should help the pupils as soon as they face difficulties). The interpretation of this should be sough in the social role reproduced by them.

The study of the open questions ascertains that the girls lay more stress on the teacher's role born as regards teaching and his attitude. The boys seem to be more interested in Mathematics since they link their positive experience with their interest in the subject to a greater extent. Besides the boys refer to the use of new technologies more than girls.

We observe that the need for a supportive teacher and for a better classroom environment increases from one form to the next whereas the pupils' negative experiences that are related to the impediments to the subject decrease.

To sum up, what is significant is the observation that the pupils state their need for a teacher who is both supportive and competent as regards his/her teaching. The pupils' positive experiences are related to the teacher's teaching practice and to their interest in the subject. Their negative experiences are linked to the teacher's attitude and to the cognitive impediment which are caused by the cognitive object.

Moreover, the pupils state their need for new learning environments.

We can therefore claim that the deeper analysis of *beliefs* helps us transform current teaching practices.

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Metaphors and Mathematics Curricula

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Abstract

'Education is a purposeful activity, and the curriculum is the main means whereby the purpose of education are achieved' [ROBITAILLE 1993, p. 4]

It seems plausible that the conception of mathematics education has evolved over the last 50 years in Germany. Curricula dramatically changes at the beginning of the 70th worldwide. There has been a shift from the highly theoretical to the more practical - away from „new math", and „back to basics". Looking into old schoolbooks mathematics appears to be the same as it has always been.

It is the purpose of this paper to highlight some aspects of the role of metaphor by presenting some qualitative and some statistical results of a content analysis of curricula and to relate the findings to more general opinions about the nature of educational metaphors.

In my research I studied the evolution of curricula to mathematics teaching by tracing changes in the kind of metaphors. The chief result was that the statistical view on elected metaphors can indicate conceptions of mathematics education.

1. Metaphors and Mathematics

Our understanding of mathematics is bound up with a view of reason.

The dominant tradition has been to see reason as abstract, unemotional, universal, disembodied and hence formal and unchangeable. The view that there does exist one unique 'correct' mathematics independent of any minds indicates objectivist view of mathematics.

The dominant tradition has been to see metaphors as a figure of speech with a genuine illuminating force, emotional, depending on models of the mind and hence unformal, quite the opposite to mathematics.

I hope to show how concepts of metaphor can highlight aspects of the nature and principles of mathematical discourse which is known as 'doing mathematics'.

1.1. Metaphors

Although there exist radically different ideas of what metaphors are Aristotele's definition is fundamental: „Metaphor consists in giving the thing a name that belongs to something else..." [Poetics, Chap. 21,1457b]

1.2. Metaphorical Concepts

Lakoff/Johnson [1980. Metaphors We Live By, p. 46-49] gave metaphors and ordinary expressions that are special cases of the 'conceptual' metaphor:

<u>THEORIES ARE BUILDINGS</u> (p. 46): Is that the *foundation* for your theory? .. Here are some more facts to *shore up* the theory... The theory will *stand* or *fall* on the *strength* of that argument. ...

<u>IDEAS ARE PLANTS</u> (p. 47): His ideas have finally come to *fruition*.
It will take years for that idea to come to *full flower*. The *seeds* of his great ideas were *planted* in his youth. ...

<u>UNDERSTANDING IS SEEING</u> (p. 48): I *see* what you're saying. .. That is an *insightful* idea. ..

1.3. Definitions and explanations in German curricula

In German Mathematics curricula ('Richtlinien und Lehrpläne Mathematik, Nordrhein-Westfalen') we find some metaphors like these ones:

- *Mathematik, das einzige Beispiel einer in sich <u>gegründeten</u>[3] Wissenschaft* .. [1952, p. 9]
  (Mathematics, the single example of a science founded in itself..)
- *<u>Aufbau</u> des <u>Denkgebäudes</u> der Mathematik* [1963, p. 31]
  (the structure of a building of mathematical thoughts)
- *Die besondere Aufgabe der Mathematik im Rahmen der übrigen Wissenschaften als <u>Lieferantin</u> von geeigneten Theorien.* [1972, p. 168]
  (the special task of mathematics in respect to the other scientific disciplines is the one of a <u>supplier</u> of suitable theories..)
- *<u>Gebiet</u> der Mathematik* [1975, S. 72]
  (<u>region</u> of mathematics)
- *<u>Gegenstände</u> der Geometrie* [1993, p. 34]
  (<u>objects</u> of geometry)

Although we will have many difficulties in finding only one conceptual metaphor 'MATHMATICS ARE BUILDINGS' seem to be the best one describing concepts of mathematics. On the other hand we shall try to integrate most of these explanations as 'buildings', 'regions', 'objects' and so on which are strongly associated with mathematics. Probably we can draw a picture in mind which looks like a town with buildings, streets, townships, ... ; probably we can say that structural thinking of mathematics can be indicated by these metaphors.

2. Metaphorical change in curricula

If indeed metaphors in curricula reflect the way educators and teachers have conceived of the domain, the changes in the kind of metaphor used may provide an unobtrusive measure of changes in conceptual paradigms.

To test a hypothesis of change during a period of 50 years and to get more precise information I examined 29 meaningful words and metaphors which seems to carry enough information about curricula.

The following list gave an overlook of elected words and metaphors occurred in nine mathematics curricula for the 'Gymnasium' of 'Nordrhein-Westfalen' edited in the last 50 years.

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[3] Metaphors are underlined

|	'52	'63	'72	'73	'75	'83	'84	'93	'99
Development	8	18	36	13	13	82	15	79	97
Unfolding	0	3	0	0	1	9	1	9	17
Growth	1	1	6	3	0	17	5	11	9
Function of Growth	0	0	0	0	0	9	1	8	0
Building up	6	25	36	4	22	17	19	29	40
Examination	1	5	15	2	0	23	28	28	23
Labour	9	89	72	39	33	238	45	144	306
BASIC	14	106	123	60	116	212	81	125	141
'Basic-Course'	0	1	61	3	6	88	5	14	50
Basic- Set	0	5	18	13	32	0	2	0	1
Constitution	6	38	93	39	42	109	47	59	70
Education/*PICTURING'*	3	43	75	51	87	92	26	70	98
Diagram /*'Depiction'*	3	78	136	18	132	38	22	6	29
FORMING	0	9	14	1	7	54	4	27	40
Object	0	51	11	6	28	60	32	18	34
ORIENTATION	0	9	14	29	8	24	16	36	66
Orient.-Course	0	3	0	25	1	0	0	0	0
Stage	6	112	69	17	44	170	61	127	159
Curriculum	0	0	8	6	3	31	1	2	0
Teacher	0	41	28	20	9	106	28	34	30
Schoolboy	4	88	77	47	89	253	99	172	217
Mathematics	26	93	156	68	123	189	79	183	299
Lesson /*'Informing'*	11	170	128	33	108	297	119	277	283
Leading/*Transporting*	16	88	215	37	156	189	75	68	104
Application	9	28	114	10	49	139	57	75	76
Treatment/*'Handling'*	12	47	118	4	88	93	61	27	48
Significance	7	64	99	24	90	154	40	87	135
Vision	12	56	43	3	16	47	19	29	31
Insight	0	8	4	3	5	19	7	16	12

This method of counting metaphors is not really new. It had been successfully used in psychology by Pollio 1977, by Stählin 1914, in respect to curricula and education by Cheverst 1972, and Drakenberg 1995.

Looking for correlation between curricula to find out which curricula have a similar profile, correlation coefficients were determined.

Correlation between Curricula (NRW 1952-1999) (based on 29 terms and metaphors)

Corr. Coeff.	'52	'63	'72	'73	'75	'83	'84	'93	'99
1952	1,00	0,65	0,79	0,59	0,69	0,63	0,69	0,62	0,67
1963	0,65	1,00	0,72	0,62	0,76	0,87	0,88	0,85	0,78
1972	0,79	0,72	1,00	0,61	0,92	0,70	0,76	0,56	0,57
1973	0,59	0,62	0,61	1,00	0,67	0,68	0,65	0,67	0,72
1975	0,69	0,76	0,92	0,67	1,00	0,66	0,75	0,56	0,56
1983	0,63	0,87	0,70	0,68	0,66	1,00	0,91	0,91	0,88
1984	0,69	0,88	0,76	0,65	0,75	0,91	1,00	0,87	0,79
1993	0,62	0,85	0,56	0,67	0,56	0,91	0,87	1,00	0,94
1999	0,67	0,78	0,57	0,72	0,56	0,88	0,79	0,94	1,00

Correlation in respect to Curriculum 1993
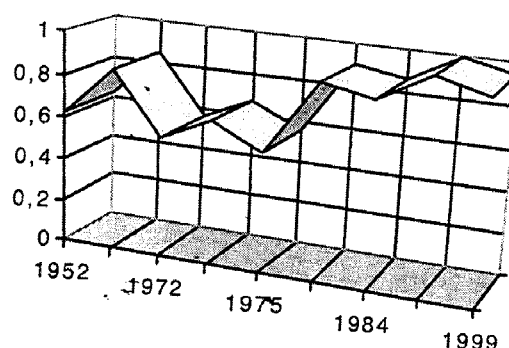
Curricula belonging to New Mathematics(1972-1975) have a special and different profile in respect to the other ones. The second observation is that a general development of curricula only interrupted by New Mathematics is visible.

The counting method seems to be an adequate method to identify this fundamental change. This research method gave us an argument in respect to the first hypothesis of change in mathematics during the last years. A factor analysis of my data gave another very interesting information: Only one factor can be identified. There may be a great common view of mathematics and education in our curricula. This helps the other thesis I mentioned at the beginning.

3. Conclusion

Although '... most of the important and interesting information is hidden from view ...'[Robitaille 1997, p. 17] metaphor analysis really offer a behind-the-scene-look giving information that not always be directly observable. I wanted to demonstrate an unusual method to analyse mathematics curricula and construct consideration, probably you may say beliefs of mathematics, in a new way.

4. Curricula

1952: „Richtlinien für den Unterricht in Mathematik und Physik an Gymnasien im Lande Nordrhein-Westfalen"

1963: „Richtlinien für den.Unterricht in der Höheren Schule, Mathematik"

1972: „Curriculum Gymnasiale Oberstufe Mathematik" (2.Ausgabe)

1973: „Richtlinien und Lehrpläne für die Orientierungsstufe Klasse 5 und 6 in Nordrhein-Westfalen"

1975: „Sekundarstufe I - Gymnasium Mathematik Unterrichtsempfehlungen"

1983: „Richtlinien für die gymnasiale Oberstufe in Nordrhein-Westfalen Mathematik"

1984: „Vorläufige Richtlinien für das Gymnasium - Sekundarstufe I in Nordrhein-Westfalen Mathematik"

1993: „Richtlinien und Lehrpläne Mathematik Gymnasium Sekundarstufe I"

1999: „Richtlinien und Lehrpläne für die Sekundarstufe II - Gymnasium/ Gesamtschule   in Nordrhein-Westfalen Mathematik"

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Teachers' Beliefs about Mathematics and the Epistemology of their Practical Knowledge

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Abstract

Humans are reasoning beings. We have the capacity to engage in practical reasoning and to act on these reasons. Practical reasoning – conscious or tacit – underlies most of teachers' decisions in teaching. The ground and framework for this reasoning used here is 'practical knowledge' or 'knowledge in use'. The participants in my research project were primary school teachers who were willing to develop their mathematics teaching and had therefore participated in 6-8 days long in-service courses in mathematics. The participants – altogether 37 – filled in a questionnaire which consisted a beliefs-inventory. Using the gathered data I chose six teachers for closer follow-up. The criteria for the selection of these teachers was that pairs would be of same sex, would have equally long teaching experience, would be teaching about the same grade level, and have opposite values in the variable open-approach. The results of the study seem to indicate that memories from childhood and early school years have influenced the teachers' beliefs about the nature of mathematics and about the way it should be taught properly. The in-service mathematics courses gave new insight and new more open methods to the teachers' practices in the math class.

Background

During the last decades much attention has been directed towards teachers' professional development in the teaching of mathematics. In order to achieve lasting improvements attention has to be drawn not only to the theoretical education of pre-service teachers, but to the implementation of the knowledge gained in teachers' everyday teaching practice. Good teachers usually reflect on what they do, but it is difficult for them to examine the assumptions and beliefs underlying their actions. In this study I want to explore these assumptions and the reasoning that rise from in-service teachers' own experiences both in school and in the teaching profession.

In the Handbook of Research on Teaching FEIMAN-NEWSEN AND FLODEN challenge the "dim view of teachers' knowledge" and point out that: "Instead for searching for professional knowledge or technical knowledge, they (researchers) have looked more broadly at teachers' practical knowledge - that is, those beliefs, insights, and habits that enable teachers to do their work in schools." Thus practical knowledge is time-bound, and situation-specific, personally compelling and oriented toward action. (Feiman-Newsen and Floden 1986, 512).

In the eighties research made a distinction between theoretical knowledge and practical knowledge. Thus FENSTERMACHER uses terms *knowledge production* and *knowledge in use*. Knowledge production arguments consist wholly of assertions, and end in a statement that is tightly connected to the preceding premises. The arguments of knowledge in use lead always

to action, to do something that is consistent with the preceding premises. (Fenstermacher 1986, 41-43.)

This distinction between theoretical and practical knowledge had its origin already in the writings of ARISTOTLE. In his sixth and the seventh books of the *Nicomachean Ethics* (*Ethica Nicomachea*) he speaks of theoretical wisdom and practical wisdom. I understand that a man's knowledge, or in Aristotelian terms, wisdom, constitutes the pool from which his reasoning arises. In the *Nicomachean Ethics* Aristotle gives formulations and examples of two types of logical arguments or reasoning. Aristotle distinguished theoretical reasoning from practical reasoning. He expressed the latter in the form of a syllogism which is often called practical syllogism. His syllogism is a series of premises which lead to a conclusion. He underlines that practical syllogism leads to concrete action and writes: "When two premises are combined, just as in theoretical reasoning the mind is compelled to affirm the resulting conclusion, so in the case of practical premises you are forced at once to do it."(von Wright 1963, 159).

ARISTOTLE gives a good example of practical syllogism in chapter vii in Book VI of the Nicomachean Ethics:

- Health is a worthy aim
  - Circulating blood promotes health
  - Morning runs circulates the blood
    - This is the morning

    [ACTION: Running]

These premises express

- a desired state of affairs, a value premise (**V**),
- an empirical character that can be evaluated to be either true or false through careful observation and study (**E**),
- a description of the context or situation in which the agent finds himself (**S**).

(Fenstermacher and Richardson 1993, 102; in addition to these three premises they include a forth type of premise, the stipulate premise.).

FENSTERMACHER AND RICHARDSON build on Aristotle's practical syllogism in their study. They use the above mentioned formalisation of premises when they analyse teachers' practical reasoning in a classroom setting. They found that this formalisation helped teachers to examine the premises that generate their practices. They conclude that this process may encourage teachers to conduct their own inquiry about their actions. It helps them to change the truth value of the premises of their practical arguments. Thus "the process allows teachers to take control of their justifications, and therefore to take responsibility for their actions." (Fenstermacher and Richardson 1993, 112)

Research questions

I found the Aristotelian formalisation of practical reasoning very helpful in my attempt to analyse the epistemology of in-service teachers' beliefs about teaching mathematics and to help them to reflect on their actions in the mathematics class situation. In her study VÁSQUEZ-LYVY refers to the definition written by Audi and gives a comprehensive description of the role of practical reasoning. (Vásquez-Levy 1993, 125.) In the following summary I condense and adjust these ideas as a list of theses. These propositions include my assumptions about the role of teachers' practical reasoning and form the ground for this study.

> **Theses**
>
> - Humans are essentially coning beings.
> - We act on the arguments provided by our reasoning.
> - Practical reasoning combines reason, beliefs, goals, and action.
> - Teacher's capacity to reason is used and can be observed when:
>   - they choose what to teach,
>   - they choose how to present the content,
>   - they choose tasks for learners,
>   - they maintain organisation and control,
>   - they assess student learning.

I started the process of my pragmatic inquiry with these theses in mind and set the following questions:

1. What is the content of the practical knowledge of primary school teachers who attend in-service mathematics training?
2. What kind of practical reasoning can be found in their teaching profession?
3. What factors have influenced the content of the teachers' practical knowledge?

Method

The participants in my research project are primary school teachers who are willing to develop their mathematics teaching and have therefore participated in at least one in-service course in mathematics. In the beginning of this year I attended two of these courses. These 6-8 days long courses had been going on since the fall 1999, and were given by two different teachers. Both teachers were men and can be assessed as very constructivistic and inspiring. The participants – altogether 37 – filled in questionnaires which contained the same beliefs-inventory I used in my earlier studies. It also included open questions about the background of teachers' beliefs and practical reasoning, and an assessment of the course.

Using the gathered data I chose six teachers for closer follow-up: two female teachers with long teaching experience, two male teachers with long teaching experience and two female teachers with short teaching experience. The criteria for the selection of the teachers was that pairs would be teaching the same grade level and have opposite values in the variable open-approach (see Lindgren 1996). The first interviews were done by phone and lasted for 35-50 minutes each and included a profound and itemised insight of themes in the questionnaire. Next, observation was conducted in each teacher's classroom followed by an interview. This included a discussion of the decisions and reasoning which manifested itself during the math lesson and a quick oral solving of four beforehand written educational problems. The math lessons and both of the two interviews were audio-taped.

Results

As a starting point when coding the data of the groups attaining the in-service courses in mathematics (n=37), the values for the variables **open-approach** (OA) and **rules and routines** (RR) were calculated. These were means from 8 and 6 items respectively in the questionnaires. These variables are very similar to Alba Thompson's variables Level 2 and Level 0. (For of a list of items see Lindgren 1995 or 1996). For the whole target group the maintained values

were: OA: x=4.28, s =0.40; RR: x=2.58, s=0.58, where the value 5 refers to full agreement with the statement.

I

In the questionnaire there was an open-ended question: *What do you think 'teaching math' really means?* The answers of interviewed teachers were:

Ann (OA=4.88 , RR=2.00): To stimulate and to give opportunities to children to discuss different thinking strategies.
Brita (3.25,, RR=2.00): The teacher gives students tools to handle problems.
Carl (OA=4.63, RR=2.17): To make the students to think and ponder, and to feel good with math problems.
Dale (OA=3.00, RR=2.33): To teach thinking.
Emily (OA= 4.25, RR= 1.84): Doing, reasoning and collaborate successful discussions.
Flora (OA= 4.25, RR= 3.17): Pondering and mastering abstract concepts.

II

To illustrate the answer to the second research question, I gave the following syllogisms.

Ann's practical reasoning

- My hope is that my students would think that math is for the whole life, and not just for the math lesson (V)
  - I have learned math in solving practical problems (E)
  - I have found out that discussions with colleagues have helped me to solve problems (E)
    - My students often have unpractical and unprofitable solutions (S).

  ACTION: I discuss practical problems with my students.

Carl's practical reasoning

- My hope is that my students would get more self-confidence and courage to take up problems (V)
  - When studying at the University I found that math can be very difficult, and my eyes were opened up to understand students who have difficulties in math (E)
  - I have found that a child's world of concepts differs from that of a mathematician, and therefore I should not always use exact terms (E).
    - Some boys find math very difficult (S).

  ACTION: I give interesting and visually concrete problems and try to 'hold on' everybody.

Dale's practical reasoning

- My goal is that my students would like math (V)
  - I was never encouraged in school and I was not interested in math (E)
  - If I abuse the students as being dull, I will depress them (E)
    - Earlier (before the course) I had a harsh attitude towards my students, and I was irritated when they did not understand (S).

  ACTION: I encourage my students.

While I was following these teachers' reasoning and their mathematics lessons, I found out that these statements are not just fine-sounding speech but a reality and reflect firm and settled principles for teaching mathematics. The above formulated syllogisms were very plainly deducted from the answers to the open-ended questions in the questionnaire and from the statements in the interviews done. The syllogisms above show significant development in the teachers' teaching practices and lead into a positive action. But we all know that there exists practical reasoning that could lead to unacceptable actions. I think that in order to correct defective actions in the class it is useful to study the ways the teachers use reasoning.

III

Next I shall record some phrases from the interviews that illustrate the answers to the question about factors that had influenced the content of the teachers' practical knowledge. The teachers who had quite high values in the variable OA (**Ann** and **Carl**) expressed the following memories of early childhood experiences and their math teachers in comprehensive school:

- Studying with mother the tape-measure, and assessing measures and volumes while baking in the kitchen.
- Discussing with father numbers, prizes, and measures.
- Teachers were gentle, logical, and firm, and gave clear instructions. A good feeling.
- The teachers were very enthusiastic. Math was easy and appealing.
- The teacher had 'inner authority'. Had very good lessons and everybody was quiet and studied.

The other teachers had some memories that seemed to have caused them some frustration or unpleasant feelings. **Dale** said that he was never encouraged in school and that he had poor or insignificant teachers. **Brita** liked mathematics, but she was often discouraged in competition situations. In the interviews it became apparent that in shaping the teachers' view of mathematics and teaching mathematics some separate happenings and memories were not so significant as an overall feeling of confidence and pleasure in learning mathematics. Feelings of frustration or of being unskilled and incompetent in early years at school would show as lack of confidence in one's own teaching of mathematics.

The feeling of incompetence and inadequacy was one of the reasons why teachers attended in-service training courses. They also wanted to find new methods and receive inspiration for teaching mathematics. The interviews show that the teachers have really gained help and clarity for their teaching profession. In **Dale**'s words: "A revolutionary course. It really opened up my eyes."

Carl had studied philosophy and he said it had given him a good approach to mathematical thinking. Philosophy also gave him an understanding of the nature of a child and of a human being in general. He had also experienced that discussions with fellow students while studying mathematics were extremely important in overcoming the difficulties confronted in learning mathematics. Now he wants to give his pupils the same possibility.

Conclusion

As a summary of the factors which have influenced the content of the teachers' practical knowledge the following things can be listed:

- Memories from childhood and of the overall feelings during math lessons at school.
- In-service mathematics courses
- Discussions with colleagues

- Math education at the University (Carl).
- Experience in teaching profession

There were no male teachers with a short teaching experiment. When male and female teachers with long teaching experiments were compared some differences were found. Both of those teachers who felt confident with mathematics (Ann and Carl) used a variety of teaching methods and could easily react to pupils' questions and needs in learning mathematics. Those who were not so confident and whose values in the variable open-approach were low, used rather traditional methods in their math teaching. The difference between Brita and Dale was that Brita seemed to be more formal, and uncertain in her teaching behaviour in the class, while Dale was quite relaxed and had a firm belief in his methods for teaching. The epistemology of the teachers' practical knowledge lies in their early childhood years, and it grows through educational experiences. This practical knowledge can be revealed – and improved - in the teachers' practical reasoning.

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The Impact of CAS on the Development of Beliefs, Conceptions and Skills in Elementary Linear Algebra

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Abstract

Empirical research by Grigutsch/Törner has shown a negative belief structure with regard to 'Formalism/Process/Application/Schema' and 'riged Schema' aspects of mathematics in 17-year-old students in base mathematics (hereafter called GK-12). After discussing some possible reasons for that structure in traditional instruction format I propose to design ample CAS-supported learning environments in Elementary Linear Algebra (eLA) to try to change that belief structure. I propose to focus on three fundamental ideas and mathematising patterns to emphasize informal visual presentation/ argumentation of concepts and semi-automated algorithms using Computer Algebra Systems (CAS) as a tool. Three examples are presented here: directed graphs (digraphs), regular Linear Systems and GAUSS-algorithm, whereby an alternative visual representation of a matrix is demonstrated.
Keywords: belief structure, Linear Algebra, fundamental ideas, visualisation, semiautomating.

1. Results of Grigutsch/Törner

In empirical research GRIGUTSCH [4] studied the belief structure of students in GK-12 courses in German secondary schools; the students were given 3 lessons of math a week, i.e. approximately 100 lessons a year. He found five main aspects in their belief structure: for those GK-12 mathematics is seen as *highly formal* (+F), mainly *schematic* (+S), *less processual* (-P) and *without significant application* (-A); they showed, in particular, a *rigid schematism* orientation (+rS), i.e. they only learned for their next assessment:
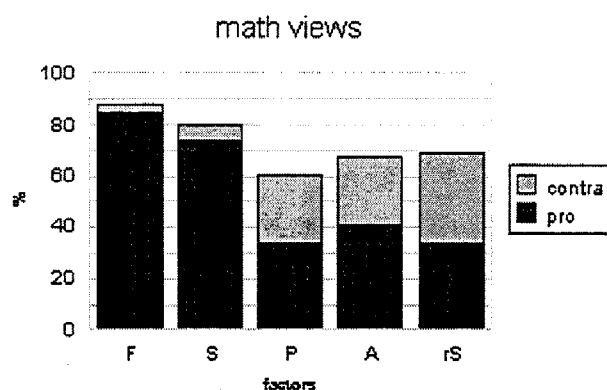
math views

Figure 1: Belief structure of students (N=338) in GK-12 at May/June 1994. Formal means strictness, exactness, preciseness. Processual describes problem-orientation, reinventing, searching. Application aims at practical use, direct applications or maths used in everyday life. Schematic is associated with use of rules, toolboxes, formulae and the rigid schema orientation is mechanical or manipulative use of concepts which are often misunderstood or not understood at all; see also[6, p.10]

Grigutsch stated that this should be evaluated as negative:

> [...] for pupils in base course mathematics is formal exact learning and making use of schemata and algorithms, often without essential progress of understanding and finding and which can seldom by applied or useed. [..] This is reflected in a self-concept of little pleasure, poor math achievements and low personal estimation. [...] A view of mathematics, which does not emphasise the algorithmic [better: rule oriented – wL.] aspect too strongly, but which emphasises the character of mathematics as an application-oriented process of understanding and finding, should be evaluated as positive. This view of mathematics should be mediated in math education.

In this preliminary report of an actual research project at the University of Duisburg I intend making some research hypotheses on reasons for that belief structure and proposing an alternative curriculum eLA-design to influence this +F+S-P-A+rS belief pattern. First steps towards an constructive implementation follow.

2. Traditional Textbook-Instruction and Classroom Formats

In the educational phase, observed by GRIGUTSCH German students study Calculus II and Linear Algebra/Analytical Geometry I (eLA). Two main influence factors of math-belief structure are the traditional textbook format (cf. TIMSS) combined with the traditional instruction format (including the teachers role) in the classroom.

The traditional textbook-instruction format is typically characterised through examples of newly published M-books for GK-12 in Germany: for a period of three years there have been two or three books of approx. 800 pages with 2000 problems; on a base of 100 lessons a year this results in a selection from 2-3 pages with 6 problems for a 45 minutes instruction period (plus homework). From pupils point of view this would seem too long (frightning and offputting), too many problems (plantation of rule based routine exercises forcing technical skills) and too fast (no time to think *in* class).

A typical German eLA-book content [8, p.9-10] concentrates on traditional themes like Cartesian coordinate system in 3D, vectors, parameter/coordinate equation of lines and planes, relations of lines/planes, orthogonality and distance problems.

Therefore it must be argued that these new books will conserve the belief pattern +F+S+rS-A-P. The danger lies in the thematic rigidity, usually leading to bundles of similar solvable routine/standard problems (problem "islands", exercise didactic), while important aspects of mathematics are missing or are not fully implemented throughout, e.g. an implementation of algorithms by means of CAS or application orientation.

The traditional classroom instruction format and the role of the teacher often reflect the above situation, see e.g. STIGLER [14] et al. for contrasting traditions of classroom maths instruction (5th graders):

> Although constructivist approaches to teaching in the US often emphasise small group and individualised instruction, Japanese teachers, surprisingly, combine the almost total use of whole-class instruction with their strong emphasis on students' thinking. [14, p.150] [...] Finally, what we see when we observe in Japanese elementary classrooms looks quite different from what apparently goes on in Japanese secondary schools, where students talk little and where the focus is on rote memorisation. [14, p.152]

3. How to change Belief system? Alternative Instruction and Course design

One possible strategy to change belief system is to use CAS as a constructive problem-solving tool. This project is based on a CAS supported problem-oriented approach, therefore leading also to curriculum design questions. Following the advice of WEINERT [15] in *Claims on Learning in this time*, the didactic principles of FREUDENTHAL [3] in *Revisiting Mathematics Education* some design decisions are [8, p.12]:

- *central concepts* and techniques of elementary Linear Algebra should be developed successively from interesting stimulating ample problem contexts
- The *individual concept net* ('mindmap') of the learner should be constructed using fundamental ideas, central mathematising patterns and local solving strategies
- *insight* should be gained via informal, visual argumentation and presentations using CAS supported investigations and explorations of every-day life problems
- *metacognitive reflection* on learning process and problem-solving methods should be stressed (see BUCHBERGER's creativity spiral in [5, p.82]).

The course content is mainly based on the following four alternative course concepts. ARTMANN/TÖRNER [1] focuses on application oriented view of eLA and use matrices as a universal and unifying tool. STRANG [12] emphasises the visual aspect of learning mathematics. MÖLLER [9] tries to eliminate pure existence proofs by using algorithmic constructions instead and stressing algorithmic thinking (e.g. processual thinking). Using recommendations of the US Linear Algebra Curriculum Study Group, LAY [7] recommends early introduction of key concepts, visualising fundamental ideas through geometric intuition/interpretation and stresses the impact of the computer on both the development and practice of LA in science and engineering. But due to different reasons none of these concepts makes use of a CAS as fully integrated didactical tool.

This alternative CAS oriented Elementary Linear Algebra Course design also takes into consideration the recommendations of US Math education reform, as fixed in two didactical axioms by PLETSCH [11]:

- **Rule of Four** [better: Five, wL]: (re)present every topic *numeri*cally, *graphi*cally, *analyti*cally (algebraic), *descriptive*ly [and *CAS*(ually), wL]
- **Way of Archimedes**: formal definitions and procedures evolve from the investigation of practical problems and experiments.

Besides promoting the use of CAS, he [11, p.292-293] warns:

Fundamental to successful computer classroom lecturing is that material must be covered thoroughly enough that it need not be retaught in a conventional classroom later. [...] Computer classroom lecturing is a difficult conversion from traditional blackboard lecturing because the medium has changed and hence so must the message.

DUBINSKY [2] proposed in his C^4C (=Calculus, Concepts, Computers & Cooperative Learning) project an **A.C.E. cycle**: Activities in lab computer environment, followed by Class discussion with modified Socratic approach in small groups and traditional Exercises to reinforce material. But we also often use a modified C.A.(E±C). cycle or BUCHBERGER's epistemological orientated *creativity spiral*:
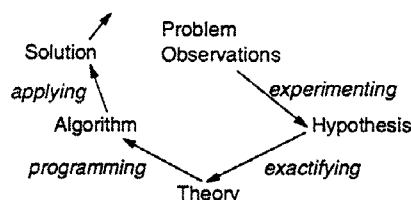
The strategic goal oriented professional software construction on a large scale using *software construction spiral* [10, p.290ff] as new paradigm (instead of the so called *waterfall model*) also shows profond similarities to the problem solving process in mathematics.

For details concerning design decisions for fundamental ideas in eLA in this study and their strategic use as central mathematising patterns (e.g. concepts, suited as model for predicting or

explaining) or local context specific problem solving strategies (besides general heuristic strategies) or in reducing complexity see [8, p.22-28].

4. Research Questions

In the above-mentioned circumstances I pose the following research questions:
- How are central basic concepts/intuitions (e.g. concept of inverse matrix) and methods (e.g. GAUSS-algorithm) of elementary Linear Algebra developed by students using fully integrated CAS with informal visual representation/argumentation and stressing algorithmic/ constructive genesis of concepts? (*cognitive system aspect*)
- How do the mathematical belief system and the self concepts of students change in such CAS supported ample learning environments? (*belief aspect*)
- What impact does a CAS as learning environment and didactical tool have on the logical/argumentative reasoning (e.g. existence/unique questions of inverse matrix), the explorative learning or problem solving behaviour and the attained skills? (*processual aspect*)

This research plan will be realised through constructing CAS-redesigned and resequenced instruction material (to enable reproduction), using it in a pilot study in a long-time learning process of approx. one year until 2000/01 fully audio monitored. A test phase with other teachers will follow in 2001. GRIGUTSCH'S questionnaire will be given to all groups at the beginning and the end of this course; first results are expected in the summer of 2001.

5. First Steps in Implementation

The instruction material will be presented to the students as prestructured problem sheets to enable individual enlargement by students or teachers e.g. CAS-notebooks with solutions. The whole material will be published together with the results of this research. I will sketch three examples of CAS-integrated learning environments to illustrate some of the above ideas.

Ample Learning Environment I: Digraphs [8, p.31-49]

A model situation of street maths (distance table of Sicily/Italy) is used to introduce the concept of matrix and the corresponding set of elementary matrix operations. Digraphs (i.e. directed graphs), adjacency matrices and digraph-operations abstract the model situation of a symbolic 'town plan' with 4 linked positions A, B, C, D allowing different interpretations (bus-, train-net, preference-relation,..) for the students:

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = M$$
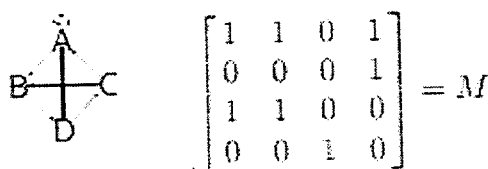
Figure 2: Digraph and associated adjacency matrix M with 0/1 entries to represent '..linked/not linked..'. The entry $M_{12}=1$ is interpreted 'A is linked to B' and entry $M_{34}=0$ is interpreted 'C is not linked to D'.

In contrast to the standard introduction of matrices the necessity to reduce the result of a matrix operation to 0 or 1 in order to ensure that the result is again an adjacency matrix requires the implementation of the operations in CAS, because only the standard matrix operations of addition, multiplication etc. are predefined.

If two adjacency matrices X and Y are added, every resulting non-zero element of X+Y is set equal to 1 giving in effect the increase of links in combining both digraphs. Subtraction X-

Y generates a digraph in which all links common to X and Y are eliminated. The elementwise matrix multiplication X·Y selects links only common to both X an Y. Especially interesting is the interpretation of usual matrix multiplication X*Y ('folding'). In the special case of repeated multiplication $M^n := M*...*M$ (n times) of the adjacency matrix M of a digraph by itself the exponent n measures the *length* (number of edges) *of a path* and the (nonreduced 'weighted') matrix entries itself gives the *number of the paths* joining two positions in the digraph. This leads to the concept of 'Pathmatrix':

$$\text{\#1:} \quad \boxed{\text{PATH(A)}} := \quad \overset{\text{DIMENSION(A)}}{\underset{i=1}{\Sigma}} \quad A^i$$

$$\text{\#2:} \quad \text{PATH} \, , \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 5 & 9 \\ 2 & 2 & 1 & 3 \\ 6 & 6 & 4 & 6 \\ 3 & 3 & 3 & 4 \end{bmatrix}$$

Figure 3: How many ways of arbitrary length lead from B to A in the above adjacency matrix M in Fig. 2? The path-matrix entry PATH(M)$_{21}$=2 gives the answer. These two prognosed paths are to be seen in the digraph of Fig 2: B>D>C>A and B>D>C>A>A (watch the sling as link!).

Thus the pathmatrix is a useful operation, but hard to calculate per hand (many iterated multiplication's and additions); the students strongly feel how useful CAS is in this situation to delegate work. I mention that the CAS-implementation process of the digraph(matrix)-operations is comparable with POLYA'S 4 phase problem solving cycle and leads to a semi (or fully) automated calculation [8, p. 41-45]. I often use simplified *semi*automatic CAS-functions to focus on the crucial step in programming (=construction) process, to avoid involvement in administrative programming specific technical considerations (F-belief aspect!) and to save simplicity and clarity of the approach. So *in contrast to a traditional CAS-free consideration we gain a **constructive runable concept** of a pathmatrix.*

Learning Environment II: concept image of the inverse matrix [8, p.50-63]

A given linear system A.X=B can be written and visualised in 3 different ways: by rows, by columns and by matrices (in two forms). Each representation has its merits and the *ability to switch between different representations is central mathematical competency*, because it raises insight and links different concepts - and often leads to new concepts:

$$\begin{bmatrix} 1x+2y = 3 \\ 4x+5y = 6 \end{bmatrix}$$ Row image

$\downarrow$ 1. compression via concept of vector (addition)

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} x + \begin{bmatrix} 2 \\ 5 \end{bmatrix} y = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$ Vector image

$\downarrow$ 2. compression via concept of matrix (multiplication)

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$ Matrix image

$\downarrow$ 3. compression via symbolisation

A.X=B

$\downarrow$ ...problemsolving via concept of analogy [1, p.10] *using CAS*

$$X = A^{-1}.B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$\downarrow$ ...and abstraction gives

●   Concept image

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -5 & 2 \\ 4 & -1 \end{bmatrix}$$ of **inverse** A^{-1}

Figure 4: Stages in forming the concept of inverse matrix: at the critical point ● the 'grundvorstellung' [6] or the concept image of the 'inverse' in the sense of TALL/VINNER is constructed and the inverse matrix is encapsulated from the dynamic process into a static mental object [2, p. 101].

What is new in contrast to a traditional consideration without CAS [1, p.42-43]? *Firstly* after transforming the given linear system into condensed symbolic form A.X=B the students can *experiment* in order to solve this matrix equation with CAS e.g. trying X=B/A (which is correct in MatLAB in the form A\B) or X=B.A^{-1} etc. leading to fruitful discussions. *Secondly* the students realise that the concept A^{-1} is existent as mathematical object in the memory of the CAS, so it can be *used* to explore properties of the inverse or to apply the new concept in diverse problem solving situations e.g. LEONTIEF-Input-Output modelling [7, p. 148ff]. *Thirdly* being convinced of the utility of the concept of inverse, students can re*construct* it to make the CAS-black box A^{-1} white via the following:

Learning Environment III: animated visualised GAUSS-algorithm [8, p.72-75]

In contrast to the traditional formal equation view of the famous GAUSS-algorithm to solve systems of linear equations we propose a CAS-animated visualisation. This visual graphical representation of GAUSS-Algorithm is being explored and investigated by the students in an open situation in 2D and 3D, leading to a *runable construction* of the inverse matrix and giving a starting point to deeper analysis of linear systems, e.g. the L.U-decomposition of a matrix.

To sum up: CAS is not only a mathpad, but a mathematical problem solving tool. *Using CAS fully integrated (and not only for demonstration purposes) in the right way should therefore transform traditional instruction and classroom formats to more experimental problem-solving habits and should enable constructive concept images leading to a more positive math belief structure.*

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Changes in pre-service primary school teachers' beliefs about teaching and learning mathematics

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Abstract

   The purpose of this study was to study the changes in pre-service primary teachers' beliefs during a mathematics methods course. The different class activities undertook by the student teachers were developed to influence the pre-service primary teachers' overarching conceptions about nature of mathematics, and about teaching and learning process. We studied the complexity of changes in pre-service primary teachers' beliefs across different domains The pre-service elementary teachers responded a Belief Scale two times, beginning and end of the mathematics methods course. The Belief Scale consisted of 28 items designed to assess beliefs in four domains: mathematics as a school subject, mathematics teaching, how mathematics is learned; and the role of mathematics teacher. Respondents rated each item using a 7-point Likert scale. We used several multivariate analysis to examine changes in the pre-service primary teachers' beliefs. We analysed the changes in the beliefs from the perspective of Green's theory (1971) and we think that the results obtained provide us with a new understanding of the learning and the development of pre-service primary school teacher.

1. Introduction

   In the literature on learning to teach, pre-service teachers' beliefs are considered as important factors in their professional growth (Pajares, 1992). Although there are a lot of meanings of beliefs (Furinghetti & Pehkonen, 1999), it is basically assumed that beliefs are constructs that describe the structure and content of mental states that are thought to drive a person's action. In the learning to teach it also is important both what student teacher believes and how he/she believes it (Green, 1971). Beliefs that student teachers bring to a teacher education program strongly influence what and how they learn and are also targets of the change within the process of learning to teach (Borko & Putman, 1996). On the other hand, the student teachers evaluate their beliefs in the new situations in which they are (compare their beliefs with new experiences and with beliefs held by others) and therefore the beliefs can change, develop or strengthen. Some research questions are focused specifically on the issue of changes in student teachers' content and structure of beliefs during teacher education programs (Philippou & Christou, 1998).

   This paper focuses on one study that examine changes in student teachers' beliefs during a mathematics methods course in the third year of their primary teacher education program. This mathematics methods course belongs to the primary teachers education program at the University of Seville. The cohort of student teachers had had a 50 hours mathematics course in their first year in the program. This mathematics course is focused on general mathematics. The content of the mathematics methods course was designed to focus on four topics: problem

solving, notion of school problem, problem posing and problem solving teaching. Different class activities were developed in a 10-week mathematics education course (30 sessions with a last of one hour and half.

2. Method

Questionnaire

In this study a questionnaire (belief scale) was developed to measure the student teachers' beliefs about four domains: A. Nature of school mathematics; B. Mathematics teaching; C. Mathematics learning; and D. The role of the mathematics teacher.

Each of the domains was described from two perspectives, the traditional and the constructivist. We adapted some items from different studies focused on the measuring of beliefs about teaching-learning of mathematics (Klossterman & Stage, 1992; Peterson eta al., 1989; Schoenfeld, 1989). Others items were newly formulated (see in Llinares et al (1995) the validation and the test of the set of belief scales obtained).The final questionnaire consisted of 28 items that were randomly arranged to avoid recognition of the perspectives and the domains. Respondents rated each item using a 7-point Likert scale ranging from "strongly disagree" (1) to "strongly agree" (7). The 28 items described the characteristics of a traditional view and a constructivist view in each of the four domains. The pre-service primary teachers responded to the Belief Scale two times, before and after of the mathematics methods course. Then, student teachers' responses were compared.

Subjects

Sixteen student teachers (7 female and 9 male) had entered in the experimental course. The student teachers had undergone a five week period of general practice teaching during their second year in the program, but they had not been yet in their period of mathematics education practice teaching.

Data Analysis

We used two multivariate analysis to examine changes in the pre-service elementary teachers' beliefs. A hierarchical cluster analysis and a factor analysis with varimax rotation were executed to look empirically for the different cluster and factors that underlie the answers to the questionnaire. On the basis of six factors yielded by the factor analysis (with an explained cumulative variance over 60 %), the items with factor loadings below .50 were removed.

3. Results

Hierarchical cluster analysis

The clusters that included all the items were more rapidly formed (at a shorter distance) after than before of the course. Either before and after the course, the clusters that are formed in the first place reflect a traditional beliefs system. Data show that the pre-service primary teachers have defined their position more clearly after the course as regards the nature of mathematics, the learning-teaching process, and the role of the teacher as identifying the two perspectives contemplated: the traditional and the constructivist.

* how are these items grouped in the formed clusters?*

Before the course.- The cluster that groups constructivist items with some more traditional beliefs on the nature of mathematics is formed by joining together two beliefs systems. The first of them is formed by 12 items and describes the *teacher's goals* (the role of the teacher is not to repeat exercises of the same type) as a dichotomy relation acceptance-rejection as regards **classroom management** (organising discussion without bothering about class order) and the **learning-teaching process** (focusing on comprehension and on the exchange of the ideas, and not so much on the number of right answers). This beliefs system incorporates some features of school mathematics (*school mathematics do not consist of rules and procedures to memorise, and formal proof are not such a basic element*).

The second beliefs system, that gathers constructivist perspective items, is made out of 4 beliefs, three of which deal with the nature of mathematics and another one with learning. This beliefs system characterises **what mathematics are** (a set of rules and procedures useful to solve everyday life problems and that always have a rule that can solve them). It also characterises **how mathematics should appear** in the classroom (solving problems for which no procedure is previously known). This beliefs system is supported by the belief that one can have good marks in mathematics without memorising rules and formulas.

After the course.- The cluster that groups constructivist beliefs is formed later than the traditional cluster. This constructivist cluster appears when a belief-item related to the role of the teacher (9;D42, *keeping order and firmness in the classroom should not be the teacher's main concern*) is added to a previous cluster that had already grouped all the rest of the constructivist belief-items.

This fact is worth noting. The belief that relates the role of the teacher to keeping order in the classroom is the last one to be incorporated to a cluster that groups all the beliefs that characterise a more innovative approach to the teaching of mathematics. It is also worth noting that its dual belief (16;D41, mathematics teacher must lead the class with steadiness, keeping order to give the lesson fluently) is the last one to incorporate to the traditional cluster as well. This probably suggests that any question related to control of discipline and classroom management are beliefs in which pre-service primary teachers do not hold a very clear position. That is to say, after the course they do not have a clearly positionation about what the role of the teacher should be as regards classroom management and discipline from a constructivist or traditional point of view.

Factor analysis

The factor analysis allows us to identify the references that the group of pre-service primary teachers uses to interpret (give a meaning) to different beliefs. The changes in the number, explained variance and content of factors will indicate the nature of the changes in the pre-service primary teachers' beliefs systems. The content of the factors will show us through which domains the preservice elementary teachers interpret a given situation.

Data show that the same amount of variance is explained by a smaller number of factors after the course. Whereas the initial questionnaire shows that the 6 first factors (defined according to 20 beliefs) explain 63.111% of the variance, in the final questionnaire a similar amount of variance (61.114) is explained with 4 factors, the content of which is defined only by 15 beliefs. Furthermore, in the final questionnaire, the first factor explains a forth of the total variance. This may indicate that, after taking the course, the preservice elementary teachers had built a better defined reference system, in the sense that they were able to interpret situations with a more reduced number of beliefs.

In relation to the content of factors, a first reading of this data may show that, after taking the course, an important reference for the pre-service elementary teachers (1st factor to explain more that a forth of the variance) through which they interpret other beliefs, came from the way they should deal with classroom procedures, from the point of view of the teacher and the teaching process. Nevertheless, before the course, the most important reference (1st factor to explain a fifth of the total variance) was the connection between the role of the teacher and learning features.

Another important change as regards this first factor is that the beliefs about the role of the teacher that provid meanings before the course (D31, D41, D42, the teacher must firmly conduct the lesson and set out many repetitive exercises so that students can memorise the steps), are no longer significant after the course (in fact, they only hold a significant role when defining factor 6 after the course, which explains only 6.595% of the total variance, when the rest of the factors already explained 67.992% of the total variance). Furthermore, after the course, the beliefs about learning are less likely to give a meaning to the pre-service primary teachers beliefs system, since only 2 beliefs about learning are kept in the definition of the 2nd factor.

4. Discussion and implications

Our findings indicated different types of changes in student teachers' beliefs. The evidence of the change in student teachers' beliefs came from the two multivariate analysis whose results we presented in the above section. The changes in student teachers' beliefs consisted in a better identification of constructivist view of school mathematics, and a different definition of the reference system of beliefs. These two types of changes are related to the two of the characteristics of the beliefs system: what and how they believe.

The detected changes in the speed of clustering show a more integrated and better differentiated beliefs system. This allows for a better characterisation by pre-service primary teachers of the traditional and the constructivist views. Another aspect that is worth noting is that the last beliefs to be incorporated to each of these large clusters (traditional, constructivist) correspond to the domain of the role of the teacher and, more specifically, to keeping order in the classroom. After the course, their beliefs about the way the class should be controlled by the teacher, both from the traditional and from the constructivist perspectives, are the last ones to be incorporated to their respective clusters. This leads us to think that the preservice elementary teachers are concerned about classroom management, and especially, about discipline. This probably shows the pre-service primary teachers' concern in connecting their beliefs about classroom control and discipline with other beliefs linked to teaching, learning and the nature of mathematics.

Considering factors as the reference that provides a meaning to most of the beliefs (the way in which some beliefs provide meaning to the rest), the fact that after the course the same variance is explained through a smaller number of factors shows a better structure of the beliefs system of the student teachers. This is suggested by the fact that a smaller number of factors gives meaning to a similar number of the beliefs. Addition, it is worth mentioning that as regards the content of the beliefs system, there was a change from the beliefs about the role of the teacher and learning into beliefs about teaching and role of teacher.

Using Green's theory (1971) as a framework, in relation with the dimensions that feature beliefs systems (from the point of view of the way in which beliefs are held), in this research we assume the idea of the existence of a beliefs system held by the group of pre-service primary teachers, moving from the beliefs system of an individual into the beliefs system of a group that

is culturally identified. In order to deal with the features of beliefs systems, Green introduces three dimension that refer to the way we belief: quasi-logical relations (where he distinguishes between primary and derivative beliefs), psychologically centred beliefs (that lead to the concept of more or less strong psychological beliefs, from the point of view of how they are supported), and the idea of the cluster.

Granted these three characteristics, our data allow us to think about one aspect of the quasi-logical dimension and about clustering dimension. As regards the former, a beliefs structure with a smaller number of factors and the change of its content, allow us to think the course has affected their primary beliefs (considered to be the factors content) from which pre-service primary teacher make sense to other beliefs (derived beliefs). This interpretation questions the widely assumed belief that modifying the pre-service primary teachers' beliefs is very difficult. However, what we cannot measure is the duration of this change. Nevertheless we can point at the need that these pre-service primary teachers continue to develop activities that allow them to reinforce their new beliefs system, initiated during the course.

In the second place, the way the course has influenced cluster formation leads us to think that the traditional and the constructivist clusters generated allow to link the beliefs about the nature of mathematics, the learning-teaching process and the role of the teacher, provided in some of these perspectives. This can help pre-service primary teachers to grant a coherent meaning to the situations they come across. Still unsolved is the question of whether their action will derive into one or the other beliefs cluster and the issue related to keep order in the classroom.

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The interest and difficulty of the mathematical problems as pupils' belief in Hungary

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Abstract

This talk deals with pupils' conceptions about interest and difficulty of the mathematical problems. It consists of the choosing of this topic, the framework of my talk, research method and problems, some results, and summary of it.

1. The choosing of this topic

The development of problem solving plays main roll in mathematics education to introduce by the new mathematics curriculum in Hungary. Among the development methodological ways is one of the most important rule is to practice the own problem solving and to develop the ability of the mathematical tasks solving.

The motivation is a fundamental condition to practice of the thinking. If the pupils known the rules, definitions, theorems to solve problem then they will not solve it because they are not interested in àt all. So it means that teachers have to use psychological aspects making the problems. I think so that two psychological aspects are very important. The first is the difficulty of the tasks. That is the best if this is not too easy and not too difficult, the pupil's knowledge is enough but they have to think because they are not able to solve it without thinking.

The second aspect is the interest of the mathematics problems because it helps them to make a basic motivation system.

The teachers have to create mathematics problems sequences (for competitions, exam tests for mathematics lessons). They have to select them and create some new ones at all levels of the mathematics education. These are the real reasons of my choosing of this topic.

2. The framework of my talk

I have read some papers from G. Törner, E. Pehkonnen, B. Zimmermann, A. H. Schoenfeld, R. Borasi, K. Tompa. As far as I known their research work deals with MAVI anyway. (I would like to thank you for E. Pehkonnen. He sent me some papers and articles which were very useful for me.)

I'm interested in the problem solving and pupil's view, but my method is not same as they have used.

I was a member of group which organised a math. competition for 10-15 ages pupils we have created the problems. Our group planned that let all problems be interesting and first will not be too difficult, the second more difficult and the last the most one.

3. The participants pupils and schools

There was a mathematics competition in 1998-99 term. It was organised by our group (which consists of 5 professional maths. teacher) in Mezocsat and Tiszaujvaros. 784 pupils took part in this competition. It is called "With more mind..." Pupil's were 5-8 grade, from 10-15 years old, from different elementary schools such as towns, villages.

Table 1. The table of the participants data

Grades	Boys	Girls	Sum total
5 th	101	87	188
6 th	106	74	180
7 th	102	84	186
8 th	69	50	119
Sum total	376	297	673

4. Method and our purpose

We used a test which consisted of 4 opened mathematics problems and a short questionnaire on it. The solving time was 60 minutes for all grade. When the time was up we asked the children to give scores for the interest and difficulty of all problems next way:

1. If they think that is ... .
    -nqt interesting: give 1 score,
    -a little interesting: give 2 scores,
    -interesting: give 3 scores,
    -more interesting: give 4 scores,
    -the most interesting 5 scores.

2. If they think that is ... .
    -not difficult: give 1 score,
    -a little difficult: give 2 scores,
    -difficult: give 3 scores,
    -more difficult: give 4 scores,
    -the most difficult: give 5 scores.

Now I would like to show you some results deals with 5th grade.

5. The problems for 5th Grade

1. The ant is so strong that can carry an object which is 50 times heavier as own mass. Assume a pupil is 3 times strong as an ant. How many horses can he carry. We known that the mass at pupil is 36 kg, the mass of the horse is 450 kg?

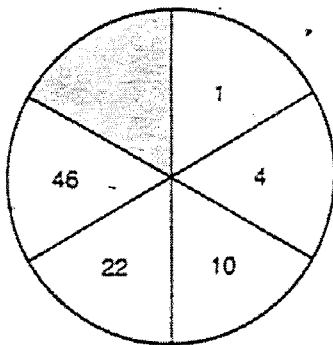2. Create rules and fill in the missing numbers. (See figure 1.)

a)

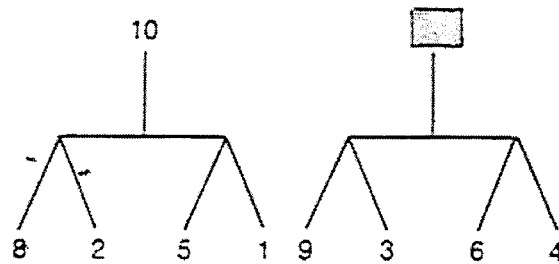| 9	81	90
8	64	72
5		

b)
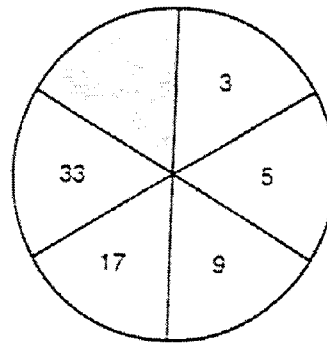
c)

d)

Figure 1

3. A little elephant and the big mouse take part a running competition. The distance is 2400 ms long. They live at the same time from the start line. After 10 minutes the little elephant will get to 2/5 part of the way and the big mouse by 100 ms fair from the 3/8 part. Which of them take larger distance? How many distances was of them after 20 minutes?

4. The volume of the rectangled prism is 18 dm^3. The length of the edges are all integers. Give the all rectangled. Determine the length of the edges count the area of all prisms.

6. Some results and summary of it.

Table 2. Data of Interest

Interest	Sum	Mean	Deviation
1.	752	4.000	0.953
2.	762	4.053	1.122
3.	704	3.744	1.249
4.	639	3.398	1.461

We can read out of the 2 table that pupils give high scores for interest of all problems. Table 2 shows the problems in order of interest. The most interesting is 2. problem the 1, 3. And the last is the 4. It means that pupils are interestid arithmetic, algebraic and functions. They are not interested in geometric calculation problem.

Assume that the background of the reason is new curriculum. This topic is not so main part of mathematics education nowadays. The pupils' answering shows the hierarchical picture. We can find the reasons of this fact is in the special ages, characteristics. The pupils haven't enough critical sensitive in this ages.

Summary: The pupils conceptions' show the professional teacher's conceptions about interest.

Table 3. Data of difficult

Difficult	Sum	Mean	Deviation
1.	508	2,702	1,299
2.	538	2,862	1,388
3.	656	3,489	1,322
4.	704	3,744	4,000

Maximum scores: 940   N=188

We read out of 3. tables that the problems in order of difficult as pupils think. The 3. table shows that 1. and 2. problems are not difficult and the 3. , 4. Problems are more difficult as on an average. Pupils think that 4. problems is much more difficult as the first one.

Summary: Pupils' view shows the real difficult order of the problems.

Table 4. Pupils' achievement

Scores	Sum	Achievement (%)
1.	1504	35.50
2.	1880	48.29
3	2256	31.21
4.	2632	19.18

N=188

Using of the scores we can determine the pupils' achievement. It can be seen that percentage is above 50. The best is 2. problem with 48.29%, next is the 1. with 35.5%, the third with 31.21%, and the last is 4. with 19.18%. These results mean the real difficult levels of the problems.

Correlation values between achievement scores and difficult

1S – 1D	-0,191**	$P < 0.01$
2S – 2D	-0.047	non sign.
3S – 3D	-0.060	non sign
4S – 4D	-0.192**	$P < 0.01$

The negative correlation values mean that pupils achievement scores are lower in those problems from which they think that are difficult. It means that we can give approximately the pupils achievement if we now their conception in advance.

Correlation values between achievement scores and interest

1S – 1I	0,204**	$P<0.01$
2S – 2I	0,234**	$P<0.01$
3S – 3I	0.255**	$P<0.01$
4S – 4I	0.046	non sign

The positive correlation values mean that pupils achievement scores higher if they think that the problem is interesting for them.

Summary: The pupils' conceptions help to choose the math problems for the teachers. We think so that the difficult and the interest are very important psychological aspects of the making of mathematics problems.

What kind of mathematics for prospective primary school teachers?

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Abstract

In this paper I shortly describe those things I have learnt while trying to find out what mathematics prospective primary school teachers should learn during their teacher education.

Main result is that more important than pure mathematics seems to be knowledge of how to teach mathematics and how children learn mathematics - and then, mathematics (contents) which is important and meaningful for future teachers arises from problems of teaching and learning mathematics.

Background information

In Finland primary school teachers work at grades 1-6 where pupils are from 7 to 12 years old. They are "classteachers" and so they have to be able to teach every subject and, for example, primary school teachers teach over 70 % of mathematics which is taught to children during compulsory education.

Future teachers do their studies at Universities, at Departments of Education and their studies include min 160 weeks of studies (one week of studies = 40 hours of work). Mathematics has 3-4 of those weeks of studies, compulsorily. Then students have a possibility to choose 15 weeks of mathematics, optionally, and these studies are offered by Department of Mathematics.

I have been mostly interested in those optional courses - what should they include? What mathematics could help a future teacher to be a better teacher of mathematics?

Step 1

In the beginning the question was: 'What mathematics?'. I had an idea that mathematics exists in different forms at different levels; there are lower, middle and higher versions of the same mathematical thing. For example, children can concretely play with a model of Towers of Hanoi but in the same time those towers include some high level mathematics. And I assumed that I can go somehow from top to down - that means to take things taught in university level and find out how those things are used in primary school.

Based on that assumption, I interviewed ten students who had done optionally 15 weeks of studies in mathematics and asked them 'which parts of that mathematics they had learned on courses, they found to be useful when they taught at school?'.

When listening trough interviews, I was first rather disappointed because there were basically only two little things which students have found to be useful: knowledge of different number systems has helped them when teaching adding, subtraction, multiplication and division in ten-based system and knowledge of polygons and polyhedrons has helped when teaching those same things to pupils.

Two little things from 15 weeks of studies - I though that was nearly same as nothing. BUT, something was different than before: These students said they like mathematics, they are willing to teach it and want to show to their pupils that mathematics is something else than rules, routines and mechanical calculations, and that mathematics can be fun and enjoyable.

So it seemed to be something in the way they thought about mathematics, about ways of teaching mathematics and something in their attitudes which had changed - and I begun to call that kind of things 'view of mathematics'. After awhile I found out that some others had also noticed that not only pure knowledge but also other things, called persons mathematical beliefs, influence strongly when she/he is teaching mathematics.

Step 2

In Ernest's model (Figure 1) a view of nature of mathematics provides a basis for teachers' mental models of the teaching and learning of mathematics. Ernest is well aware that social context can disturb this system, but the main idea is that practices cannot be changed without a change in the individual's view.
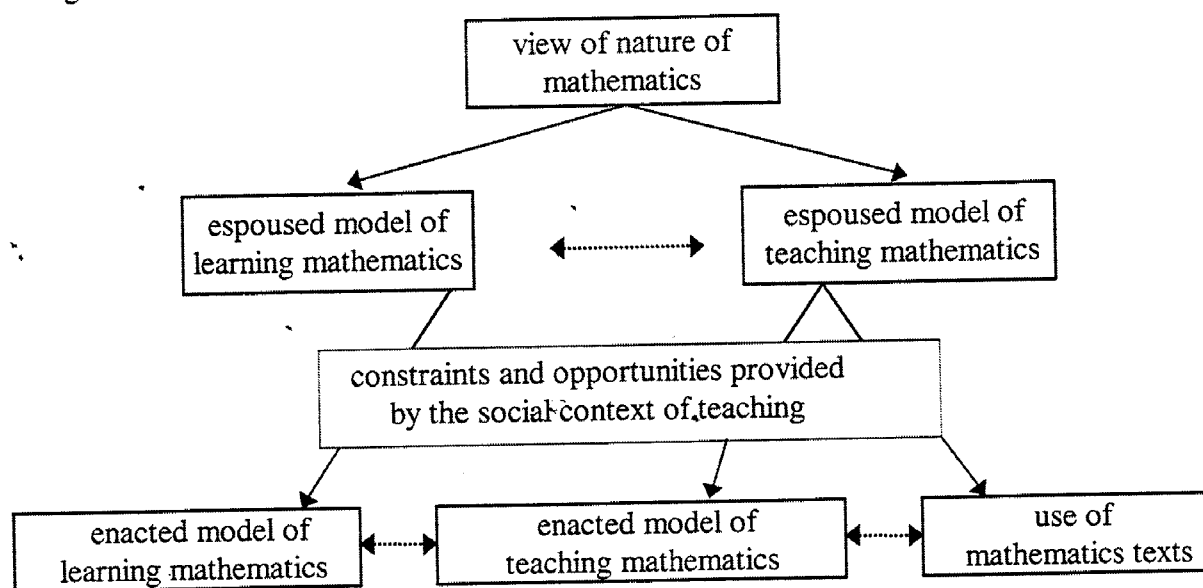

Figure 1: Relationship between Beliefs, and their Impact on Practice (Ernest 1989, 252)

Ernest uses a 3-level model for teachers' views of mathematics:

1. Instrumentalist view: mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts.

2. Platonistic view: Mathematics is seen as a static but unified body of certain knowledge. Mathematics is discovered, not created.

3. Problem-solving view: Mathematics is seen as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of inquiry and coming to know, not a finished product, for its results remain open to revision.

The descriptions below give a specification as to how these views are related to practices through the teacher's role and the intended outcome of instruction:

Teacher's role	Intended outcome
1. Instructor	Mastery of skills with correct performance
2. Explainer	Conceptual understanding with unified knowledge
3. Facilitator	Confident problem-posing and problem-solving

The use of curricular materials in mathematics is also of central importance in a model of teaching. Three patterns of use are:

1. the strict following of a text or scheme;
2. modification of textbook approach, enriched with additional problems and activities;
3. teacher or school construction of the mathematics curriculum.

(Ernest 1989, 250-251)

These three philosophies of mathematics, as psychological systems of beliefs, can be conjectured to form a hierarchy. At the lowest level is instrumentalism, at the next level is the Platonist view and at the highest level is the-problem-solving view. One can also say that the two first ones represent traditional mathematics teaching and the third one represents modern mathematics teaching, and naturally the goal is that future teachers have the modern view. But how can they reach that kind of view and with what mathematics?

Step 3

After listening interviews couple of times I got some hints what things at mathematics education could be such that they develop prospective teachers' view of mathematics to "right" direction.

A) The first conclusion was that use of concrete and visual representations was something which students had found to be very helpful, useful and worth for using also in their own teaching. Thus knowledge of ways of representing mathematics seemed to be important mathematical knowledge for a teacher. This item is included in Shulman's concept **Pedagogical content knowledge:**

"It includes the most regularly taught topics in one's subject area, the most useful forms of representation of those[content] ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word, the ways of representing and formulating the subject that make it comprehensible to others.
It also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students for different ages and backgrounds bring with them to learning".
(Shulman 1986, 9)

B) The second conclusion was that concerning to some contents, students were more conscious of their own metacognitions and also of children's cognition; the problems in their own learning had taught them what problems children might have and when they were teaching they were also more open to listen to children's ways of thinking. This item is seen in Petersons (1988) framework, where she argues that in order to be effective, teachers of mathematics need three kinds of knowledge: How students think in specific content areas, how to facilitate growth in students' thinking and self-awareness of their own cognitive processes.

C) The third conclusion was that students had learnt also some mathematics - mathematics which was rather difficult to specify - not particular contents or subjects of mathematics but

some knowledge which Shulman (1986) calls **Content knowledge or subject matter knowledge.** Turner-Bisset (1999) has specified Shulman's concept and divided it into two parts:
1) *Substantive subject knowledge* consists of the facts and concepts of a discipline, and the organising frameworks used to marshall what may appear to be a profusion of disparate bits of information
2) *Syntactical knowledge* ways and means by which the propositional knowledge has been generated and established.

Step 4

Thus there are three different kind of knowledge which are essential for a teacher: knowledge of mathematical thinking and learning (teacher self and pupil), knowledge about teaching mathematics and knowledge about mathematics. These three parts can be represented simply with the following triangle

"teaching"

"mathematics" ———— **"learning"**
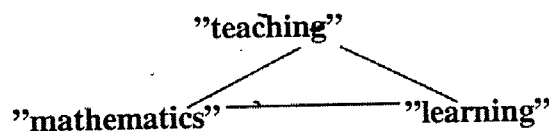
Or, with the terminology which Repo (1996) uses in her model (figure 2) 'Basic elements for the constructivistic learning environment': *knowledge construction, didactic construction* and *mental construction.*
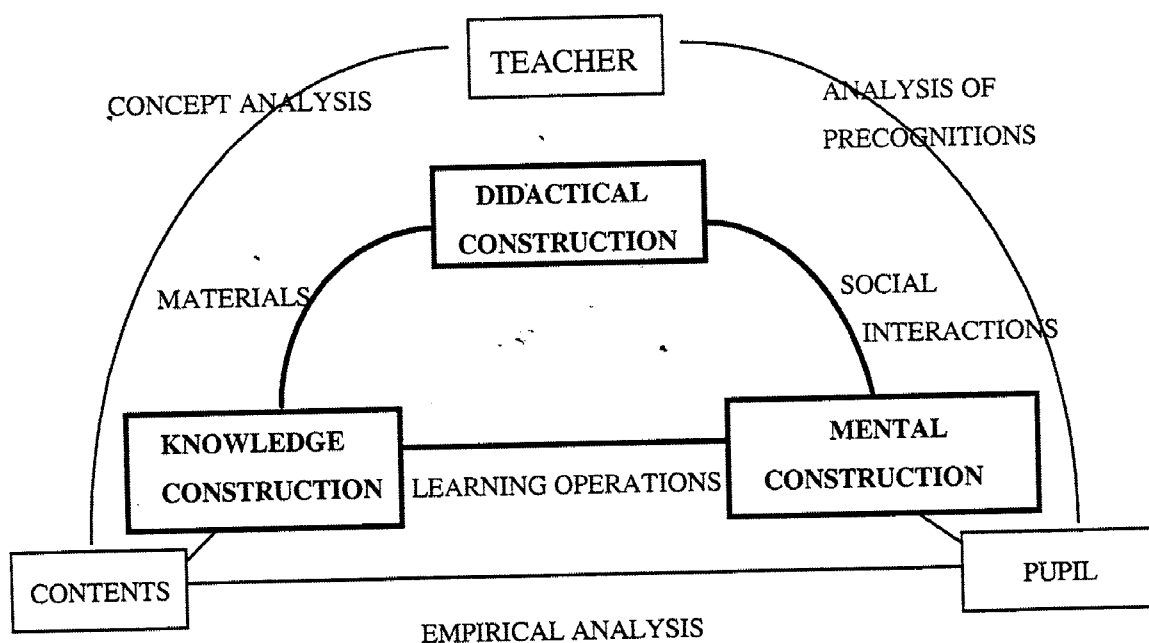

Figure 2: Basic elements for the constructivistic learning environment (Repo 1996, 182)

Step 5

The main conclusion is that "mathematics" for a teacher is mathematics which is very tightly related to teaching and learning of mathematics. Its roots are on contents which are meant to teach to pupils and it starts from problems and questions of teaching and learning mathematics. Thus it is not so reasonable to go from top to down but viceversa from down to top.

But - when we start from problems and questions of teaching and learning we probably find the whole field of mathematics if we go far enough. So the questions are: How far should a primary school teacher go? Which are the most important things she/he should find? OR as the question was in the beginning: What mathematics?

Step 6

I must admit that I have not found an answer to the question I had in the beginning, but I have learnt alot and if I reformulate the question to be What kind of mathematics? then I can give one answer: Mathematics which starts from problems of teaching and learning mathematics, and this kind of mathematics I call **'didactic mathematics'**.

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Efficacy in Problem Posing

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Abstract

We examine the efficacy beliefs of prospective teachers' with respect to problem posing (PP). Specifically, we used questionnaires and interviews to study the structure of efficacy beliefs in PP, and to examine the relationship between efficacy in PP, in teaching PP and performance in constructing problems. We found that high efficacy students were more able to construct problems of more advanced complexity than low efficacy students. Significant differences were also found in the level of efficacy beliefs in terms of the subjects' background, prior involvement in related tasks, and gender.

The affective system and problem posing

After decades of research at the international scene, we have some understanding of the role played by affective variables in human development. Schoenfeld (1992), however, claimed that "we are still a long way from a unified perspective that allows for a meaningful integration of cognition and affect or, if such unification is not possible, form understanding why it is not" (p. 364). He encouraged the search for new methodological and exploratory frames in the effort of conceptualizing the arena of beliefs Subsequent research resulted in expert consensus that affect is an essential factor in learning, which interacts with cognition during problem solving activities. The affective system is considered as the most important among the five representation systems in a unified psychological model proposed by Goldin (1998) for mathematical learning and problem solving. The other four systems are the verbal syntactic, the imagistic, the auditory, the formal notational, and the system of planning, monitoring and executive control.

The affective system includes beliefs, conceptions, views, attitudes, emotions etc. related to mathematics and mathematical learning. Thus, if we define learning as the development of one's "competencies", then affective competencies can be learned and consequently taught in the same way as cognitive competencies (Goldin, 1998). Regarding the teaching of affective competencies, the teacher's own belief system is of primary significance. Several components of this system have been so far investigated, including self-confidence, self-esteem, self-concept, and self-efficacy. In general, feeling efficacious to perform a certain task was found to be the most reliable predictor of one's behavior in the course of achieving this task (Bandura, 1997. In particular, the ability to construct problems and one's confidence in that ability are among the most important competencies in mathematics learning (English, 1997). Teacher education programs should, therefore, enhance both cognitive and affective development of the prospective teachers.

Bandura (1997) defined self-efficacy as one's conviction that he or she is able to achieve a certain task. By analogy, teaching efficacy can be defined as ones' belief in his/her capability to

achieve learning outcomes. Self-efficacy is a context-specific construct in contrast to self-esteem, which is more global. That means that the study of teacher efficacy is more meaningful when carried out in terms of specific teaching tasks rather than in general.

Several researchers found that student's efficacy beliefs are closely related to mathematics achievement. For instance, in models using path analysis the direct influences of efficacy beliefs on students' performance were estimated to range from .349 to .545 (Pajares, 1996). Furthermore, Pajares and Miller (1994) asserted that efficacy in problem solving had a causal effect on students' performance. They found that efficacy beliefs are better predictors of performance in problem solving than beliefs about the usefulness of mathematics, the involvement of students in mathematics, students' gender and experience with mathematics.

The debate about reforming mathematics education was directly affected by new conceptions of the nature of mathematics and of the meaning of knowing mathematics. Knowing mathematics is widely identified as "doing mathematics" and learning mathematics as equivalent to constructing meaning for oneself and the ability to handle non-routine problems. In this context, PP becomes a primary factor that contributes to enhance students' ability to solve mathematical problems (Leung, 1994). Moreover, the growing support for some kind of constructivist epistemology has led to increased emphasis on teachers' ability to construct problems. Indeed, the main responsibility of teachers consists of constructing and/or selecting pedagogically rich problem situations, which facilitate a group of students to do mathematics on their own.

PP tasks may take different forms, and thus students can be involved in PP through a variety of situations. In this study, we asked students to pose problems based on a given mathematical situation, on a number sentence, or by modifying the structure, the data or the information given in certain problems. The goal was to explore relationships between the prospective teachers' efficacy beliefs on PP in each of the situations and their competence to complete the tasks. In this respect, we sought for (a) relationships between students' efficacy beliefs in PP and their ability to construct problems, (b) relationships between students' efficacy beliefs to pose problems and efficacy to teach problem posing, and (c) significant differences in efficacy beliefs PP in terms of gender, prior involvement in problem posing, and mathematical background?

Methodology

A 31-item questionnaire was administered to 115 fourth year students, during the final stage of teaching practice, aiming at the clarification of their involvement in PP and problem solving activities, and of their efficacy beliefs with respect to these tasks. Next, we selected and interviewed 25 students according to their efficacy level, ten from the low efficacy (LE) group, eight from the average efficacy (AE), and seven from the high efficacy group (HE). The interviews involved tasks similar to those in the questionnaire and were conducted by one of the researchers. The tasks and the directions to students were:

Number sentence. "Construct three different problems, which could be solved using the equation 56: 6 = n".

Mathematical situation. Construct three problems based on the following story: "Michael, Nicolas, and John drove in succession on their way back from a trip. Michael drove 80 km more than John. John drove double the distance that Nicolas did. Nicolas drove for 50 km ".

Problem modification. Construct up to seven different problems modifying the problem: "The students in a certain school were talking about their favorite singers. One fourth of them voted

for singer A, one sixth for singer B, one eighth for singer C, and one twelfth for singer D. If 90 students were undecided, find the student population of the school".

The students were given ample time to construct problems on their own; when a student got stuck, the interviewer provided appropriate hints about possible ways out. For instance, in the case of the number sentence, a regular hint was "find a problem in which the answer is not fractional", in the case of problem-modification the students were advised to "insert new information", or "change the unknown", "impose new constraints", etc. Non sensible or impossible problems were considered wrong proposals.

Results

Analysis of the questionnaires. Students' responses were factor analyzed using Principal Axis factoring with varimax rotation. A five-factor solution was identified as the most appropriate structure of efficacy beliefs (explained variance = 75%). The loadings of the items on each factor were large and statistically significant.

The first factor, which comprised of six items indicating efficacy in PP **given a number sentence or a mathematical situation,** explained 21.4% of the variance. The items comprising this factor and the loadings were "Given a number sentence, I can construct, one problem (.67), two different problems (.72), three different problems (.67). "Given a mathematical situation, I can construct one problem (.82), two different problems (.85), three different problems" (.75). The second factor consisted of seven items reflecting confidence in one's ability to pose problems by **modifying a given problem.** This factor explained 19.71% of the variance and consisted of the following items. "I can modify a given problem by adding information (.66), removing information (.78), changing the unknown (.53), changing the context (.80), changing the values of some variables (.61), using the strategy "what if" (.79), and imposing new constraints (.78)". The third factor, which reflected the subjects' efficacy to **teach** PP strategies, explained 12.58% of the variance. This factor is comprised of the following three items: "I am able to integrate problem posing in my teaching"(.86), "I am able to teach primary students how to construct problems (.81) and "I feel competent to help primary pupils construct problems on their own" (.67). The fourth factor reflected the subjects' **involvement in** PP activities and explained 11.7% of the variance. This factor consisted of the following four items: "After solving a problem, I usually consider a related problem"(.78), "During my field experience, I attended mathematics lessons, which involved problem posing" (.77), "I have dealt a lot with problem posing"(.72), and "When I am given a problem, I usually try to find other related problems" (.77). Finally, the fifth factor reflected students' **experience in** PP and explained 10.26% of the variance. This factor consisted of the following three items: "I have spent considerable time in problem solving" (.70), "During my field experience I have attended mathematics lessons which involved problem solving" (.82). "During my field experience I taught PP " (.77).

We used the five extracted factors to explore differences among students, which reflect efficacy beliefs as dependent variables and the students' mathematical background (high-school strand[4]), prior involvement in related tasks, and gender as independent variables. ANOVA showed that students from the science strand had more desirable efficacy beliefs than students in both the classical and the economics strands in all three cases of problem construction

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[4] The Department accepts students from the science section (mathematics and science), the economics section with (additional mathematics), and the classical strand (students take only the core mathematics curriculum).

(number sentence, mathematical situation, or problem modification). The same pattern was also found on the teaching efficacy factor. There were no significant differences among students from the economics strand and the students from the classical strand. Taking into account students' prior involvement in related tasks, we found that students with extensive experience in such tasks expressed higher efficacy beliefs in their ability to construct problems and teach problem posing than those students with limited or no experience. It should further be noted that all participants in general expressed significantly higher involvement with problem solving than with PP activities ($\bar{X}_{ps}= 2.01$ $\bar{X}_{pp.} = 1.66$, p < .01).

Significant differences were also found between male and female preservice teachers on the factor of teaching PP ($\bar{X}_m= 3.11$, $\bar{X}_f = 2.71$, p < .05). These differences can partially be attributed to the male students' superior efficacy beliefs in their ability to construct problems from a given number sentence over female students ($\bar{X}_m = 3.62$, $\bar{X}_f = 3.10$, p < .05). Finally, the students' efficacy beliefs in constructing problems from supplied information were significantly higher than their efficacy beliefs in teaching PP ($\bar{X}_{pp.} = 3.27$, $\bar{X}_{tpp.} = 2.81$, p < .001).

Significantly correlations were found between the factors "efficacy to construct problems" and "efficacy to teach problem posing" (r = .62, p < .01), as well as between the factors teaching efficacy on the one hand and the factors posing problem from a number sentence and a mathematical situation, and modifying a given problem on the other hand (r = .58, r = .55, r = .52, respectively, p< .01).

Analysis of the interviews. The analysis of the interviews showed that all participants realized the importance of developing PP competencies and considered problem posing as harder than problem solving. They valued, however, problem posing as the ultimate goal of mathematics learning, since "a thorough understanding of problems and problem solving is evidenced when teachers and pupils reach the level of problem posing".

The majority of the students (52%) felt uneasy when asked to construct a problem of their own (two of the LE students felt even anxious when assigned such a task). Five of the AE students said that even "listening the term problem posing makes them feel insecure", and they consider problem posing to be "a very complicated process". The majority of the HE students felt quite comfortable with the task, though one of them mentioned "I do not have any real experience and hence I would not to undertake such a task". The differences among the three efficacy groups were more obvious when they focused on PP based on a mathematical situation, a number sentence, or on the modification of a given problem.

LE students were less confident in PP than AE and HE students. Specifically, three LE students felt that they were "not well prepared to involve their students in PP activities"; they stated that they themselves "faced so many troubles" in this activity and were "not confident in undertaking such a task". Two others expressed "reservations..." and "felt more comfortable in teaching PP in the lower school grades". For instance, one of the LE subjects mentioned "what I really lack is the confidence in myself, ... that I will succeed in making a problem. My prior experience was so far in solving problems, ... not to make up a good problem to assign the to others". The AE subjects were more or less ready to pursue the task, though "they needed more experience with PP". On the contrary, five of the HE students stated that they were well prepared to integrate PP in their teaching, while two others held the same beliefs as the AE students, i.e., they said that they needed "additional experiences".

PP from a mathematical situation. We adopted the linguistic and structural criteria to classify the problems constructed by students namely (a) the type of the question, and (b) the number of important relations involved in the problem structure (English, 1997; Silver and Cai, 1996).

The problem questions can be classified as **conditional, relational, and assignment**. The conditional and the relational questions are considered as more complicated than the direct assignment questions. Further, the number of relations involved in a problem indicates the complexity of the problem in the sense that a large number of relations in a problem adds to its complexity (English, 1997. The students' answers were classified according to the above mentioned criteria.

Bandura (1997) asserted that efficacy beliefs is a good predictor of the quality of peoples' work meaning that the complexity of the problem and the type of the question should be related to the efficacy level. Indeed, we found that the LE students constructed in total 30 questions, 23 of the assignment and seven of the relational type, while the AE students constructed 17 assignment questions, four relational questions, and three conditional questions. Finally, HE students constructed eleven assignment questions, eight relational, and two conditional questions. An increasing trend was also observed in complexity as indicated by the average number of relations. per proposed problem; it was 2.33 for LE subjects, 3.25 for AE subjects, and 2.95 relations for the HE subjects. Table 1 shows indicative problems constructed by students in each efficacy group.

Table 1. Examples of problems constructed by the subjects of the three groups

	Low Efficacy	**Average Efficacy**	**High Efficacy**
Assign.	How many km did John and Michael drive?	How many km did the three friends drive altogether?	How many km did each of them drive?
Relat.	How many more km did Michael drive than John?	Did both Nicolas and John drive more km than Michael did alone?	How many less km did Michael drive than Nicolas did?
Cond.	none	If the average speed of the car were 50 km/h, how long would the journey last?	If the distance of their journey was X km, how many km would each of them had driven in order to reach their destination?

PP from a number sentence. The students had great difficulties in constructing problems given a number sentence. Most of the students explained their difficulties saying that" there is no story to start with...one has to start from the beginning, to create everything in his/her own mind". Three LE students were unable to construct problems, based on this number sentence, eliciting answers other than 9 2/6, despite hinds by the interviewer. The AE and the HE students were able to do so, but it was evident that this task was more difficult even for them.

Modifying a given problem. Constructing a new problem by modifying a given one was the second easier task for the majority of students, next to the problem situation, which was generally judged as the easiest. About 64% of the subjects initially thought to change the story of the problem, 56% to change the values of the variables or the unknown, 28% to introduce new information and 28% to delete some information. However, only two of the subjects thought to impose new constraints or extend the problem using the "what if" strategy.

Discussion

The findings of this study support the claim that the main source of efficacy beliefs comes from the individuals' experiences with similar and related tasks (Bandura, 1997). The following

extract from a LE student is quite indicative "I really lack the confidence in myself, that I will succeed in doing a problem; my prior experience was so far to solve a problem, not to find a good problem for the pupils". The science strand students (with an overall deeper mathematical education than the students from other strands) exhibited a higher level of efficacy beliefs. Males were found to hold higher efficacy beliefs than females. One possible explanation could be the masculine "aggressive" attitude to overestimate own capabilities against the feminine moderate attitude, influenced by the well-known role stereotypes.

The correlations between efficacy beliefs in problem posing and the ability to construct problems indicate that efficacy beliefs is decisive factor influencing, possibly predicting the subjects performance in PP, at least in the type of sources examined in the present study. Furthermore, these beliefs provide a clue about the quality of the results in such a task. The AE and the HE students were able to construct more problems and of higher complexity, as indicated by the type of the questions raised and the number of relations involved. In addition, these subjects used to test their problems and felt more comfortable with this task of problem posing, in contrast to the LE students who felt anxious, when facing a problem posing task and they seldom attempted to test the problems they constructed.

In conclusion, our findings suggest that developing efficacy beliefs in problem posing should be an integral part in any preservice teacher education program. Efficacy beliefs constitute "an important component of motivation and behavior" (Pajares, 1996, p. 341) and consequently are important for integrating problem posing and problem solving in classroom instruction. The correlation found among the efficacy in problem posing and the students' beliefs about teaching this activity suggests a possible focus for further research.

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Teachers' Beliefs about Girls and Boys and Mathematics

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Abstract

The paper deals with the question: what do Finnish lower secondary school teachers of mathematics believe about girls and boys as learners of mathematics.

In this study teachers (112 female and 93 male) were to classify a list of characteristics as more typical among girls or among boys (aged 13-15 years). In this paper I analyse the answers to items on the questionnaire that deal with the question of mathematics as a male/female domain. The results suggest that a great majority of teachers see differences between girls and boys as learners of mathematics. Even many of those teachers who did not see any differences in inborn abilities believed that boys more often than girls succeeded in mathematical tasks demanding higher cognitive ability. There is some evidence that some of the teachers, though a minority, have a tendency to stereotype mathematics as a male domain.

The preconceived beliefs of the teachers concerning gender-based learning styles – boys use their brain and girls are just hard workers – may reinforce and sustain differences.

1. Background

Since the early 1970's there has been an increasing research activity in the field of gender and mathematics education especially in the English-speaking Western nations [3]. In Finland only lately, more than twenty years later, has this issue become current. In the year 1995 an evaluation of the availability and equality of Finnish comprehensive schooling was carried out. In this evaluation, equality between boys and girls was one of the themes. Boys did better than girls in the mathematics assessment. However the boys' math marks were slightly poorer than the girls'.

The current problem in Finnish upper secondary school is that far less girls than boys take the extended course in mathematics (and especially in physics). The question is, what it is in the teaching of comprehensive school mathematics and in the learning experiences of students that causes such a strong gender commitment. The low participation rate of girls in mathematics studies is a disadvantage for girls and gender equality because of its consequences for girls' later career possibilities. The extended course in mathematics is a "gatekeeper" to many faculties at the universities. Though it must be admitted that gender equality is not the only reason for the present interest in girls and mathematics. In Finland a more important reason is connected with employment and shortage of labour in the information technology. Therefore there is a growing need for students in higher education of mathematics and technology. Girls are an underrepresented group and therefore they are seen as a resource and a potential group for these fields of study.

In Finland equality between boys and girls at school is generally considered so self-evident that the principle is not written into the school curriculum. Schools tend to be "gender-blind", and teachers' gender-neutrality is often merely superficial. In principle, good teachers are

expected to promote equality, but what this means at the day to day level is unclear. If a teacher is not aware that he or she may carry unconscious beliefs concerning gender and mathematics, these discriminating beliefs can act against the teacher's good aims at equality. Thus cultural myths concerning gender, e.g. mathematics as a male domain, become self-fulfilling prophecies.

Teachers' beliefs influence their teaching practices and classroom behaviour and they in turn affect the pupils' views of themselves as learners of mathematics. Studies in Finland and also in other countries have shown that girls' self-confidence in mathematics is lower than that of boys' and that teacher-pupil interaction is gender dependent. Teachers' attitudes, beliefs and perceptions of gender differences may impact the formation of students' confidence in mathematics. Girls are a minority in advanced mathematics courses. It is possible that teachers' beliefs about gender and mathematics play an important role especially in girls' mathematics choices for upper secondary school and later choices at the university level.

Thus perceptions that educators have of real or imagined gender differences can be used as one indicator of the conditions that may influence secondary school students.

2. Central concepts

In this paper I use the term gender in the meaning of sex. Another alternative policy is to use the term sex when referring to biological distinctions and the term gender when referring to psychological and social features associated with the biological sex. However in this paper I do not discuss any deeper the issue on defining the concept gender; a student is a girl or a boy.

In the educational literature and among researchers there is no common definition for the concept belief [2]. Beliefs are on the border between cognition and affect. The latter, affect, is often more or less emphasised in a teacher's belief concerning gender and especially gender and mathematics. What a teacher sees as his or her experience based knowledge about girls and boys unavoidably reflects his or her unconscious primitive beliefs. Therefore in this paper I use the term belief even in the case the subject, the teacher, might speak about conceptions, knowledge or facts.

3. Subjects

The study is a survey. The test subjects are Finnish mathematics teachers from a sample of 150 randomly chosen schools for grades 7-9 (13-15 -year olds). In each school two mathematics teachers, one female and one male, if available, were asked to respond to a questionnaire. This was carried out in spring 2000 and material was received from 112 female and 93 male teachers. Some smaller schools in the sample might have had only one mathematics teacher and some schools did not have a male teacher; therefore the percentage of responding was approximately 72 % for female teachers and 68% for male teachers.

4. Instruments

I developed a list of characteristics that the teachers were to classify as more typical among girls or among boys in their classrooms. These characteristics were based on previous research findings and also on my own experience as a mathematics teacher. The 55 statements of these characteristics were grouped under the following headings:

A) Girls and boys in math-class
B) Girls' and boys' attitudes

C) Girls' and boys' abilities and cognitive skills

D) Upper secondary mathematics choices and career choices

E) Gender equality in school

This grouping of the statements was intended to support the teacher in answering the questionnaire. Some items that were supposed to belong to the same belief factors were mixed into different groups in order to prevent teachers answering consciously in "the socially acceptable way". In addition to this structured questionnaire the teachers answered to some open questions concerning differences between boys and girls as mathematics learners.

In this paper I analyse some of those items that deal with the question of mathematics as a male/female domain. These nine items and the abbreviations are the following:

A_math	X gets through the extended mathematics course more easily.
Inborn	X is innately mathematically more talented.
Reasoning	X is capable of reasoning.
Intelligence	X's success in mathematics is based on the use of his or her intelligence and power of deduction.
Unfamiliar	X can solve unfamiliar tasks.
Routine	X is better at routine tasks than at problem solving.
Painstaking	X's achievements in mathematics rely more on painstaking practise than understanding.
Rote-learning	X leans on rote-learning and does not even try to understand.
Unimportant	Mathematics is not crucial for X's future career choices.

In these statements the subject X was to be chosen from the following five alternatives:

G	usually a girl
g	a girl more often than a boy
±	a girl as often as a boy
b	a boy more often than a girl
B	usually a boy

Thus the item response was based on a trivial comparison between the two groups. This was aimed at to help the teachers to answer without much effort and maybe frankly as well. The quite high responding rate can possibly be attributed to the easiness in answering. Also it was expected that using the choice between a girl and a boy would make the meaning of the answer clear and unequivocal. Conventional Likert-type items answered on a scale from agreement to disagreement are not unproblematic [1]. For example, what can be referred from disagreement with the item: Girls can do just as well as boys in mathematics? Are girls doing better or are girls doing worse? This kind of problems were avoided in the present questionnaire.

5. Some first results of the belief of mathematics as a male or female domain

The nine items were answered to by the teachers in a different order than presented above and among the other (altogether 55) items. The first three of the items above concerned beliefs about gender differences in mathematical capabilities. These three items were mostly answered neutrally or "a boy more often than a girl". In the analysis the ratings were scaled -2, -1, 0, 1 and 2 (from -2 "usually a girl" to 2 "usually a boy"). The average of the ratings of the first three items were positive 0.27, 0.24 and 0.34. The positive values mean that teachers slightly tend to believe in boys' higher capabilities in mathematics.

The next items concerned gender differences in learning style and in approaching mathematical tasks. Most teachers believed that boys more often than girls use their heads, so the averages for these items were positive: *Intelligence* 0.57 and *Unfamiliar* tasks 0.69. correspondingly the averages were negative for items *Routine* -0.63, *Painstaking* -0.82 and *Rote-learning* -0.74. In short the results indicate that teachers believe that girls more often than boys work hard without understanding.

The last item *Unimportant* had also a negative average -0.58. Most teachers believe that the statement "Mathematics is not crucial for future career choices" fits more often to a girl than a boy.

6. Teachers' belief profiles

The teachers were categorized on the basis of the answers. This was done using K-means cluster analysis. The cluster centres were chosen to maximize between-cluster distances. The 3-cluster solution appeared to be reasonable in respect to the interpretation and also in respect to cluster sizes. The 4-cluster model was also considered. The 4-cluster model gave the three first clusters very much alike those in the 3-cluster model and the fourth cluster included only 7 teachers (3%). Hence the 3-cluster model was rated appropriate for a closer study. In the 3-cluster model the clusters were quite equal in size. Cluster No.1 had 30% of the teachers, Cluster No.2 had 32% and Cluster No.3 was slightly bigger with 38% of the teachers. These three clusters are given the following characterisations:

* Cluster No.1: "Boys use their intelligence, girls lean on work"
* Cluster No.2: "Mathematics is a male domain"
* Cluster No.3: "No gender differences?"

The plot of means of the item scores gives the belief profiles of different teacher clusters. They are shown for each cluster in Figure 1.
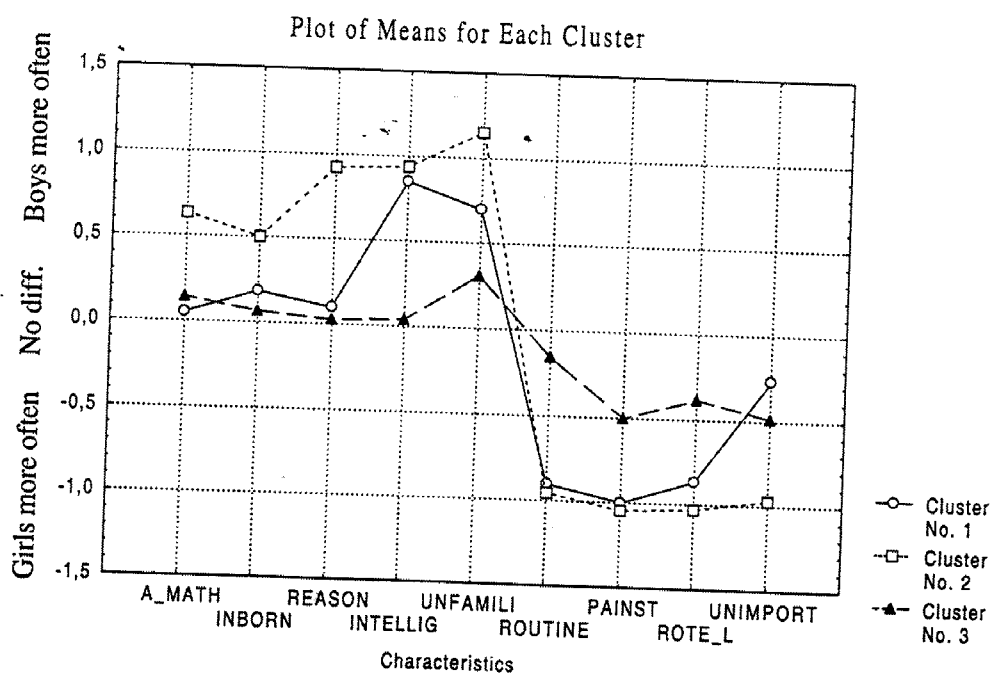

Figure 1:Belief profiles for three clusters of teachers.

In all clusters most of the means of the item scores differed from value 0, which was the neutral alternative; a boy as usual as a girl. These differences were statistically significant or very significant. There were a few exceptions: the means of item scores for *A-math* and *Reasoning* in the first cluster and *Inborn*, *Reasoning* and *Intelligence* in the third cluster did not differ from value 0 statistically significantly.

Teachers of the first cluster "Boys use their intelligence, girls lean on work" say that they believe in equal abilities and most of them believe also in equal importance of mathematics to girls and boys. Still we might ask the question: do the answers to the other items still reveal traditionally stereotyped expectations e.g., boys are smarter; girls have to work hard to succeed?

Teachers of the second cluster "Mathematics is a male domain" seem to believe that there are inborn differences in mathematical ability between girls and boys. Boys are talented more often than girls. In consistency with the foregoing, boys more often than girls are capable of reasoning, use their intelligence and power of deduction and solve unfamiliar tasks. Girls more often than boys are better at routine tasks than at problem solving and do not try to understand but lean on rote-learning. Teachers of the second cluster answered that mathematics is more often not crucial for girls' career choices, this differs clearly from the answers of the teachers of the other two clusters.

Teachers of the third cluster "No gender differences?" chose mostly the neutral alternative of the questionnaire; "a girl as often as a boy". The minor differences they had found between boys and girls were concerning learning style and importance of mathematics. Still there is a question mark: what is beneath the surface of these answers? I have a problem in interpreting the true beliefs of the teachers of cluster 3. There seem to be two alternatives. The first interpretation: The teachers of the third cluster are committed to gender equity. Nevertheless they are aware of gender differences and they are realistically and truthfully admitting that girls slightly more often than boys rely on plain work and also mathematics is not always crucial for girls' future careers. The second alternative interpretation is quite opposite to the first: The answers are to be interpreted negatively. These teachers are not able to observe and recognise gender differences or they just do not want to see any gender related problems. It is highly probable that the third cluster consists of both types of teachers: those who are supporting gender equality and those who are "gender-blind".

Later this empirical data will be extended with data gathered by teacher interviews and in-class observations. By this means I am expecting to find the answers to the previously mentioned problems in interpreting the teacher belief profiles.

7. Discussion

This research on teachers' beliefs about gender differences of student mathematics learning suggests that a great majority of teachers have different beliefs about male and female students. Even many of those teachers who did not see any differences in inborn abilities believed that boys more often than girls succeeded in mathematical tasks demanding high cognitive ability. There is some evidence that some of the teachers, though a minority, have a tendency to stereotype mathematics as a male domain.

The preconceived beliefs concerning gender-based differences in learning styles – boys use their brain and girls are just hard workers – may reinforce and sustain these differences. Teachers' beliefs lead to different expectations, different expectations lead to different teacher treatment of boys and girls.

However the impact of this phenomenon on gender differences in student performance, attitudes and choices stays unclear. One might ask what is the role of a teacher's own beliefs about mathematics and what is the influence of his or her teaching style. Is it possible that a teacher in the classroom, maybe unconsciously, gives the impression "Almost all mathematics problems can be solved by direct applications of the facts, rules, formulas and procedures shown by the teacher or given in the textbook"? Is it possible that girls as "good and obedient learners" are more submissive than boys to accept and follow models even if these models reflect an unhealthy mathematical belief of their teacher? Is it because of this phenomenon a girl more often than a boy becomes a rote-learner or a copier and a reproducer of other peoples' mathematics instead of doing her own mathematics? There is a vicious circle: The teachers' teaching practises reinforce girls' unfavourable learning practices which in turn reinforce teachers' lower expectations of girls' success in mathematics. This interesting issue is beyond this presentation and needs a further study.

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Appendix 1

Distribution of answers of 205 middle school mathematics teachers

G   usually a girl   -2
g   a girl more often than a boy   -1
±   a girl as often as a boy   0
b   a boy more often than a girl   1
B   usually a boy   2

		G	g	±	b	B
A_math	X gets through the extended math in upper secondary school more easily		3	**148**	47	6
Inborn	X is innately mathematically more talented		1	**158**	41	5
Reasoning	X is capable of reasoning		6	**127**	68	4
Intelligence	X's success in mathematics is based on the use of his or her intelligence and power of deduction		5	90	**98**	12
Unfamiliar	X can solve unfamiliar tasks.		2	74	**114**	15
Routine	X is better at routine tasks than at problem solving.	10	**116**	73	6	
Painstake	X's achievements in mathematics rely more on painstaking practise than understanding.	9	**151**	45		
Rote_learn	X leans on rote-learning and does not even try to understand.	13	**126**	65	1	
Unimport	Mathematics is not crucial for X's future career	8	**109**	81	5	1

Cultivation of student's and teacher's beliefs about mathematics education

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> "Knowing means being capable of creating, either by mind, by hand or through speech. Because everything happens either through creating or through imagining, that is by creating images and shapes of things."
>
> Jan Amos Komenský

Abstract

The didactics of mathematics realises the need to find the ways of changing the present school supporting creative teaching and learning and inhibiting the long-standing instructive teaching. The contribution discusses integrating of grasping of situation into mathematics lessons and the utilisation of projects as two of the possibilities how to develop student's and teacher's attitudes and beliefs about meaning and aims of mathematics education.

1. Introductory remarks

In the last time we meet comparatively often in our country the comment that present mathematics education is devoted mostly to the performance of students, that it does not develop in adequate extent the cognitive skills and personal capabilities of students.

Plenty of our teachers see the core of mathematics education in practising and mastering numerical and drawing techniques, propositions and instructions, how to solve certain typical problems. Students then understand mathematics as a set of rules that it is necessary to drill hard and to remember well. Some of them even make conclusion that only a few elected individuals are allowed to get down to the secrets of mathematics. This students' belief is great obstacle for effective learning mathematics.

It is possible to say that the actual process of education does not coincide in favourable extent with the opinion declared by didacticians (e. g. HEJNÝ, KURINA in [2]) that

> ... the sense of education is the cultivation of students, ... it is not the priority, that students will master certain amount of knowledge or information, but students should understand the world, they should be able to solve problems which they can meet, they should be able to search for information and evaluate it, they should be equipped for their own further development.

In the course of learning part of the "world of culture" becomes a component of "mental world of student" [2,10,11]. Important basic question of didactics of mathematics is what way along this process proceeds. Whether via transmission and instructions or by the way of construction, when the student actively approaches to the construction of his/her own mental

world [2]. In our present school predominate instructive approaches. In our opinion the memorized idea is not real knowledge. We believe that it would help to release our school from many drawbacks and to increase the quality of education, if we stressed more the constructive approach in mathematics education [1].

2. How to achieve the change of the character of mathematics education?

We participated in the following experiment with interesting results. Teachers of lower secondary school were set by five suggestions how to proceed in the course of teaching the Pythagorean theorem. These suggestions differed by involving constructivistic ideas at various levels. The teachers' task was: (1) to decide what suggestion they find to be the most suitable and (2) to choose the suggestion which is the nearest to their own procedure in the teaching process. Also those teachers, who chose in step (1) one of predominantly constructive approaches, characterised self-critically their own procedure at least by one degree closer to instructivism and transmission. It can indicate that some teachers have difficulty in changing their style of teaching.

We realise that the character and quality of education depends decisively on the teacher. Therefore we endeavour to affect and influence the teachers with the aim to cultivate their opinion and beliefs in the sense of mathematics education for the development of students' personality. We should help them to move the centre of their appeal to students from the role of someone who passes the information over to the role of a person who stimulates challenges, questions and problems, creates the climate, provokes the activity, urges the students to learn actively and as users. Aiming at this goal it comes to the forefront of our interest the "open approach" method [9]. In connection with it we started to study two areas – grasping of situations [3,4,6] and utilisation of project [7,8].

3. Grasping of situations

"Situation" means for us mathematical or non-mathematical (open) domain, which we want to study and from which questions and problems grow up.

As we formulated in more detail e.g. in [3,6,13] we mean by the grasping of (real) situation process of thinking that involves: perception of situation as a section of real life; discovering the key objects, phenomena and relations between them; setting-up certain direction of grasping real situation; realising and formulation of problems (cluster or cascade interfaced problems with different grade of difficulty) growing from the situation and of questions, which could be asked. These activities are directed to the investigation and understanding of the situation and their aim is the problem posing. In the following stages of grasping of situations predominates the problem solving, i.e. mainly searching for answers to questions, solving the posed problems and formulating results; interpretation and evaluation of the results and answers; identification of the new situation and its grasping using the experience enriched by the current activity.

When we deal with grasping of situations we place the teachers as well as students in an "unknown world." Therefore, we prepare scenarios, e.g. [5,12], designed mainly for the teachers. We treat them as an inspiration, never as an instruction. The components of such scenarios are especially: presentation of the situation; set of questions to help remember previous experience; introductory problem with complementary questions; supporting collection of problems growing from given situation (some of them are usual, some of them

awake feelings that they do not belong into the mathematics); comments for teachers that outline various possibilities of grasping of given situation; remarks relating to individual items that not only show the mathematical solution but also put stress on the role of introspection.

The aim of scenarios is to show the possible ways for analysing a situation and it's grasping. The experience shows that it is appropriate to start with narrowed "oriented" situation. It means that already at the beginning we outline the domain of investigation, the area of interest, orientation of questions.

4. Sample of situation

We will show sample of situation, which is according to our experience interesting for both, teachers and students. Investigation of this situation does not require any great mathematical knowledge. Therefore we dealt with it from the 4th grade (10-year-old) [13].

- Presentation and description of the situation
  We commute to work in our own car every day. Our friends who live in the nearby towns and villages and who we work with use their own cars as well.
- Orientation of grasping situation
  We consider if it is possible to save travel expenses and to what extent.
- Simple introductory open problem
  Adam and Bob go to the same place of work. Each of them goes every day with his own car. The cost of Adam's journey is 18 crowns (Kc) and the cost of Bob's journey is 6 crowns (Kc).

  Adam   *Bob*   *work*

  o--------------------------o------------o
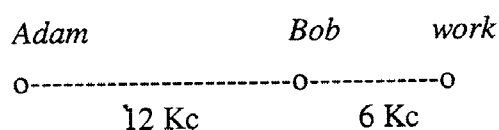
        12 Kc   6 Kc
- Outlined sequence of questions
    - *What can you calculate using numbers in the picture?*
    - *Consider if Adam and Bob could spare some money. How will they do it?*
    - *Find out how much money would they pay if they went together with Adam's car. How much money would they spare?*
    - *Work in pairs. Imagine that one of you is Adam and the other Bob. Suggest what amount of money each of you should contribute to the total expenses.*
    - *Evaluate from Adam's (Bob's) point of view if the suggested sharing expenses is advantageous or disadvantageous for you, if it is acceptable or not. If one of you does not agree with suggested sharing expenses, try to find a compromise acceptable to both of you.*

The situation that concerns individual transport enables to "vary (release) parameters" and scenario proceeds by suggestion of possibilities of it. This phenomenon consists in that we change some of initial data in the given situation, e.g. distances, the number of travelling persons, the shape of travelling route, expenses per kilometer, the shape of map, etc. in such a way that from each change a new situation, new series of problems grows - from very simple to more difficult ones.

In the course of work on the grasping of situations (and similarly on the projects) we saw by younger students (10-year-old) greater involvement, enthusiasm, creativity, interest in work and more willingness to gain new knowledge. The older students (14-year-old) often worked without interest, they observed the situation from "outside", and they were not able to put

oneself into the position of acting persons. They were shiftless, uninventive, e.g. they found only one (if any) suggestion how to share expenses (usually by halves), and they looked for pattern. They did not understand why should they deal with this problem when such problems are usually not subjects of exams and they believe that they learn, after all, for exams.

Our experience showed that great obstacle arises if the teachers do not understand our scenario as an inspiration and endeavour to proceed precisely according outlined items and questions. Teaching process was considerably more successful and satisfying for teachers in case that the teacher understood the prepared scenarios as a source of ideas and as a starting point for his/her own work. Such teachers read through the submitted scenarios and then they modified them or created their own situations and prepared for them their own scenarios. On the other hand there were teachers who did not see in such work any sense and were afraid of losing time with it. The teacher has to believe to the usefulness of this approach to the mathematical education and only in such case success can come.

5. Oriented situation and project

As we mentioned earlier we start the work on the grasping of situations with narrowed "oriented (or focused) situations" that could have character of "open problems". We concluded that it is possible to understand grasping the "oriented situation" as a project. On the other hand we realised that the part of the work on the project is the grasping of situation. It appeared here that many viewpoints coincided. For this reason we linked these two areas of research, i.e. the work on grasping of situations and the work on projects, together [13].

Projects comprise, in our interpretation, various activities in which the students discover mathematical concepts and laws and/or in which they learn about possibilities of the use of mathematical concepts outside mathematics. When working on a project, a teacher only leads them or, even, they work on their own.

A project, which is well prepared and realised in mathematical lesson, can enable the student to get over possible obstacles while learning mathematics; help him/her to understand various relationships in mathematics and in reality; cultivate student's ability and competences in various activities (collecting data, sorting, classification, argumentation, abstractions, communication etc.); teach students how to deal with stressful situations, etc. (see [7] for more details).

6. Sample of project

The project "Mathematical calendar" was carried out with 14-year-old students. The aim was to prepare the calendar for the year 1999. The calendar should have twelve pages; each page should contain a picture and two word problems.

At first the students drew pictures. The colours of the pictures should represent feelings, which a particular student linked with the given month. After that the students created word problems. Firstly they chose the theme in which they formulated the word problem as follows: Based on their own experience (from their point of view) they chose something characteristic for certain month (e.g. for February: it is cold, therefore children wear stocking caps). Or - by means of encyclopaedia they chose important event in the Czech republic or in other countries, which happened in given month, for example: July - USA - Independence Day - the opening of Pentagon. When deciding upon the theme of problem, the students should concurrently have in

mind also the second step – to find suitable mathematical topic of the problem, e.g.: Pentagon – area of regular pentagon, percentages, fractions.

At the end they formulated problems, e.g.:

There are 1000 pupils in a primary school. Of them, there are 56% girls. How many boys had a stocking cap on February 7th in the primary school in case that each tenth boy forgot his stocking cap at home?

On July 4th 1943 a new precinct called Pentagon was opened to celebrate the Independence Day. The ground plan of the building is a regular pentagon, which is inscribed to circle with a radius of about 260 metres. What part of the circle does the pentagon occupy?

When students meet in the course of mathematics lessons the task: "Formulate the word problem ..." then the situation or the mathematical topic is usually already set. In this project the student were set neither the situation nor the mathematical theme. Therefore this activity was for the students completely new and in many cases evoked their interest. The students learnt in the course of work on the project not only to perceive but also to search for "mathematics around them". Such activity hence contributes to the students' appreciation of the sense of mathematics education.

After finishing the work on the project it was proved to be helpful to organise the exposition of students' works or to present attained results in the form of student conference. In the conference took usually part not only the mathematics teachers but also the teachers of other school subjects. Enthusiasm of students with which they spoke about their work stimulated those teachers to the involvement of project method also in their own teaching, in their subjects. In this way the student conferences then became one of the means of cultivation of teachers' belief. So gradually (slowly but surely) changes the character of education (teaching process) at this school.

7. Concluding remarks

Our current experience with teachers' approach towards the involvement of grasping situations and utilisation of projects in mathematics education shows that the main obstacle for including of those topics in the educational process is the common style of work and teachers' routine which is focused mainly on mastering of numerical and drawing techniques. The same it is possible to say on teachers' belief about the necessity to decrease instructional character and to increase its constructive character. Some teachers are afraid that this approach distracts the students from "proper" mathematics. This opinion in fact expresses their narrow view of the importance of mathematics education for the development of personality. Many teachers are afraid of lengthy discussions whose contribution towards mathematics they doubt. Others are afraid of the time and intellectually intensive preparation and the difficulty to predict students' reaction in advance. Many also do not know how to evaluate students on such assignments.

On the other hand if we manage to overcome the initial distrust of teachers, they are able to work very creatively – to suggest and prepare their own scenarios and projects. Also their belief in suitability and necessity of reinforcement the constructive character of teaching has increased. The work on scenarios focused on grasping situations and preparation of projects thus contributes towards the cultivation of teachers' opinion, conception, and beliefs concerning sense and aim of mathematical education.

We discussed with the teachers their reactions on including problems of grasping situations and utilisation of project method into mathematics education. At first it is necessary to mention, that it is the topic with which our present school has only a little experience if any. On the other hand many teachers start to realise its sense and usefulness. Important role of course plays their motivation, their volition to try something new. We will show some short parts from teachers' introspection, enunciations about their experience.

"The preparation is very demanding, it is necessary to think out each step and to estimate in advance possible students' trains of thoughts and questions. ... Students accepted with pleasure that they are allowed to determine themselves on formulation of new problems, to influence new questions, to assess convenience of solution. In this activity took part also those students who are not much successful in mathematics."

"In the preparatory phase I was afraid how would the students accept such approach to the problems from practice ... even that before I convinced myself that those non-traditional problems activated students. But attempt to start discussions of the type "What if ..." filled me by distemper."

"It seems to me, that it is really motivating, if the teacher is able to utilise students' experience in the course of problem solving. It is gratifying to see the eagerness in students' eyes with which they started to work ...".

It was possible to follow also the development of cognitive competences of students. The students started to investigate, discover the phenomena and the relations between them, to create hypotheses and to verify them, to seek for links etc. Concurrently they started to appreciate that the sense at mathematics education consists not only in mastering techniques but also in development of competences in problem solving (and also in other things). Mathematics slowly becomes meaningful, useful and interesting activity that satisfies students and they started to get confidence in their mathematical competence and to believe that they are able to solve problems.

Our main intentions for further research in this field are the following:

- To study social interaction between the students as well as between the students and the teacher in the course of grasping situations and work on projects.
- To investigate mutual relations between pedagogical interactions and acquiring pieces of knowledge.
- To search for main obstacles in cultivation of teachers' as well as students' beliefs about mathematics education and for phenomena that enables to find those obstacles.

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The conception of mathematics among Hong Kong students and teachers

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Abstract

Though Hong Kong students outperformed their Western counterparts in international comparisons in computational problems, their performance in non-routine mathematics questions is not that brilliant. This could be related to the conceptions of mathematics both among the students and among the teachers. When mathematics is regarded as an absolute truth or a set or rules for playing around with symbols, students would tend to treat doing mathematics as the memorisation of algorithms and learning mathematics as a process of transmission. With the use of situations similar to those in Kouba and McDonald (1991), we found that students possess a relative narrow conception of mathematics. Later, we proceeded to make more in depth investigations by the use of open-ended questions. Again, it was found that students did not perform well with open-ended questions and in general, they tried to approach a mathematical problem by searching a rule that fits by identifying what is given, what is being asked and the topic the problem belongs. Not only that, evidence has shown that such a way to approach mathematical problem is largely shaped by the way they are experiencing learning, task demands, and classroom environment. In other words, such a narrow conception of mathematics which exist both within the students and in the classroom culture has led to students tackling mathematical problems by the search of rules rather than via a conceptual understanding of the context.

1. How we launched the endeavour

The research team (other members: Chi-Chung Lam and Ka-Ming Wong) started off the investigation as a curriculum issue. Students' conception of mathematics was taken as part of the "attained curriculum" (Travers & Westbury, 1989). First of all, Asian students outperformed their Western counterparts in international comparisons in mathematics, (Cai, 1995; Leung & Wong, 1996, 1997; Stevenson & Lee, 1990), but we suspected that they may lack conceptual understanding. As one facet of it, we suspected that students only possess a narrow conception of mathematics. Secondly, the mathematics curriculum was under reform in which the author was actively involved. As a first step, we need to make a full situational appraisal, to identify the strength and weaknesses of the current curriculum. There, students' conception of mathematics was again one of the foci of investigation (Wong et al, 1999). Thirdly, we made some tried out the use of history mathematics in teaching (Lit & Siu, 1998). To evaluate the effectiveness of such a curriculum, we saw that conventional tests do not suffice. We incorporated assessments in the affective domain, including beliefs. All these led us to a more in depth investigation on the conceptions of mathematics. At the same time, we realised that a well-established methodology for such a series of studies is in need.

2. Our project

Initially we targeted at the development of a methodology, an instrument, that could be used in future research. As we moved on, why Hong Kong students possess such a narrow conception of mathematics become prominent. In brief, it was shaped by the long time indulgence of a learning environment that lacks variation. It was thus a natural step forward to study the conception of mathematics among teachers.

(a) *Prologue* (1992-93): use of open-ended questions to tap when did students regard themselves as having understood some mathematics (Wong, 1993, 1995).

(b) *Phase 1* (1996-97): use of hypothetical situation and asked student to judge whether it is "doing mathematics" in each case (Wong, Lam, & Wong, 1998).

(b') *Test trail of results obtained in phase 1* (1997-98): testing of reliability of a questionnaire developed from the results obtained in phase 1 (Wong et al, 1999).

(c) *Phase 2* (1997-98): use of open-ended mathematics problems to tap students' approaches to tackling these problems in relation with their conceptions of mathematics (Wong, Marton, Wong, & Lam, in preparation).

(d) *Phase 3* (1998-99): investigation of teachers' conceptions of mathematics and mathematics teaching by questionnaires and by interviews (part of the result can be found in Wong et al, 1999).

3. Students' conceptions of mathematics

3.1. When did students regard themselves as having understood some mathematics ?

Two hundred and forty one Grade Nine students in Hong Kong were invited to respond to the open-ended questions on approaches towards mathematics problems: (a) "What would you do first in facing a mathematics problem ? (b) "What methods do you usually use in solving mathematics problems ?" (c) "What is the essential element in successful mathematics problem solving ?" (d) "Which do you think is the most important step of a successfully solved mathematics problem ?" and (e) "If you were to score a completed mathematics problem, which do you think is the most important part ?". It was found that "trying to understand", "revise and work hard" and "asking others for help".

Another set of five open-ended questions on understanding mathematics was asked of 356 Grade Nine students. They were: "(a) When will you consider yourself to have understood a certain mathematics problem ?", (b) "When will you consider yourself to have understood a certain topic ?", (c) "Before actually tackling a mathematics problem, how can you be sure that you can solve it ?", "(d) When do you discover that you don't understand a certain topic thoroughly enough ?" and "(e) Which part of the topic do you think you must understand in order to solve problems successfully ?" It was found that over half of the respondents took understanding to be "getting the correct answer".

These 356 students were later requested to recall any instance in which they understood, realised, grasped or comprehended a mathematics topic, formula, rule or problem, after they responded to the open-ended questions. Results revealed that, for Hong Kong secondary school students, understanding mathematics, may mean the ability to solve problems, the knowledge of underlying principle, the clarification of concepts, and the flexible use of formulas.

3.2. Use of hypothetical situations

Twenty-nine students were confronted with ten hypothetical situations in which they were asked to judge whether "doing mathematics" was involved in each case. Most of the situations were taken from Kouba and McDonald (1991). Some examples are "Siu Ming said that half a candy bar is better than a third. Was he doing mathematics ?", "An elder sister lifted her younger brother. She said that he must weigh about 30 pounds less than she. Did she do mathematics ?" and "Dai Keung and Siu Chun went to take a photo at the spiral staircase at the City Hall. When the photo was processed, Dai Keung discovered that the staircase looked like a sine curve. Did he use mathematics when he looked at the photos ?". Results revealed that students associated mathematics with its terminology and content, and that mathematics was often perceived as a set of rules. Wider aspects of mathematics such as visual sense and decision making were only seen as tangential to mathematics. In particular, they were not perceived as "calculable." However, students did recognise mathematics as closely related to thinking. Views of mathematics were also sought from sixteen mathematics teachers. Mismatch between students' and teachers' views was found. Some of the views among the teachers were self-conflicting.

3.3. Test of reliability of a questionnaire

A questionnaire comprising the three subscales of "mathematics as calculables", "mathematics involves thinking" and "mathematics is useful" was developed from the above research (Section 4. above). They consists of 14, 6 and 6 items respectively, put in a 5-point Likert scale (strongly disagree, disagree, slightly agree, agree, strongly agree). It was administered to a total of 6759 (2630 from Grade Six, 1357 from Grade Nine, 1419 from Grade Ten and 1419 from Grade Twelve). Satisfactory reliability indices (Cronbach' alpha) were obtained (Grade 6: .71, .59, .70; Grade 9: .73, .69, .75; Grade 10: .72, .71, .78; Grade 12: .73, .69, .76). In general, the students agreed that mathematics is something calculable (mean scores of 3.38, 3.27, 3.32 and 3.21 for Grades 6, 9, 10 and 12 respectively), involves thinking (mean scores of 3.90, 3.92, 3.94 and 4.04 for Grades 6, 9, 10 and 12 respectively) and is useful (mean scores of 3.72, 3.24, 2.99 and 3.22 for Grades 6, 9, 10 and 12 respectively). As we move up the grade levels, the extents to which they think mathematics involves thinking increased and decreased for usefulness.

3.4. Use of open-ended mathematics problems

Nine classes (around 35 students each) of each of Grades Three, Six, Seven and Nine were asked to tackle to a set of mathematical problems. Each set comprised 2 computational problems, 2-4 words problems and 4 open-ended questions. Two students from each class (2 $\times 9 \times 4 = 72$ students) were then asked how they approached these problems. The original hypothesis was, a narrow conception of mathematics (as an absolute truth, say) is associated with surface approaches to tackling mathematical problems and a broad conception is associated with deep approaches (Marton & Säljö, 1976; Wong, Lam, & Wong, 1998). Crawford et al (1998a, 1998b) already found that fragmented view is associated with surface approach and cohesive view is associated with deep approach, but they were investigating general approaches to learning rather than on-task approaches (Biggs, 1993).

Consistent with what was found in previous research, students repeatedly showed in this study a conception of mathematics being an absolute truth where there is always a routine to solve problems in mathematics. The task of mathematics problem solving is thus the search of such routines. In order to search for these rules, they look for clues embedded in the questions

including the given information, what is being asked, the context (which topic does it lie in) and the format of the question. Students held a segregated view of the subject too. Writing is not mathematics and mathematics is calculations with numbers and symbols. Many of them thought that by letting the answer to be found as an unknown x and by setting an appropriate solution, virtually all mathematical problems can be solved by such kinds of routines. Furthermore, one should only write down those things that are formal and that you are sure to be correct (otherwise, marks will be deducted for wrong statements). Therefore, leaving the solution blank is common found in their scripts which does not mean that the student did not tackled the problem nor that the student had no initial ideas of solving it. It is also clear from this study that students' conception of mathematics is shaped by classroom experience. Most problems given to students lack variations, possess a unique answer and allows only one way of tackling them (Wong & Lam, in preparation). It is thus not surprising that students see mathematics as a set of rules, the task of solving mathematical problems is to search for these rules and mathematics learning is to have these rules transmitted from the teacher.

4. Teachers' conceptions of mathematics and mathematics teaching

4.1. Questionnaire

An instrument developed and validated by Perry, Tracey & Howard (1998) was translated into Chinese and administered to 369 Hong Kong primary mathematics teachers, 275 Hong Kong secondary mathematics teachers. The questionnaire was also administered to 105 primary mathematics teachers in China Mainland (Changchun) and 156 primary mathematics teachers in Taiwan for possible cultural comparison. The questionnaire consisted of 7 items on "mathematics instruction as transmission" and 11 items on "child-centredness of mathematics instruction" and were put in a 5-point Likert scale (strongly disagree, disagree, slightly agree, agree, strongly agree). Satisfactory reliability indices (Cronbach' alpha) were obtained (Transmission: .64, .61, .63 and .65 for Hong Kong primary, Hong Kong secondary, China Mainland and Taiwan respectively; child-centredness: .73, .69, .73 and .73 for Hong Kong primary, Hong Kong secondary, China Mainland and Taiwan respectively).

The status of undergraduate education (mathematics major, education major, other degrees, no degrees), teacher education (mathematics major, non-mathematics major, no teacher education) and year of teaching experience were also asked. Correlation analyses revealed that the more transmission-oriented and less child-centred, those teachers who were mathematics majors at undergraduate studies inclined less to taking teaching as transmission and the more the year of teaching experience, the more was transmission orientation. However, transmission orientation was not related significantly with teacher education and child-centredness was not related with undegraduate studies, teacher education nor year of experience. Analyses of variance also showed that Taiwan teachers were most child-centred and Hong Kong primary mathematics teachers were most transmission oriented while Taiwan the least.

4.2. Interview

We confronted 12 secondary mathematics teachers in Hong Kong with the same set of hypothetical situations as in Section 4. We asked them what would be their reactions if students have different reactions (whether taking them as doing or not doing mathematics) to these situations. In addition to these, we confronted the teachers with some quotations of mathematicians like

(a) "Mathematics has nothing do with logic" (K. Kodaira)

(b) "The moving power of mathematical invention is not reasoning but imagination" (A. DeMorgan)

It was found that the conception of mathematics among the teachers is broader than that found among the students, in which, "mathematics involves thinking" was unanimously agreed. Other facets of mathematics, as reflected by the teachers, include "mathematics involves logic", "mathematics emphasises conceptual understanding", "mathematics is a problem solving process", "mathematics is a language", "mathematics is calculable", "mathematics is identified with content", "mathematics is applicable", "mathematics is a structured set of truths", "having a mathematics sense is important" and "mathematics is a cultural activity". Many teachers saw the two sides of the story too, for instance, mathematics involves logic but logic is not mathematics. Linkages between different facets were also expressed. For example, one teacher said, "mathematics is computation with reasoning" while another said "computation is a way to train thinking".

5. An outlook to possible future research

There are a number of areas that worth further research. First, we may need a more powerful and more established instrument to tap the conceptions of mathematics and mathematics learning both on the student' and on the teacher' side. The work done by the Australian group could a direction that we can proceed (Crawford et al, 1994, 1998a, 1998b). The distinction between the conceptions of mathematics and of mathematics learning, though intertwined, should be well aware of. Naturally, it is easier to tap teachers' conception of mathematics instruction than of mathematics itself. The inter-relationship among students' conceptions of mathematics and mathematics learning, their antecedents and consequences worth further exploration. By antecedents we mean the shaping of such conceptions by their exposure to mathematics, the mathematics problems they encounters and the conceptions of mathematics and mathematics learning of their own teachers. And by consequences, we refer to their approaches (surface vs deep) to tackling mathematics problems, their performance in solving routine and non-routine mathematics problems. The work of the Australian group (Crawford et al, 1998a, 1998b) certainly form a good basis towards this end. Cultural comparison can be useful in such an investigation too. Intervention programs to broaden students' conception of mathematics could be another exciting area of research too.

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