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Current State of Research on Mathematical Beliefs IV

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Editor's Statement

The papers in this volume – which was prepared by the Finnish-German research group MAVI (MAthematical VIews on beliefs and mathematical education) – contain the abstracts of talks given at the fourth workshop on "Current State of Research on Mathematical Beliefs". The conference took place at the Gerhard-Mercator-University of Duisburg on April 11–14, 1997. The aim of this research group, being the initiative of my colleague Erkki Pehkonen and myself, is to study and examine the mathematical-didactic questions that arise through research on mathematical beliefs and mathematics-education.

The next workshop will take place at the University of Helsinki on August 22–25, 1997.

A bibliography containing more than 750 titles around mathematical beliefs has been published by the MAVI group:

Törner, G. & Pehkonen, E. (1996). *Literature on Mathematical Beliefs*. Schriftenreihe des Fachbereichs Mathematik, Preprint Nr. 341. Duisburg: Gerhard-Mercator-University.

Again, the initiators would like to encourage all interested colleagues to join our network and to participate in our activities.

Duisburg, April 1997 *Günter Törner*

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Designing Qualitative Research

Experiences and Suggestions

Introduction

This paper contains some reflections on qualitative social research, based on the author's own experience with investigating the belief systems of teachers concerning computers and computer science applying qualitative methods. The aim of the paper is far beyond providing a comprehensive overview of qualitative methodology or a philosophy of the 'qualitative paradigm'. It is rather aimed at giving some suggestions that might be useful to a novice researcher planning his or her doctoral dissertation within the framework of qualitative research. The interested reader is referred to the publications by Tesch (1990), Lincoln & Guba (1985) and Mayring (1993).

What is qualitative research? At present, the only generally accepted statement seems to be that "qualitative research means different things to different people" (Tesch) and there is little evidence that this will change in near future. What we can say, is "that since the 1970's more and more researchers have become interested in a 'new paradigm' that moves us away from numbers and back to asking people questions and to observing" (Tesch). Although differently labelled as *qualitative*, *naturalistic, ethnographic, interpretative, phenomenological, subjective, hermeneutic,* etc., these approaches share certain basic beliefs about the status and function of research in the human sciences and about how it should be done. It is possible to understand these basic beliefs by having a look at their antagonist positions, i.e. the 'quantitative' and the 'positivist' positions.

Aspects of the 'qualitative paradigm'

The transition from the pre-scientific to the scientific period at the end of the Middle Ages may be described as a change of the ways in which questions were answered. To get answers, more and more people preferred putting questions to nature rather than to authorities such as Aristotle and Albertus Magnus. The success of the new *empirical approach*, i.e. analysing nature by disassembling and dissecting, which vigorously changed even the thinking about the nature of reality and knowledge, owed a great deal to the power of quantitative methods, i.e. modelling reality by counting and measuring. Under the leading metaphor of numbers, mathematics in a long process changed its role from a successful instrument to the actual 'language of knowledge', a generally accepted epistemological authority. In fact, during the development of the natural sciences, the original empirical paradigm was more and more shaped to be a *quantitative paradigm*.

The 'objectivity' of the results gained in the natural sciences also incited, from an early stage onwards, the human sciences to imitate the successful model and to assimilate their own paradigms. With the one exception of anthropology, many researchers in human sciences for a long time believed only those phenomena to be explorable and accessible to scientific investigation that could be measured. While the only accepted non-numerical instruments had been the 'case studies' in psychology and the 'participant observations' in sociology, most work had been done by applying the tool kit of statistics.

> Unfortunately, not many phenomena in the human world come naturally in quantities. [...] Sigmund Freud discovered plenty about the way human beings function, and so did Jean Piaget. Neither of them tested hypotheses, or used large and representative enough samples of people to satisfy the rules of statistics. Yet they both made important assertions about human beings and created many psychological constructs for use in the description of their theories. Freud employed a perplexingly simple way of finding out why people acted and thought or felt the way they did. He asked them. Sometimes they didn't know. Or they were ashamed to tell, or they were afraid to acknowledge the matter to themselves. So Freud observed. [...] When we ask questions about human affairs, the responses come in sentences, not numbers. (Tesch 1990, pp.1–2)

On a methodological level, qualitative research can be defined as *non-quantitative research*. Here, analysis is the descriptive and interpretive process of making sense of 'numberless' data, which we may call narrative or textual. While quantitative research is based on the 'monolithic concept' (Tesch) of statistics, qualitative research has a wide spectrum of non-codified procedures developed by psycholinguistics, communication research, educational psychology, and cognitive science.

Basic beliefs about ...	*Positivist Paradigm*	*Naturalist Paradigm*
the nature of reality	Reality is single, tangible, and fragmentable.	Realities are multiple, constructed, and holistic.
the relationship of the knower to the known	Knower and known are independent, a dualism.	Knower and known are interactive, inseparable.
the possibility of generalisation	Time- and context-free generalisations (nomothetic statements) are possible.	Only time- and context-bound working hypotheses (idiographic statements) are possible.
the possibility of causal linkages	There are real causes, temporally precedent to or simultaneous with their effects.	All entities are in a state of mutual simultaneous shaping, so that it is impossible to distinguish causes from effects.
the role of values	Inquiry is value-free	Inquiry is value-bound.

Table 1. Positivistic vs. qualitative (naturalist) basic beliefs (Lincoln & Guba 1985)

From a pragmatic point of view, qualitative research may be seen as a widening of the repertoire of research instruments and perspectives rather than as a change in research paradigms. And in fact, analyses are often performed by combining quantitative and qualitative methods.

From a more rigid philosophical point of view, however, there is every indication that the appearance of qualitative approaches complies with the characteristic features of a general change in research paradigm. Lincoln & Guba describe, from ontological and epistemological perspectives, the qualitative (naturalist) paradigm as a *post-positivist paradigm* with a fundamental change of basic beliefs about the nature of reality and of knowledge (cf. Table 1).

The authors point out that in natural or human sciences research "has passed through a number of *paradigm eras*, periods in which certain sets of basic beliefs guided inquiry in quite different ways". The main periods are called the *pre-positivist era*, ranging over a period of more than two thousand years from Aristotle to David Hume, the *positivist era*, and the *post-positivist era*. From today's perspective, the mere titles of the periods illustrate the pervasiveness and predominance of the positivist approach.

Critics always owe a great deal of their ideas to what they criticise, and even the followers of a new paradigm should be aware that they are 'standing on the shoulders of giants', i.e. on those of the successful adepts of the old paradigm.

Characteristics of qualitative research

Tesch (p.2) points out that "conducting scientific investigations is not a matter of following recipes. Research does not take place in a neutral environment. It is guided by assumptions about the nature of knowledge, and it has political antecedents and consequences." As implications of the post-positivist axioms for doing research, Lincoln & Guba (pp. 39–43) list fourteen characteristics of operational qualitative (naturalistic) research, the central aspects of which will be quoted in the following:

> *Natural setting.* N (the naturalist) elects to carry out research in the natural setting or context of the entity for which study is proposed because naturalistic ontology suggests that realities are wholes that cannot be understood in isolation from their contexts, nor can they be fragmented for separate study of the parts (the whole is more than the sum of parts); [...]
>
> *Human instrument.* N elects to use him- or herself as well as other humans as the primary data-gathering instruments (as opposed to paper-and-pencil or brass instruments) because it would be virtually impossible to devise a priori a nonhuman instrument with sufficient adaptability to encompass and adjust to the variety of realities that will be encountered; [...]
>
> *Utilization of tacit knowledge.* N argues for the legitimation of tacit (intuitive, felt) knowledge in addition to propositional knowledge (knowledge expressible in language form) because often the nuances of the multiple realities can be appreciated only in this way; [...]
>
> *Qualitative methods.* N elects qualitative methods over quantitative (although not exclusively) because they are more adaptable to dealing with multiple (and less aggregatable) realities; [...]
>
> *Purposive sampling.* N is likely to eschew random or representative sampling in favour of purposive or theoretical sampling because he or she thereby increases the scope or range of data exposed (random or representative sampling is likely to suppress more deviant cases); [...]
>
> *Inductive data analysis.* N prefers inductive (to deductive) data analysis because that process is more likely to identify the multiple realities to be found in those data; because such analysis is more likely to make the investigator-respondent (or object) interaction explicit, recognisable, and accountable; [...]
>
> *Grounded theory.* N prefers to have the guiding substantive theory emerge from (be grounded in) the data because *no* a priori theory could be possibly encompass the multiple realities that are likely to be encountered; [...]
>
> *Emergent design.* N elects to allow the research design to emerge (flow cascade, unfold) rather than to construct it preordinately (a priori) because it is inconceivable that enough could be known ahead of time about the many multiple realities to devise the design adequately; [...]
>
> *Negotiated outcomes.* N prefers to negotiate meanings and interpretations with the human sources from which the data have chiefly been drawn because it is their constructions of reality that the inquirer seeks to reconstruct; [...]
>
> *Case study reporting mode.* N is likely to prefer the case study reporting mode (over the scientific or technical report) because it is more adapted to a description of the multiple realities encountered at any

given site; because it is adaptable to demonstrating the investigator's interaction with the site and consequent biases that may result (reflexive reporting); [...]

Idiographic interpretation. N is inclined to interpret data (including the drawing of conclusions) idiographically (in terms of the particulars of the case) rather than nomothetically (in terms of lawlike generalisations) because different interpretations are likely to be meaningful for different realities; [...]

Tentative application. N is likely to be tentative (hesitant) about making broad application of the findings because realities are multiple and different; [...]

Focus-determined boundaries. N is likely to set boundaries to the inquiry on the basis of the emergent focus (problems for research, evaluands for evaluation, and policy options for policy analysis) because that permits the multiple realities to define the focus (rather than inquirer preconceptions); [...]

Special criteria for trustworthiness. N is likely to find the conventional trustworthiness criteria (internal and external validity, reliability, and objectivity) inconsistent with the axioms and procedures of naturalistic inquiry. Hence he or she is likely to define new (but analogous) criteria and devise operational procedures for applying them. [...] it is worth noticing that the conventional criterion of internal validity fails because it implies an isomorphism between research outcomes and a single, tangible reality onto which inquiry can converge; that the criterion of external validity fails because it is inconsistent with the basic axiom concerning generalizability; that the criterion of reliability fails because it requires absolute stability and replicability, neither of which is possible for a paradigm based on emergent design; and that the criterion of objectivity fails because the paradigm openly admits investigator-respondent (or subject) interaction and the role of values.

A novice researcher, at the stage of planning his or her own work, will usually be looking for orientation. Within the framework of quantitative research, orientation will be sufficiently provided by applying the 'codified recipes' of statistics.

I	Whatever the research questions may be – all research is focused on and aimed at subjects, i.e. human beings.
II	The research subjects should be observed in their natural environment.
III	All research is based on individual cases.
IV	Generalisation of results can never be done by applying a general method; generalisation in any individual case requires individual arguments.
V	Establishing results in any case requires interpretation.
VI	All interpretation must be based on exact and comprehensive description.
VII	The research process has to be open for changing and developing both the research questions and the research methods.
VIII	Each change or development in the research process must be extensively justified and documented.
IX	All analysis is constructed by the researcher; all analysis requires an introspection (self-analysis) of the researcher.
X	All research is interactive – it changes both the research subject and the researcher.

Table 2. Personal 'decalogue' of qualitative research

With qualitative research, however, matters are different as recipes are obsolete. The more texts on qualitative research the newcomer consults, the more similar but miscellaneous aspects will be explicated and the more often he or she will be confronted with the advice to

follow his or her own ways and not to listen to others' advice. It is a question of economy to provide a list of subjective research guide-lines by extracting the texts according to personal needs. Far from being a recipe, such guide-lines may serve as landmarks of orientation. Permanent re-arrangements and re-formulations of the list according to the growth and change of experiences will accompany the research process and establish it as a permanent process of reflection. The author's guide-lines took the form of the following 'decalogue' (cf. Table 2).

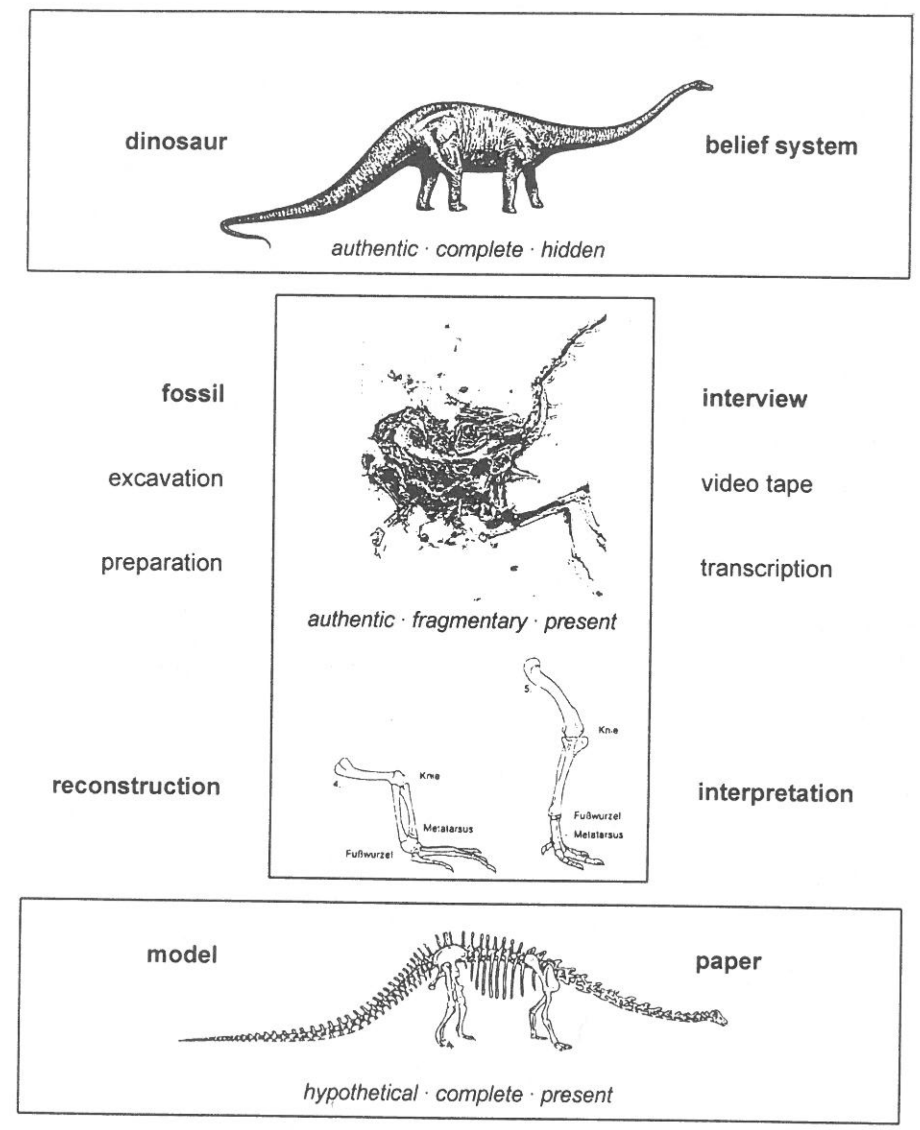

Figure 3. The 'palaeontology paradigm'

Exploring belief systems by interviews

To start with an anecdote from an interview with a teacher: "You are teaching both mathematics and computer science. Do you prefer one of those subjects?" — "No. Definitely not. No." The respondent seemed to have a clear self-concept as a teacher. With a questionnaire, things might have been clear at this point, however, it was an interview and after giving a detailed explanation of his view of the two subjects for about ten minutes, the respondent ended with the remark: "All that you simply can't do in math classes and that's why I so tremendously like to teach computer science."

The teacher apparently did not realise a contradiction, and in fact, there was none. It was only a normal inconsistency of conscious and subconscious attitudes which became evident, or in terms of beliefs, between a 'surface belief' (I do not prefer any of my subjects) and a 'deeper belief' (I tremendously like to teach computer science).

Interviews provide an appropriate method of gathering data within qualitative research. In her above quoted statement referring to Sigmund Freud, Tesch emphasised the techniques of asking and observing. Both are put to use in interviews. Even what the interview partners will not tell can be made accessible for observation by a thorough discourse analysis.

Exploring belief systems has some analogy with doing research in palaeontology (cf. Figure 3). What we are looking for, i.e. a person's belief system, may be authentic and complete, but it is hidden, just like a dinosaur. The only thing accessible to us is a fossil, an authentic and present, but fragmentary approximation to the original, an imprint or mark left by the original. So is the interview. After excavation and preparation, video taping and transcription, modelling of the original will be possible in some cases by the aid of reconstruction or interpretation. The model that we construct may be complete and present, but unfortunately it will be hypothetical. The fact that beliefs did not die out is of little help. We will never be able to lay our hands on a belief, as we will never encounter a dinosaur. They may both be real, but nevertheless they are just mental constructs.

Data gathering: interviews

Interview-based beliefs research can be organised according to a six-phase model:

Phase 1. Developing research questions

Phase 2. Developing interview techniques (adapting the 'human instrument')

Phase 3. Interviews (data gathering)

Phase 4. Transcriptions of the interviews (data processing)

Phase 5. Interpretation (data evaluation)

Phase 6. Writing a paper

According to the research characteristics described by Lincoln & Guba, adapting the *human instrument* and allowing the research design to *emerge* may be realised by an occasional backtracking between or by the combination of phases. Especially for novice researchers, substructuring the interview phase (or combining phases 1–3) will give way to both refining the research questions and to improving the interview techniques. With the author's own research project, the following sub-structure has proved to be well-suited for the purpose (cf. Figure 4).

The preliminary set of interviews provides the researcher with an empirical basis for designing the main set, i.e. a 'pool of themes and topics' and experience of what respondents will bring up on their own account. It allows to standardise the catalogue of interview questions of the main set without risking the loss of relevant information. At the same time, it offers the opportunity of training the researcher's skills in conducting interviews.

Within the framework of qualitative research a questionnaire may serve as an instrument of 'incubation' which prepares the respondents for the interviews. Qualitative inquiry is not

aiming at taking the respondent by surprise. An interviewee who is familiar with the project's topics is preferable by all means.

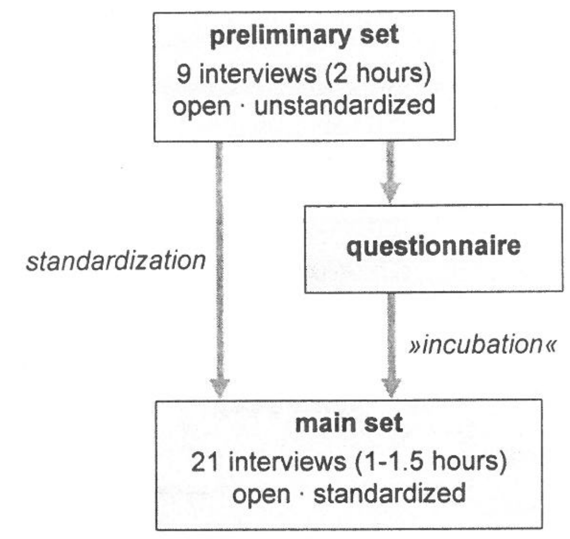

Figure 4. Sub-structuring the interview phase

Data processing: transcriptions

To guarantee a high degree of authenticity of the empirical data yielded by the interviews, those should preferably be video taped. Audio taping should be used only if demanded by circumstances. In any case, the taped interviews have to be transcribed.

Producing interview transcriptions means an arduous and lengthy work. The transcriptions, however, are necessary because written texts allow easier handling, documentation, and analysis of the interviews than video tapes would do.

Video taped interviews have their own 'dramaturgy' which may

- restrict the researcher's view
- capture the researcher's eye
- guide the researcher's understanding
- direct the researcher's attention to surface information
- make the researcher watch the respondent's statements, failing to notice what she or he does not say
- seduce the researcher to be passive (consuming), instead of being active (observing, analysing).

The same may apply to simply reading transcriptions as a whole text. As a thorough analysis of the interviews requires a detailed exploration from various points of view, transcriptions should, for interpretational aims, be processed in manifold ways. This can be done by rearranging and re-structuring the transcribed texts, for instance by sampling or extracting quotes referring to a certain topic (cf. *Data evaluation: interpretation*).

The transcriptions will consist of about 35'000 characters per hour of interview, however, transcribing means more than just typing – as is shown by the mere fact that punctuation may clarify or deform the original. Transcriptions have to be produced with a very high degree accuracy and, moreover, repeatedly require *proofs of the consistency* of the transcribed text (internal text control) as well as *verification* of the text on the basis of the video tapes (external text control) as shown in Figure 5.

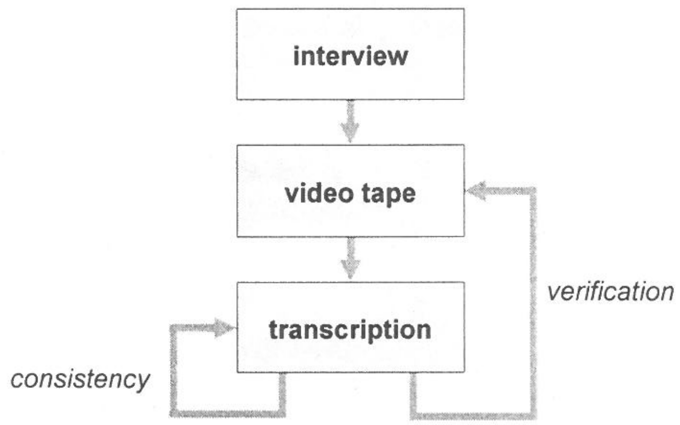

Figure 5. The process of making interview transcriptions

The cycles of consistency test and verification are the main instruments of approximating the transcription to the original interview. Yet, for economical reasons, one is bound to cut short these processes (after some cycles of verification and consistency testing) and to take the transcriptions as a basis for evaluation.

Although the transcription and interpretation phases should not be merged, even transcription may occasionally require some interpretation (because of insufficient intelligibility, audibility, and clearness of the respondents' statements). It should therefore be done by the researcher him/herself rather than by a typist (this holds especially for the process of verification).

Data evaluation: interpretation

The qualitative paradigm has some vital affinities to another set of powerful new ideas, i.e. the concept of *radical constructivism*. The constructivist approach provides ways of better understanding the nature, origin and development of knowledge. It may help the researcher to deeper conceive the respondents' ideas – and even improve the researcher's introspective insight into his own understanding. For a short outline of some basics of constructivism we quote from Noddings (1990, p.7):

> Constructivism is a popular position today not only in mathematics education but in developmental psychology, theories of the family, human sexuality, psychology of gender, and even computer technology. [...]
>
> Constructivism may be characterized as both a cognitive position and a methodological perspective. As a methodological perspective in the social sciences, constructivism assumes that human beings are knowing subjects, that human behavior is mainly purposive, and that present-day human organisms have a highly developed capacity for organizing knowledge. These assumptions suggest methods – ethnography, clinical interviews, overt thinking, and the like – specially designed to study complex semi-autonomous systems.

> As a cognitive position, constructivism holds that all knowledge is constructed and that the instruments of construction include cognitive structures that are either innate or are themselves products of developmental construction.

At current constructivist discourse, various conceptual differences are discussed. However, there is agreement about the following (Noddings, p.10):

1. All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
2. There exist cognitive structures that are activated in the process of construction. These structures account for the construction; that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.
3. Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.
4. Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism.
   a. Methodological constructivism in research develops methods of study consonant with the assumption of cognitive constructivism.
   b. Pedagogical constructivism suggests methods of teaching consonant with cognitive constructivism.

The research process of asking and observing people, of interviewing and interpreting, is structured in phases for methodological reasons. The phase model, however, involves the risk of what we may call a 'methodological routine-blindness'. The research stages *person*, *interview*, *video tape*, *transcription*, and *interpretation* form the layers of a more and more constructed reality (cf. Figure 6). To prevent blindness, it will be necessary to keep the research subject in view, i.e. the human being. And that means to 'keep the layers transparent' and to conceive of the stages as facets of a whole with multiple realities.

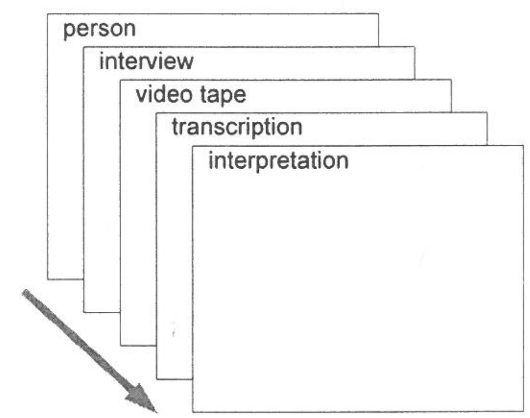

Figure 6. Layers of a constructed reality

To provide an introduction to the art of interpretation would be far beyond the scope of this paper. In the following, we therefore confine ourselves to giving some technical hints. For a general survey of hermeneutics, the reader is referred to the profound publications of Beck & Maier (1994), Oevermann (1986), Oevermann et al. (1979), and Titzmann (1993); a deep understanding of the process of interpretation is provided by the work of the French philosopher Paul Ricœur (Ricœur 1975 & 1985).

The processing of the transcription texts mentioned above can be described as 'cutting' it from one context and 'pasting' it into another, according to Marton (1986, p.43): "Each quote has two contexts, [...] first the interview from which it was taken, and second, the 'pool of meanings' to which it belongs." From a more formal point of view, these operations are referred to as *de-contextualization* and *re-contextualization* (cf. Figure 7). They provide one of the main techniques of interpretational qualitative analysis.

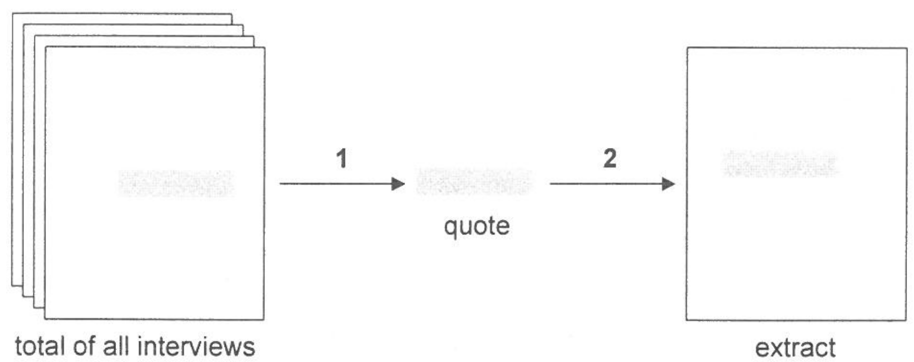

Figure 7. De-contextualization (1) and re-contextualization(2)

Especially with standardised interviews, extracts from the total of all transcriptions will allow the researcher to re-read the original interview texts. The 'multiple realities' encountered in an interview will often only surface when the perspective of observation is changed, e.g. by analysing a quote within a new, but relevant, context. At the most basic level, the extracts take the form of a whole transcription ('vertical extract', cf. Figure 8).

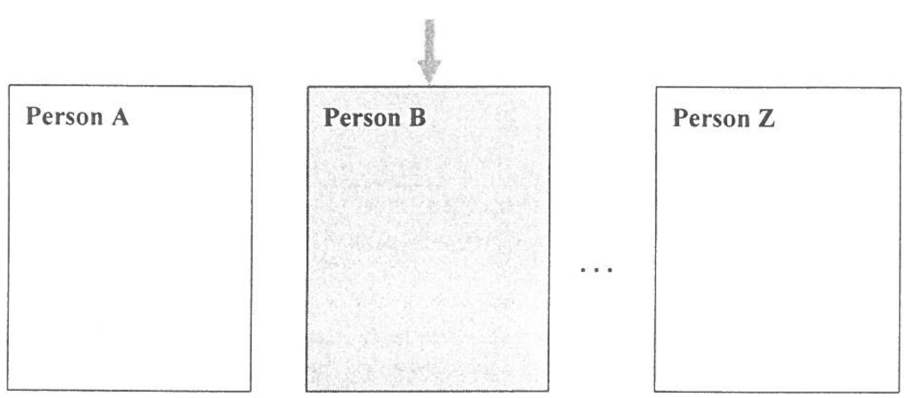

Figure 8. Vertical extract

The second type of extracts is provided by sampling all the contributions to a specific question ('horizontal extract', cf. Figure 9). This can easily be done with a computer by using the standard 'cut' and 'paste' features of text programs.

The third type of extracts ('discrete extract of keywords', cf. Figure 10) consists of all transcription contexts containing a certain keyword. This, too, can be done by a text program, using the standard features of 'searching' and 'sorting'. All those procedures can be automated by macros.

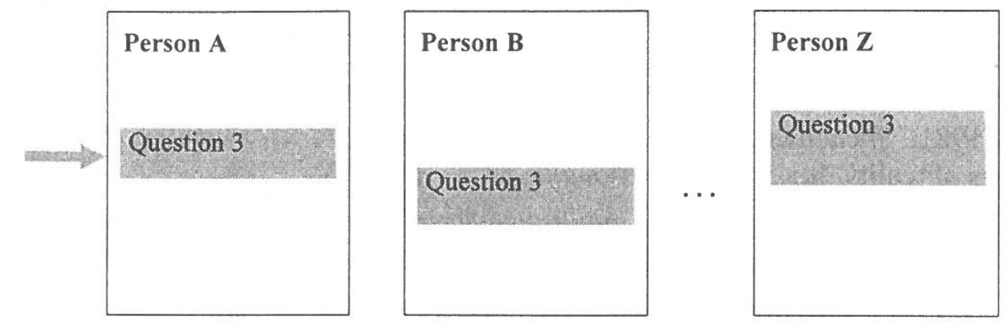

Figure 9. Horizontal extract

These three varieties of extracts can be sampled by simple syntactic operations (pattern matching). Yet one of the most important principles of interpretive discourse analysis requires extensive semantic operations and can therefore not be automated, or at the utmost only partially, i.e. here we encounter the *principle of exhaustion.* To gain a profile of the respondents' views concerning a certain topic, *all* meaning units (statements, phrases, etc.) referring to this topic have to be sampled. Following the maxim that no meaning unit is meaningless, each instance has to be found and registered. Interpretation has to consider each single instance by assigning it to at least one 'interpretational category'. As meaning units are semantic, and not syntactic, entities, this requires a thorough non-automated interpretation by the researcher.

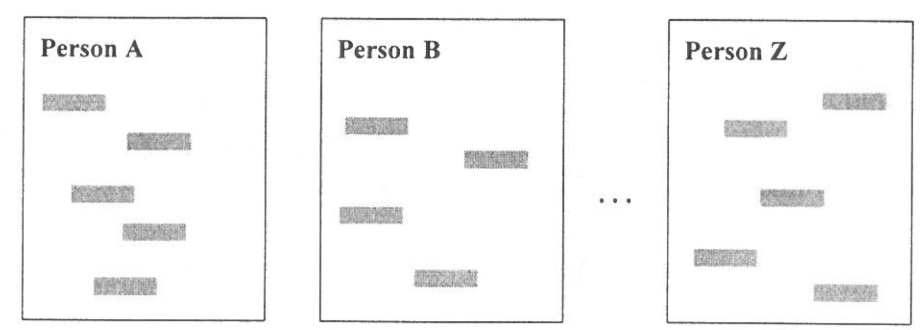

Figure 10. Discrete extract

So there is a fourth, and most important, type of extracts, the 'discrete extract of meaning units', which demands all the interpretive skills and creativity of the (human) researcher. Computers may often increase or even encourage human creativity – however, it is one of the author's conscious and deeply rooted beliefs that before he will see a computer doing qualitative research he will encounter a dinosaur.

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Constantinos Christou & Georges Philippou

The Mathematical World Views: The Case of Cypriot Middle School Students (Grades 6-9)

Introduction

Studies of affective issues have always been central to the goals of mathematics education, since students' beliefs about mathematics affect their enthusiasm for studying the subject and their decision whether they will continue on to more advanced studies in the field. One major concern of research on beliefs was the quality of the instruments that were being used. A particular problem of the use of such instruments was the difficulty of finding adequate explanations of the relationship between beliefs and achievement as well as the causal relationships among them.

A recently published article (Pehkonen, 1996) reported on the development of an instrument by Zimmerman and Pehkonen for measuring student beliefs concerning the teaching of mathematics in different countries. According to Pehkonen (1996), an individual's belief system is divided into four main categories: Beliefs about Mathematics, beliefs about oneself within Mathematics, beliefs about Mathematics Teaching, and beliefs about Mathematics Learning. These main categories of beliefs are also reflected to a certain degree in the Zimmerman - Pehkonen questionnaire. In this article we focus on the cognitive level of the belief system, assuming that the subjective knowledge of mathematics and mathematics teaching and learning embraces the above mentioned categories of beliefs. Specifically, the main goals of this study are:

1. To determine whether the Zimmerman - Pehkonen instrument can identify the cognitive level of the students' belief system.
2. To explore Cypriot students' "mathematical world views".
3. To find out the differences (if any) among students with regard to some personal characteristics such as their sex, grade level, and type of school (urban vs. rural).

Theoretical Background of the Study

The prevailing ideals of the nature of mathematics and the superordinate basic philosophies about the teaching and learning of mathematics can be classified into two categories, namely the static and the dynamic. From the static point of view, mathematics is an abstract structure of knowledge, which consists of accumulated rules, formulas, terms, and algorithms. Consequently, the learning of mathematics is seen as the application of definitions, facts and procedures, and of logical, formal deduction to verify hypotheses and systematize results. Hence, students must be provided with "ready" or "prepared" knowledge, and what is needed from them is to memorize and re-produce facts, and apply rules in order to solve problems.

This contrasts with the notion of mathematics derived from the dynamic point view according to which mathematics is an active way of thinking about problems and reaching conclusions. Students, from this point of view, need to invent and re-invent mathematics by understanding facts and making connections to previously discovered principles. Re-inventing mathematics means developing the ability to do mathematics, which is the actual goal of teaching the subject. Mathematics is neither only memorization and application of definitions, formulas, facts and procedures, nor is it drill and practice.

Definitions of beliefs include one's subjective experience which is based upon implicit knowledge of mathematics and its teaching and learning (Pehkonen, 1997). The spectrum of an individual's beliefs is very wide, and form a structure with multiple components, which influence each other. The individual's belief system consists of three components: cognition, affection, and action (Grigutsch, Raatz, & Törner, 1997). The cognitive component can be considered as the subjective knowledge of mathematics, the affective component refers to the emotional relationship with mathematics, and the action component is relevant to the readiness or tendency of a person to act in a certain manner. Thus, beliefs towards mathematics constitute a very complex and multi-layer system that enables individuals to find orientation in their environment. In this way, we assume that there is a hypothetical construct "world of beliefs", which is a system of beliefs towards mathematics. There are two levels from which we can gain information about the "mathematical world views" of students. The first source refers to the information that can be gathered from students' expressions on single belief items, and the second refers to the information that can be inspired by the relationships among single belief items. Our research focuses on the overall spectrum of "mathematical world views", and not on separate items.

In this study we emphasize on the identification of the structure of students' beliefs and views towards mathematics, since the establishment of the mathematical world view of students constitutes the foundation for the emergence and the development of belief factors as well as the changes in the system of their beliefs. In addition, students' beliefs may be connected with their learning behavior and the way they are learning and doing mathematics (Thompson, 1991). In many situations, students who think of mathematics from the static point of view will be more likely to consider the memorization of facts and procedures as more important than the comprehension of mathematical ideas and relationships, which are mostly favored by those adhering to the dynamic point of view.

Students working with mathematics develop personal experiences that create beliefs, which affect their future behavior, and this behavior, in turn, determines current and future experiences. This may result in cumulative processes which sustain themselves, and thus beliefs can turn into a kind of self-fulfilling prophecies, which in most cases reinforce existing beliefs. In this manner, it is assumed that there exists an interdependence among beliefs and mathematics teaching and learning.

Taking into consideration the importance of beliefs in the teaching and learning of mathematics, this study purports to investigate the "mathematical world views" of students through the Zimmerman - Pehkonen's questionnaire. Of course, it is not possible to describe the complete field of "mathematical world views" of students through a single questionnaire, and thus the present study is restricted to the cognitive level of beliefs which deals mainly with the student beliefs towards the nature of mathematics, and consequently, towards the teaching and learning of mathematics.

Method

In constructing their questionnaire Zimmerman and Pehkonen assumed that the development of students' mathematical beliefs is based on the idea that one's belief system consists of five dimensions: beliefs about Self, Mathematics Content, Mathematics Learning and Teaching (Pehkonen, 1996). Thus, we administered the Zimmerman - Pehkonen questionnaire to determine whether a similar pattern of results would also emerge in Cyprus.

Subjects

Data were collected on 1099 Cypriot students from grade 6 to grade 9, both from urban and rural schools, including 427 students in grades 6, 222 students in grade 7, 231 in grade 8, and 219 students in grade 9. Table 1 gives details of the grade, type of school (urban and rural), and sex distribution of students.

	Grade 6			Grade 7			Grade 8			Grade 9		
	Urban	*Rural*	*Total*	*Urban*	*Rural*	*Total*	*Urban*	*Rural*	*Total*	*Urban*	*Rural*	*Total*
Males	167	63	230	57	49	106	63	56	119	55	511	106
Females	143	54	197	62	54	116	66	49	112	58	55	119
Total	310	117	427	119	103	222	126	105	231	113	106	219

Table 1: The sample of the study.

Procedure

We administered the Zimmerman - Pehkonen questionnaire, which included 32 items designed to measure general beliefs on the teaching and learning of mathematics. These items were factor analyzed using Principal Axis factoring with varimax rotation. A five factor solution was identified as being the most appropriate in isolating six distinct scales relating to beliefs.

Analysis of variance was then used to determine whether differences in beliefs could be predicated from the set of variables identifying characteristics such as grade, gender and school (rural and urban).

Results

Factor Analysis

In determining how the sample matched the Pehkonen's hypothesized belief categories, a five factor solution resulted in the best distribution of items into identifiable subscales (The five factors accounted for variances greater than 1, that is the eigenvalue was greater than 1). First, the correlation matrix for all variables was computed to identify the variables that do not appear to be related to other variables. The items 3, 4, 5, 12, 15, 18, 21, 31, and 33 of the original instrument did not have large correlations (e.g. less than .3) with at least one of the other variables, and thus, we excluded them from further analysis. The resulted five factors seem to be meaningful in the sense that they describe students' beliefs towards the learning, the teaching, and the content of mathematics, as well as students' beliefs about their own role and their teachers role in the teaching-learning process.

The first factor consists of six items (29, 23, 24, 18, 19, and 21) that are homogeneous in content and thus it can be interpreted meaningfully as an aspect of mathematics that emphasizes students' beliefs about the learning of mathematics (Table 2). This factor explained 20% of the variance and had a correlation coefficient of .67. The negative loads in items 29 and 23 indicate that students, in general, do not conceive of mathematics learning as demanding much effort or much drill and practice.

The second factor, with a correlation coefficient equal to .52, explained 16% of the variance, and reflected student beliefs towards the content of mathematics (items 6, 9, and 22). All items have a positive load indicating that students believe that mathematics teaching involves a wide spectrum of topics such as problem solving and calculations (Table 2).

Item	Factor 1: Students' Beliefs about the Learning of Mathematics (a=.67, explained variance = 20%)	Load
29	Mathematics learning (ML) needs as much practice as possible	-.55
23	ML demands much effort	-.43
24	In mathematics there is more than one way of solving problems	.43
19	Mathematics teaching involves tasks of practical benefit	.42
18	ML needs as much repetition as possible	.41
21	Mathematics teaching cannot always be fun	-.20
Item	Factor 2: Students' Beliefs about the Teaching of Mathematics (a=.52, explained variance = 16%)	Load
6	Mathematics teaching involves drawing of graphs...	.63
22	Mathematics teaching involves calculations of areas, volumes...	.44
9	Mathematics teaching involves problem solving	.42
Item	Factor 3: Students' Beliefs about the Nature of Mathematics (a=.63, explained variance = 14%)	Load
7	Mathematics is a discipline that requires quick and correct answers	.57
8	Mathematics is a strict discipline	.47
12	Mathematics is learned by heart	.40
1	Mathematics involves mental calculations	.36
16	In mathematics everything is reasoned exactly	-.22
5	In mathematics everything is expressed exactly	.16
Item	Factor 4: Students' Beliefs about their Role in the Learning of Mathematics (a=.45, explained variance = 13%)	Load
31	Students work in small groups	.57
13	Students ... put forward their own questions	.45
28	Students ... construct concrete objects	.43
25	Mathematics are learned through games	.41
Item	Factor 5: Students' Beliefs about Teachers' Role in the Learning of Mathematics (a=.52, explained variance = 12%)	Load
26	Teacher explains every stage in detail	.45
27	Students solve tasks... independently	.38
15	Teacher help when difficulties arise	.32
10	There is a procedure to follow	.31

Table 2: The extracted factors with items and items loadings for each factor.

There are four items connected with the third factor (correlation coefficient =.63, and explained variance=14%) that load over .4, and two items that load less than .3. All items are homogeneous and can be interpreted as relevant to the nature of mathematics, i.e., mathematics is a subject which requires correct answers and is characterized by strictness and precision. The first statement loads very high in this factor (.57) indicating that students emphasize the speed and accuracy of calculations, while at the same time they seem to reject the idea of having to give reasons or express their reasoning in answering mathematical problems or exercises (see item 16 with the negative load).

The fourth factor (correlation coefficient = .45, and explained variance=13%) can be identified as the students' beliefs concerning their own role in the process of learning mathematics. The four items in this factor have positive loads and all of them are higher than .4. The fifth factor (coefficient alpha = .52, and explained variance 11%) reflects students' beliefs about the role of the teacher in the process of teaching mathematics. All items in this factor are homogeneous except for item 10, which also loads on the nature of mathematics factor.

The five extracted factors-learning, content, and nature of mathematics, as well as students' and teachers' role in the learning of mathematics-are dimensions that are essential to one's "mathematical world view". The homogeneity of the factors hints at these dimensions and contributes to the structuring of students' perceptions and cognitive representations of mathematics.

Differences among students

To answer the second question of the study, i.e., whether the extracted factors can be used to predict students beliefs in terms of their personal characteristics, we used Analysis of Variance (ANOVA) with the factors as dependent variables and students' sex, grade level, and type of school as independent variables.

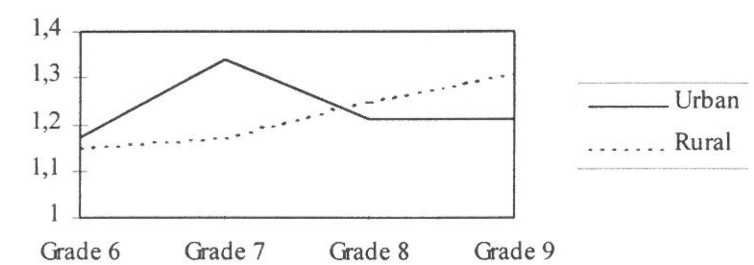

Figure 1: The interaction between grade level and type of school with students' beliefs towards the learning of mathematics as dependent variable.

As far as the learning of mathematics is concerned, there is a statistically significant main effect due to the sex of students, and an interaction between the grade level of students and their type of school. Specifically, it was found that male and female students do not have the same beliefs about the learning of mathematics. Females (m=1.18) have more positive beliefs about the learning of mathematics than males (m=1.25)[1]. In addition, the interaction between grade level and type of school (Figure 1) indicates that students in grade 6 (elementary school) do not differ in their beliefs about the learning of mathematics. However, in grade 7

[1] Number 1 in the Likert-type scales meant fully agreement, while 4 meant disagreement. Thus, small means indicate more positive beliefs.

students in rural schools seem to have more positive beliefs than their counterparts in urban schools. In grades 8 and mostly in grade 9 urban students develop more positive beliefs than rural students.

Students' beliefs towards the teaching of mathematics were found to differ in terms of their grade level and gender. A post-hoc analysis revealed that students in grade 6, and particularly girls, considered mathematics as consisting of a wide range of topics such as calculations, problem solving and drawing figures (items 6, 9, and 22). On the other hand, students in grades 7, 8, and 9, and especially male students, viewed mathematics teaching as consisting mainly of calculations. This may be explained by the fact that in secondary education what receives more emphasis is the finding of the correct solution to given exercises.

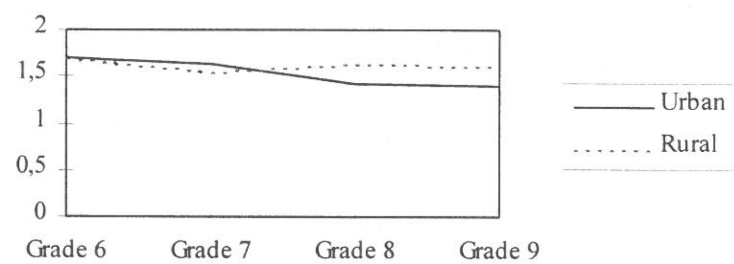

Figure 2: The interaction between grade level and type of school with students' beliefs towards the nature of mathematics as dependent variable.

The same pattern of results as with students beliefs about the learning of mathematics was found in the case of students' beliefs towards the nature of mathematics, i.e., a statistically significant interaction between the grade level and the type of school as well as a significant main effect involving the gender of the students. Figure 2 depicts that students in elementary schools do not differ in their views about the nature of mathematics. The differences are obvious in grades 8 and 9, where urban students develop more positive views about the nature of mathematics than rural students. These differences might be attributed to gender differences as female students (m=1.57) begin to conceive of mathematics in a more comprehensive way than males (m=1.67).

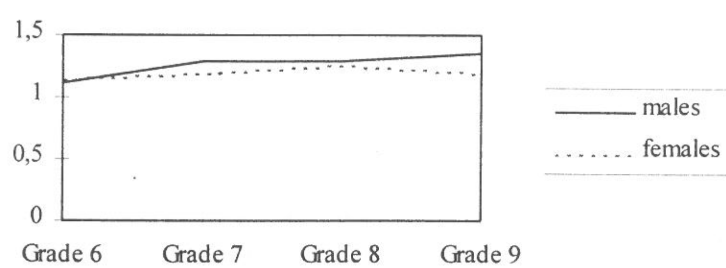

Figure 3: The interaction between grade level and sex with students' beliefs towards their own role in the learning of mathematics as dependent variable.

A statistically significant interaction also exists between the grade level and the gender of students concerning students' beliefs about their own role in the learning of mathematics. The interaction (Figure 3) indicates that students in grade 6 conceive of their role in the teaching

of mathematics in a similar manner, while the views of male students in high school are less positive than their female counterparts. Furthermore, students in rural schools (m=1.26) conceive of their role in the teaching of mathematics in a less positive manner than students in urban schools (m=1.18).

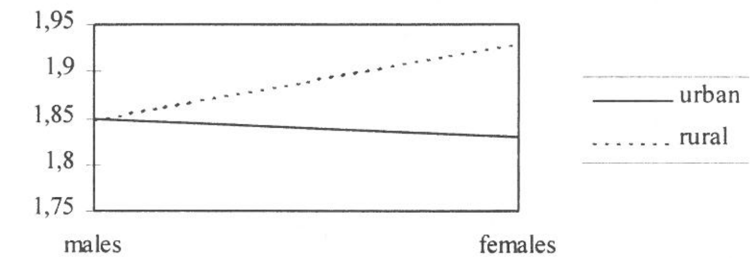

Figure 4: The interaction between type of school and sex with students' beliefs towards the role of teachers in the learning of mathematics as dependent variable.

Students' views about the role of the teacher in the process of teaching and learning of mathematics differ in terms of their grade level. Post hoc analysis showed that students' beliefs in grades 8 and 9 are less positive than those of students in grades 6 and 7. On the other hand, there was a statistically significant interaction between students' gender and type of school. Figure 4 shows that male students, independently of the type of school, hold the same beliefs about the role of the teacher, while urban female students are more positive than female rural students.

Conclusions

The five extracted factors indicate a simple structure, and, therefore, enable us to subdivide the set of statements into meaningful categories or dimensions. This in turn means that the dimensions can be operationally defined, since the statements within each factor are homogeneous in content. In this respect, the statements probably define operationally the given feature (learning, teaching, nature, students' role and teachers' role), and the measuring of these features are probably valid. Thus, the attributes learning, content, nature, students' role and teachers' role in the learning and teaching of mathematics, seem to constitute dimensions, which categorize students' perceptions, cognitive representations, experience, and their ways of responding to mathematics.

Even though we have extracted only five factors, we are certain that the "mathematical world views" is a very complex construct. It certainly contains a great deal of elements and relationships among these elements (Pehkonen, 1996). However, we assume that the five extracted factors constitute fundamental dimensions, which can characterize the subjective knowledge of students about mathematics, independently of the complexity of the hypothesized models that are under study each time.

Moreover, it seems that the five factors can be predicated by the students' personal characteristics. Specifically, it was found that the students' "mathematical world views" is not homogeneous, but heterogeneous, since students have different beliefs in each of the five dimensions, which range from rejection to approval. Differences were found, for example, among students in terms of their sex, type of school, and grade level. Female students as well as urban students seem to hold more positive beliefs towards the five dimensions than male and rural students. In other words, it was found that males and rural students believe to a greater degree than female and urban students that mathematics learning is a rigid, exact subject, and that it is a discipline with a need for accurate results and infallible procedures that require repetition, memorization, drill and practice. Furthermore, it was found that female students enjoyed mathematics more than male students and conceived of themselves as active participants in the teaching and learning process. On the other hand, male and rural students considered themselves as passive recipients of mathematical knowledge, and thus they perceived teachers as the individuals that are responsible for transmitting mathematical knowledge, and verifying that students have received this knowledge.

It was was found that there were no differences among students in elementary school (grade 6), while these differences arise in high school. This result confirms those found in previous studies (Fennema, & Hart, 1994), and reinforces the idea that students in high school begin to conceive of mathematics as a discipline that requires much effort, and that mathematics is made up of an accumulation of facts and rules to be used skillfully by the trained artisan in the pursuance of some external end. However, what is of most importance in the present study is the fact that there are differences among high school students in rural and urban schools, and that females and males differ with respect to their "mathematical world views". Gender differences still exist in personal beliefs concerning mathematics, but these differences favor females. What remains to be done is (a) to find out the direct effects of sex, as well as of the grade level and type of school on students' beliefs, and (b) to examine the direct effects of the categories of students' beliefs on each other and the relationships among these categories.

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Margareth Drakenberg

Attitudes to Mathematics in Grades 1-6

Background

During the 1980s, the interest in affective variables in mathematics education became a focus of research and many review articles were published (e.g. Reyes 1984; Pajares 1992; McLeod 1994), focusing on attitudes towards mathematics, rather than questioning the various components of the affective domain. However, some of the more frequently encountered areas of interest have included understanding of emotions, attitudes and beliefs of students, as well as of teachers, and their relation to cognitive variables. The great majority of these studies has used attitudes as a general term that includes beliefs about mathematics and about self. In this paper attitudes refer to affective responses that involve positive or negative feelings of moderate intensity and reasonable stability. This does not mean that the author is not aware of the fact that there are many different kinds of mathematics as well as a variety of feelings about each type of mathematics.

Most of the studies of affective variables in mathematics learning have focussed junior and senior high school level. Very few studies consider early elementary school students to be of interest, in spite of the circumstances that the students' attitudes towards mathematics, most probably, start their development during these early years in school. Nor does it seem to be of any interest, during those years of schooling - or later on -, to develop any kind of 'remedial' program in order to equalize growing differences between boys and girls regarding attitudes towards mathematics.

Aim of the study

That women are under-represented in occupations related to mathematics and science is, by now, a well-known fact. You can see this as early as junior and senior high school levels where the mathematical and/or science programs are heavily dominated by males. In an attempt to equalize this difference between males and females, a few municipalities in Sweden have initiated special science programs (remedial programs) for pre-school children. No scientific evaluations have, so far, been conducted and a question bothering me has been whether the pre-school years are the most appropriate for such a program? At what grade levels are differences in attitudes toward mathematics noticeable? What may influence females to be less confident than males in mathematics? Do male students send subtle messages that they, for instance, do not trust females' knowledge in mathematics?

Methodology

In order to get some answers to these questions a tentative study was conducted. However, most studies regarding sex-differences in attitudes in mathematics have focused junior high or secondary high school level and they have shown that many students have already fully developed attitudes toward e.g. mathematics at the junior high level. In this study, focus has instead been restricted to grades 1 to 6. A total of about 150 students have answered a questionnaire of attitudes in mathematics. The questionnaire is a Swedish version of the Fennema-Sherman Attitude Scales, which I became acquainted with during my post doc-year at Prof. Fennema's department (Fennema et al. 1980). This questionnaire, demonstrating attitudes towards mathematics as multifaceted rather than singlefaceted a construct, has also been used in the 80s in some Swedish studies (Drakenberg & Mattsson 1983;1984;1988) and I have also noticed the use of the Scales in Finnish studies (i.e. Malmivouri 1996).

The items used in this study was thus adapted to Swedish speaking circumstances and focused upon students' beliefs about their weaknesses and strengths in mathematics, usefulness of mathematics, mathematics as a male domain and cooperation with other students. In responding to the different statements the students had to choose between fully agree, partly agree, hesitant or fully disagree. The data, in this study, was collected this spring semester in a Finnish elementary school during a normal lesson in mathematics. I am very grateful to the students, teachers and their principal for their positive interest and cooperation.

At this state of the research the data collected has been analyzed in the simplest possible ways, i.e. only analyses of means have been used in order to get a general idea about the students' beliefs, their development and gender differences, if any. The ways of analyses can be considered justified due to the character and main purpose of the study, i.e. a tentative study conducted to give some hints about the origin and the development of elementary school students' attitudes and beliefs towards mathematics.

Results and discussion

The items in the questionnaire have been organized into themes and the results will be presented and discussed following these themes: Attitudes towards the subject; Confidence ; Perceived usefulness; Work methods; and Boys' attitudes towards girls' knowledge in mathematics.

Attitudes towards the subject.

U.S.-reports (e.g. Dossey et al. in McLeod 1992) have indicated that there is a general decline in the percentage of students who say they enjoy mathematics as they proceed through school. Also students in other countries show little enthusiasm for mathematics as they progress through school (see e.g. Foxman et al. in McLeod 1992).

All the students in this study, independent of grade level, considered mathematics to be a very important subject although it did not belong to their favorites or to the subjects they liked the least. In their responses the students also indicated that they found mathematics neither enjoyable nor interesting. As seen in Figure 1, girls from grade 3 to grade 6 indicate that their interest in mathematics is continuously declining the more they learn. I think this is serious, telling us that something has to be done about the content of mathematics during these years in order to catch the interest of the girls. Analyses of mathematics and science studybooks

have shown that there are too seldom female-related tasks in these books that could make the girls feel familiar with the circumstances described in those tasks.

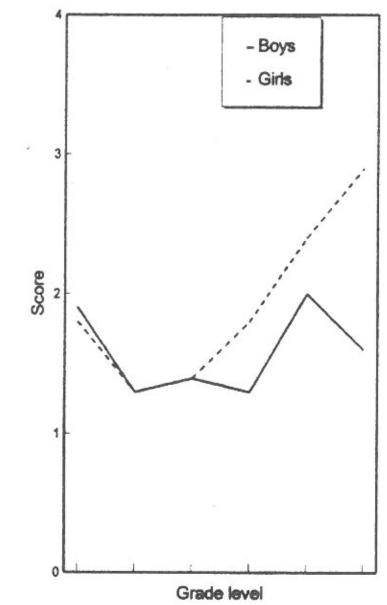

Figure 1. Boys' and girls' interest in mathematics.

Confidence in mathematics

Students' beliefs about their competence in mathematics are an important affective factor in mathematics classrooms, and over the years confidence in mathematics has been frequently studied. A substantial amount of data has been collected regarding gender differences in confidence when working in mathematics, but, so far, very little research has focused on how children develop their personal beliefs about themselves as learners. Research on confidence in learning mathematics (e.g. Reyes 1984, Reuterberg 1996) showed that in general boys tend to be more confident and estimate their own capacity to be higher than the girls.

As many studies have already shown, also this study showed that the boys' confidence in mathematics is higher than the girls.Throughout the six grades in elementary school, boys consider themselves more clever than girls, they feel they know what they are doing. They also find mathematics easy and believe they are able to solve also the most difficult tasks. In Figure 2 we can see that from grade 3 to 6 girls become more and more insecure in their working on mathematics tasks and as a consequence, their interest in the subject is continously declining.

Such a difference in perceived confidence between boys and girls has also been shown to influence the children's intentions to seek help in the classroom. Newman (1990, 77), studying elementary school children, has shown that children who believe they are competent are likely to seek assistance when needed - the implication of which is that "those most in need of help may be those most reluctant to seek help". In a complementary study, Newman & Goldin (1990), the above mentioned results are given a deepened analysis showing that girls are more hesitant than boys to ask for help because they are so embarrassed and afraid of nagative reactions to their help-seeking behavior. Such reactions might lead to girls not getting adequate help in mathematics. This may influence them in such a way that it is not very likely that they take mathematics in school when it becomes optional.

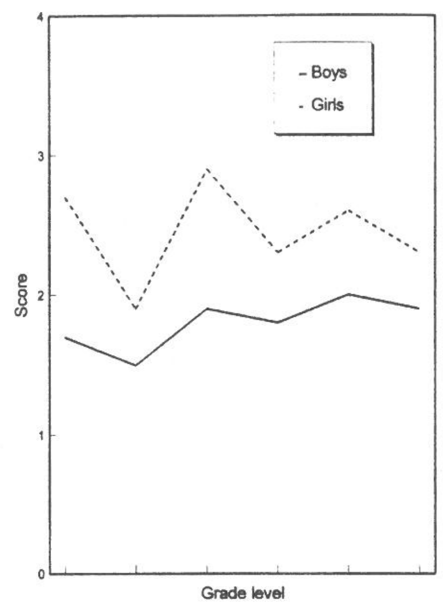

Figure 2. Boys' and girls' confidence in mathematics

Perceived usefulness of mathematics

All the students in this study agreed upon the statement that mathematics can be used in many different ways and thus that it has a great variety of uses. All the students also considered it important to get a good grade in mathematics because good grades in mathematics might lead to a good job. Most of them agreed that they would need mathematics also in their future jobs. In two items, one located in the beginning of the questionnaire and the other at the end, all the students are rather hesitant or fully disagree to the statement "I don't think I will use mathematics after I have finished the school", see Figure 3. Fennema (1989), in summarizing her results on this issue, noted that males in general reported higher perceived usefulness than females. This is the case also in this study, but the differences between the sexes are not big.

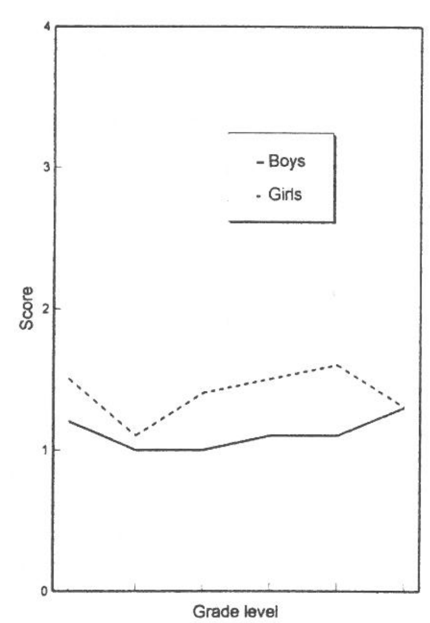

Figure 3. The perceived usefulness of mathematics.

Work methods in mathematics

Research dealing with mathematics learning has given increased attention to the social context of instruction. Here, not only classroom's social context is important, also the school as well as the home have an effect upon students' beliefs and their appreciation of cooperation.

The results from this theme of questions showed in this study that girls are more positive than boys when it comes to working in groups and to getting the possibility to discuss solutions with their classmates, see Figure 4. To let the students discuss the math problems, their possible solutions and solution-strategies seem to favour girls who indicate they understand much better what they are doing. On the other hand, to work in groups can also be conceived of as having to adjust one's own work tempo to the rest of the group, something the boys are not that fond of. Instead, the boys prefer a competition situation where they can work in their own pace.

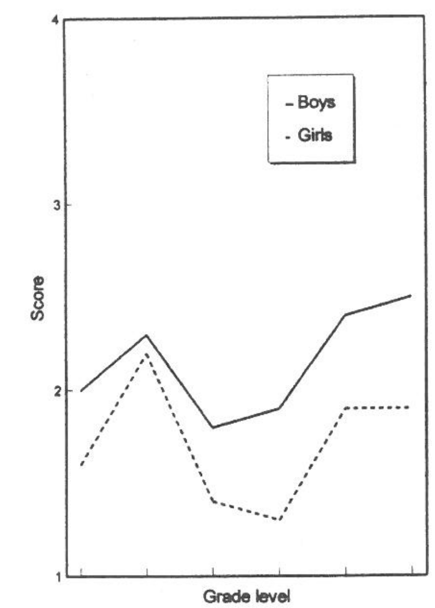

Figure 4. Cooperation in solving and discussing mathematical tasks.

In their interesting book, "The story about girls and boys", Bjerrum-Nielsen & Rudberg (1989) have shown that the two sexes approach the learning situation in different ways. Their results coinside with U.S.-results, e.g. Peterson & Fennema (1985), who have been able to show a positive relationship between, in particular, girls' achievement and cooperation activities, while boys' achievement is more influenced when elements of competition are present.

Boys' attitudes towards girls knowledge in mathematics.

As early as the first grade, the boys consider themselves smarter in mathematics than the girls and a peak is reached in grade 2, see Figure 5, where all the boys fully agreed that they were more clever than girls in mathematics. In this grade also many boys consider the girls who are interested, or clever, in mathematics to be "nuts". This under-estimation of the girls' knowledge in mathematics is further strengthened during the elementary school years, here investigated, by the boys' attitude never to ask a girl about the correct answers or for help in mathematics. The predominant reason for choosing the particular helper is, according to

Newman & Goldin (1990, 93) "the child's perception of the individual's competence". Girls' knowledge in mathematics is therefore often oversighted by the boys.

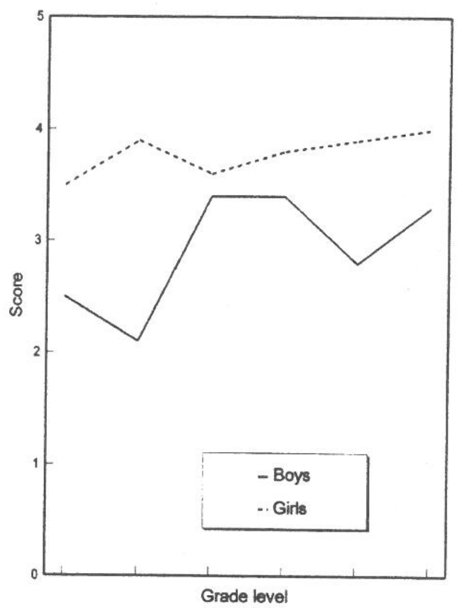

Figure 5. Boys' attitudes to girls' knowledge in mathematics I.

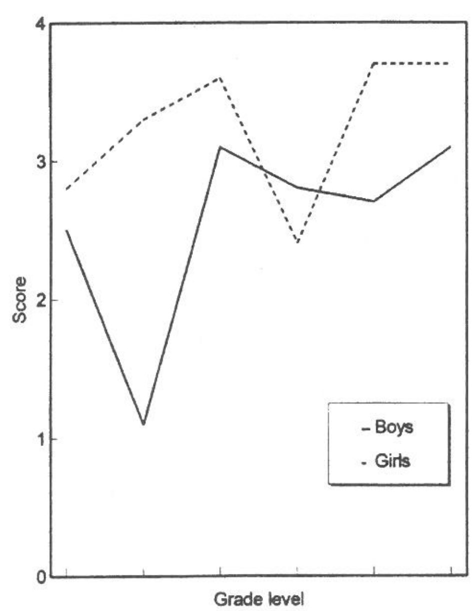

Figure 6. Boys' attitudes to girls' knowledge in mathematics II.

Summary

In most studies of attitudes in mathematics, junior or senior high school students have been investigated because gender differences have been found in both attitudes and achievement in mathematics at those school levels. But these attitudes start to develop much earlier and further develop, in this case, through students' personal learning experiences with mathematics.

Through these experiences the students will construct their beliefs about themselves, about mathematics, and its learning.

In this study, it was found that the girls' knowledge of and confidence in mathematics are influenced negatively and they lack support already from the lowest grades in elementary school. As a consequence, the girls' enjoyment and interest in mathematics is continuously declining during the elementary school years and when getting to junior high school level many girls are less likey to take mathematics when it becomes optional, or they take only the shortest or easiest courses.

So, according to this study, if some kind of remedial programs are to be offered they should be carried out during the students' first years in elementary school. However, this study is a tentative study and inferences must be made with obvious care. On the other hand, the correspondence of the present findings with well-established literature gives the results some validity and the study can be considered to provide interesting information for guiding future research.

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Günter Graumann

Beliefs of First-Year-Students about the Mathematical Education of Teacher Students

In April 1996 and in January 1997 among first-year-students which want to become primary school teacher I handed out several questions about mathematics education at school and about the mathematical education of teacher students at university. Some interesting results in respect to the mathematics education at school I offered already at MAVI 3 in Helsinki (see in: E. Pehkonen, Research report 170, Department of Teacher Education, University of Helsinki 1996). Here, I want to talk about the beliefs of the students in respect to their coming education in mathematics and didactics of mathematics.

For better understanding I must say that in the present circumstances in North Rhine-Westphalia all students which want to become primary school teacher have to study mathematics and didactics of mathematics (each topic at least 10 to 12 hours/week/semester spread over 6 semesters). When I gave to them the questionnaire they have had only one course about introduction into didactics of mathematics.

The questions about the mathematical education of teacher students are the following:

"What does belong to the mathematical study of future primary teacher ?

(41) Natural numbers and calculation with natural numbers
(42) Mastering fractions
(43) Concepts and theorems of elementary geometry and imagination of space
(44) to know more than only primary school mathematics
(45) ability to offer proofs
(46) reflections on mathematics
(47) to do exercises regular
(48) to have exemplarily a good look at one problem of elementary mathematics at one's own.
(49) to deal with history of numbers and history of mathematics
(50) to deal only with questions in respect to didactics of mathematics
(open question) wishes and remarks about the mathematical education of primary school teacher."

Each question should be answered twice - first in respect to the IS-state (they think) and secondly in respect to the 'SHALL-state'. The scale for each answer consists of -2 (fully disagree), -1 (disagree), 0 (partly-partly/undecided), +1 (agree), +2 (fully agree) and n (no answer).

The number of questionnaires with usable answers I got is 200 for April 1996 and 41 for January 1997.

Until now I have only worked out the answers of the questioning from January 1997. At that time the students have had already three semesters experiences with mathematical education for primary school teacher at the university of Bielefeld. The results of the questions (41) to (50) are the following:

No	IS-state							SHALL-state							Diff.
	-2	-1	0	+1	+2	*n*	$\overline{x_1}$	-2	-1	0	+1	+2	*n*	$\overline{x_2}$	$\overline{x_2} - \overline{x_1}$
41	0	1	3	20	16	1	+1.28	0	0	1	14	25	1	+1.60	+0.32
42	1	5	6	19	9	1	+0.75	1	1	7	15	16	1	+1.10	+0.35
43	0	0	7	27	6	1	+0.98	0	0	5	17	18	1	+1.33	+0.35
44	0	1	8	12	18	2	+1.21	0	4	7	21	8	1	+0.83	-0.38
45	2	6	12	11	9	1	+0.48	7	7	12	11	3	1	-0.10	-0.58
46	4	4	14	14	2	3	+0.16	1	1	5	19	13	2	+1.08	+0.92
47	0	1	6	19	14	1	+1.15	0	0	7	15	18	1	+1.28	+0.13
48	2	8	16	11	2	2	+0.08	1	2	13	16	7	2	+0.67	+0.59
49	5	9	11	13	2	1	-0.05	1	8	9	15	7	1	+0.48	+0.53
50	7	14	11	5	2	2	-0.49	7	8	10	7	6	3	-0.08	+0.41

The *interpretation* of these results could be the following:

1. We see (as already in MAVI 3 mentioned) that the IS-state and the SHALL-state often differ a lot.
2. Natural numbers and elementary geometry is taught already but is asked still more. Also mastering of fractions is asked more.
3. Proofs are not so much taught but still should be treated less.
4. Primary school mathematics is important but should be treated less.
5. History of numbers and mathematics is done not so often and should be done more.
6. Reflection about mathematics is important and should be done more.
7. Exercises are very important.
8. To work one problem at one's own ist done not so often and should be done more.
9. Only a few students think that their study should focus only on didactics of mathematics.

The consequences of these results can not be done in general because of the small number of experimentees on one hand and the special situations at other universities on the other hand. But I think hints for discussions about the mathematical education of future primary school teacher are given.

To complete the impression about the mathematical topics I also used an *additional questionnaire* during my lecture (called "base course of mathematics" for future primary teacher). The Idea for it I got from Mosel-Göbel & Stein at the GDM-conference 1996 (See in: Beiträge zum Mathematikunterricht 1996, S.297-300, Bad Salzdetfurth). After finishing a topic I asked the students how they cauld follow the lecture and if they think the lecture was good for their later profession. The scale went from -3 (fully disagreement) to +3 (fully agreement).

The topics have been the following:

1. Types of numbers and relations between the operations
2. Basic concepts of set formalism
3. Basic concepts of relations and functions

4. Natural numbers as cardinal numbers and systems of notions for numbers
5. Fundamental concepts and problems of elementary geometry

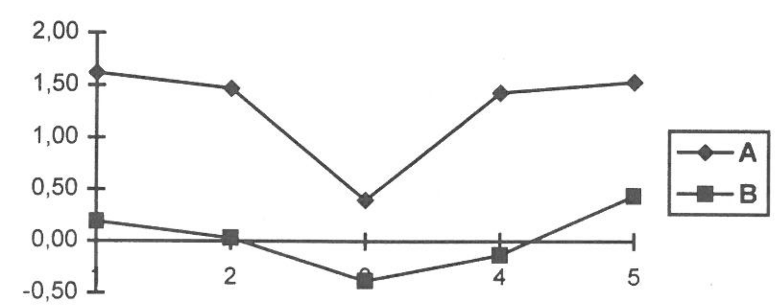
The result in respect to this questionnaire was the following (because fluctuating total numbers I always give the procentual rate):

Topic No	-3	-2	-1	0	+1	+2	+3	*Mean*
1	2.8 %	1.9 %	1.9 %	5.7 %	28.0 %	31.3 %	28.4 %	+1.62
2	1.1 %	2.8 %	4.5 %	12.3 %	25.1 %	27.4 %	26.8 %	+1.47
3	8.7 %	7.6 %	13.0 %	19.6 %	26.1%	6.5 %	18.8 %	+0.40
4	2.8 %	3.4 %	4.5 %	11.8 %	21.9 %	26.4 %	29.2 %	+1.43
5	2.3 %	2.3 %	0 %	0 %	37.2 %	46.5 %	11.7 %	+1.56

Question A: "I can follow the lecture".

Topic No	-3	-2	-1	0	+1	+2	+3	*Mean*
1	6.0 %	10.3 %	12.0 %	29.9 %	21.4 %	13.2 %	7.3 %	+0.19
2	4.5 %	12.4 %	13.5 %	33.7 %	19.1 %	15.1 %	1.7 %	+0.03
3	14.1 %	8.7 %	15.2 %	39.1 %	14.1 %	3.3 %	5.4 %	-0.38
4	12.3 %	6.1 %	19.0 %	23.5 %	26.8 %	8.9 %	3.4 %	-0.13
5	0 %	4.6 %	14.0 %	32.6 %	30.2 %	18.6 %	0 %	+0.44

Question B: "The lecture makes sense for my future profession".



These results of this questionnaire are similar to those Mosel-Göbel & Stein got: All topics but topic 3 (relations and functions) are understandable. The proposed importance for the future profession of the means of all topics is nearly 1.5 less. The best topic in both views is elementary geometry.

Finally, I will give a summery about the *open answers* the students gave in the first mentioned questionnaire. For this I evaluated all tests from April 1996 and January 1997. A lot of comments have been very similar. Therefore I built eight categories in which I distributed the comments. The total number of evaluated tests is 241, but for getting the weight of one class I counted the number of comments (if someone made two comments so these were counted both). If there was no comment I counted it as one comment - because I think "no comment" is also a comment. The total number of all comments by this counting is 295. (This is the basis for the given percentage.)

The *wishes of the students for their study* now are the following:

1. No comment	126	(42.7 %)
2. Stressing references to the practice in school	41	(13.9 %)
3. Stressing didactics strongly and giving hints for teaching	41	(13.9 %)
4. Good Explanations, good structuring and self-experiences	25	(8.5 %)
5. Not so many proofs and formalistic mathematics	20	(6.8 %)
6. Information about learning ways and problems of pupils	16	(5.4 %)
7. Learning for games, pleasure and self-experience in school	16	(5.4 %)
8. Sound knowledge, logic thinking and demanding mathematics	7	(2.4 %)

I think, these results give us some hints for a possible conception of the mathematical teacher education. But a detailed interpretation of these results I better let open for the discussion.

Markku Hannula

Pupils' Reactions on Different Kind of Teaching

Introduction

As a part of a research project I begun to teach mathematics in two classes. The framework of the project was presented in MAVI-3 (Hannula, Malmivuori & Pehkonen, 1996). I tried to imply a gender inclusive teaching based on my previous work. Guidelines for teaching have been group-work and pupil centred and discovery learning. So the teaching was different from their previous experience and different from their exeptions. Here I shall present some findings concerning the strong reactions of the pupils in one of the classes.

Description of the class.

I begun to teach mathematics at seventh grade, which is the first grade at lower secondary school. Situation is in many ways new to the pupils. For six years the groupings have been fixed and they have been taught mainly by one teacher. Now they come to a new school where lessons are taught by different teachers and even the groupings change. This mathematics class was a half of a class, the other half was taught by another teacher. In my part there were 12 girls, of whom one come from another class after six weeks. I had asked for a girls-only group for research reasons. The pupils didn't know that, and because the majority of pupils in the whole class were girls, it could have been also by chance. Their advancement in mathematics has been average or above average. Their primary grade mathematics numbers are presented in table 1.

Number	4, 5 or 6	7	8	9	10 (highest)
N	-	2	5	3	1

Table 1. Pupils' primary grade mathematics numbers (scale from 4 to 10).

Methodology of this study

This article is based on two data gathering methods. First, I kept a diary. I wrote down my feelings and the comments that pupils made during the lesson. Quotes from my diary are marked with a date at the end (14.10.). I also interviewed eight pupils in two groups in December and two pupils again in January. These are marked with (I1) or (I2) at the end.

Beliefs are very personal. That's why I gave up trying to find general beliefs of my pupils, and changed the focus to understanding beliefs of some pupils. In this article I present three stories. First one is my story, seen from the teachers point of view. The two other stories are pupils' and I have presented them in Figures 1 and 2. I have chosen which parts of the material to exclude and which to include, but I hope that the voice you hear is of those pupils, not mine. The two pupils did equally well in tests, but their experiences in the class were different.

In text there are some codes that need to be explained. [*Text in square brackets is slightly different from original, I have added, changed or left out something*]. {Comments on behaviour or tone of speech}. {* contextual information*}. (?) means part of speech that could not be interpreted. Beginning of simultaneous talk and interrupting are marked with a slash /. **Stressed words** are written in **boldtype**.

My story

In the beginning everything seemed to go really fine.
Second lesson. The working climate was excellent (16.8.)
First task was to check the homework in groups. They had no need to ask me. (19.8.)

Paula has difficulties solving a task and has persistence for over 10 minutes. I try to help her with some hints. She doesn't accept my help at once because she wanted to try solving it herself first.
Paula: *I am going to understand this.* (23.8.)

Mathematics lesson, group work:
Maria (comments another pupil): *Yes [these lessons are fun], like last year it was only: "Shad up!" and then we counted from the book.* (28.8.)

Later on, it turns out that pupils have difficulties too.
Lots of misunderstandings in homeworks. At the end of the lesson one of the brightest pupils commented: *Now I understood it all. At home I understood only half of it, now I understood it all.* (11.9.)

There becomes more and more criticism in the classroom.
Eva {complaining}: *You don't teach on the blackboard.* (14.10.)
Linda: *One can't know how to do these, 'cos you haven't taught.* (15.10)
Paula {replies to another pupil}: *Don't say mathematical, it makes me puke.* (15.11.)

The peak of the criticism was probably, when pupils and their parents complained to the headmaster and we had an open discussion in the class about my teaching (25.11.). I did some changes in my teaching, and the atmosphere become better. Some pupils however kept strongly critical approach, which caused problems for the whole class.

Anna: *And I think that Eva, she too, ought to think about her attitude, because I had similar attitude first, /*
Helena: */ Me too. /*
Anna: */ that if I didn't learn something, I blamed you. But it depends quite much on what you do yourself. And if she does only the tasks that you tell to do, and does not look for any knowledge by herself.*

Helena: *And you have no time left for teaching us others or them either, if all your time goes for defending yourself.* (I1)

Helena: *I believe, that if all from our class would take it a bit more seriously, [ ] it might begin to go well again.* (I1)

Eva is complaining that she doesn't learn. Anna replies to her: *It depends a bit on ones own attitude too.* (10.1.)

I no longer teach this class. In January I asked the headmaster weather it would be possible not to teach this class next autumn to have more time for research. She gave me an opportunity to finish it at the beginning of February.

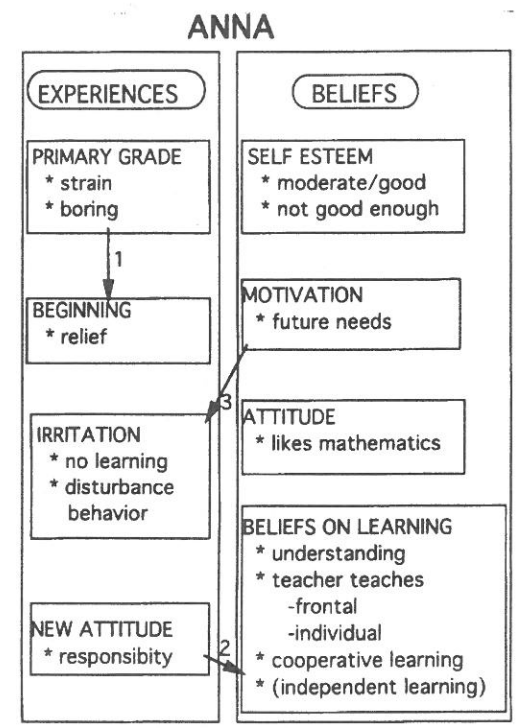


Figure 1. Anna's story.

Anna's story

At first I was so irritated...well I mean at the very beginning it was really nice, because from the first mathematics lesson I noticed, that somehow, at lower grade it was so strict, it was somehow such a relief that I no longer needed to strain in math class[1]. [ ] *And then after a while, some couple of weeks, it started to annoy me, that one didn't learn anything. Well it must have depended from my own attitude most, and I begun to feel pissed off, and I abused you and I abused all the others and I only used bad language in the class, and I didn't get anything done.* [ ] *So I changed my attitude. I thought that it doesn't help anything, that you*

[1] Small numbers refer to arrows in Figures 1 and 2.

shall be our teacher anyway and so on. So then I changed my attitude, and decided to study more myself.[2] (I2)

Notes in my diary confirm the process. I had at least two notes on Anna and Helena solving problems in good co-operation (21.8. and 9.9) and one note (13.11.), where Anna's group didn't do the assigned task.

Primary grade

Our teacher was quite demanding, so that she almost all the time had surprise tests, with awfully difficult tasks, and hardly anyone could solve those. And otherwise too, that even if you got a ten in all tests she wouldn't give more than an eight in school report if you don't keep your hand up to almost all tasks and be otherwise active too. (I1).

But it was so dry, somehow, the teaching in primary school [ ]. We always went exactly according to the book [ ]. First it was taught in theory all that, and then there wasn't much anything else, no project works or anything like. That left somehow awful traumas sort of. The teacher newer even asked who would like to do some task on the board, she just commanded [ ] The lessons caused awful traumas, 'cos one always strained somehow awfully. (I2)

She had quite good self-esteem in mathematics.

I: *Well, Anna, do you think that you are good in mathematics?*

Anna: *I don't know. [ ] .. theory went quite easily for me, but then all word problems and such were difficult on lower grade at least.* (I2)

For me it is at least so, that if you explain me something thoroughly, I do understand it, or sort of internalise it. I mean really thoroughly, so that there are no question marks left there. [ ] ... then I do remember it quite long and understand it. And then it is easy for me to solve more difficult tasks of the same thing. (I2)

Her relation to mathematics had love and hatered.

...I have tried my homework, but then I have lost my nerves, because... heh and then I throw the books to the wall. I've done that a couple of times. (I2)

I really like mathematics, but I just can't have much of it done, even though I try. (I2)

She had a motivation to learn mathematics for future needs.

...one of the pupils was quite lousy in mathematics at lower secondary school but the teacher was so strict that she had done all the homework after all. And then she went to math-specialised class in upper secondary school and it was really easy for her. But are we going to have it easy [ ] if it continues like this...[3] (I1)

She shared my ideas on teaching, at least partially.

Well, at least it has worked well, that friends have taught eachoth..sort of. like, if I didn't understand something and if [ ] Sara had understood, then she has always taught me, and that has worked well. That must have been the way you meant it, that you are the last one to ask from. (I1)

[S]ometimes I think that you have given tasks [where] we ought to figure it out ourselves. But I think that it might be better still on the seventh grade, that you teach it first in the class, and we practise it at home, [ ] It might work on the ninth grade, [ ] but now it seems to be really difficult. (I1)

[...] 'cos now, it has been sort of that we have had to look for the information a little bit too much ourselves. (I2)

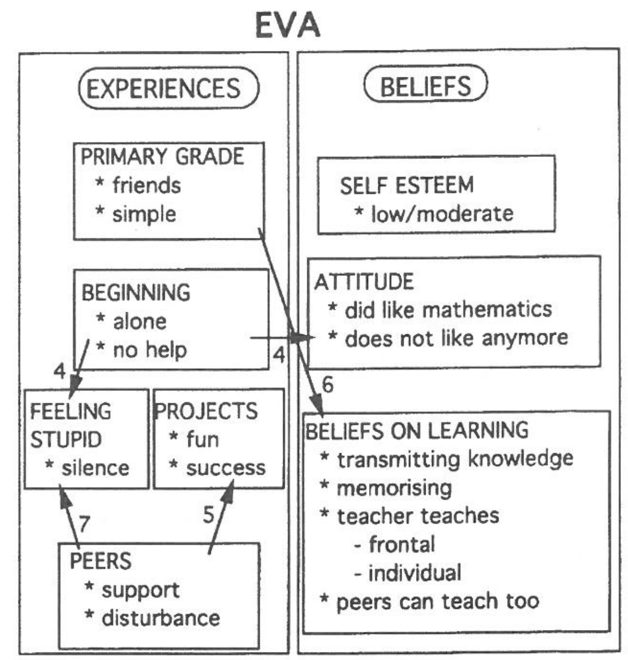


Figure 2. Eva's story.

Eva's story

Eva was another pupil in the same class. Her story was different from the very beginning.

When I came to school first time, and we had spent some three lessons. I thought that I was the only one not to understand anything, because I think that I wasn't helped too well then. Even though I asked for help, you explained somehow strangely and left at once. I tried to be really nice and all that, but I didn't know anyone, and I was too shy to ask all the time. I thought that all the others know except me, but it wasn't so. (I1)

... when we begun or when we come to school, I did try to participate in the very beginning, [ ] I asked you some advise and you walked away and said "ask your group". I didn't know them. For sure I dare ask them and I thought that I am the only one not to understand anything. So I told my mama and papa, that I don't understand anything during the lessons, and that I thought that you didn't help us, at least not me. [ ] that the others understand, but I don't at all. [ ] So it just stayed that way, and I had to be silent because I was so stupid, 'cos I don't understand it and all the others do. And so begun my attitude. So if you had helped me better in the beginning, I could be somewhat more eager in mathematics.[4] (I2)

She did have some good experience too:

I like those projects, I would like to do them.[5] (I2)

That gave me ardour. It was really fun to do the pairwork.[5] (I2)

I thought that [mathematics] was fun first, but it no longer is. (I2)

Primary grades were a happy and easy time for her.
Eva: *And I remember the one lesson, when there were girls from our class, and we just chatted and told weather we shall marry some rich man.* {laughter} (I1)

Eva: *We had a teacher* [ ], *who taught so well, that everyone got excellent marks.* (I1)

Her beliefs on good teaching seem to arise from a rote learning ideology. She wants to have explicite instructions what to do.
I: *How does a good teacher teach?*
Eva: *So, that she goes through the things so long time, that everyone understands.* [ ] *And explains in an easy way. For example numerator and denominator – forehead and nose./* {*The Finnish words meaning numerator and forehead begin with the same letter, and denominator and nose respectively.*}
Paula: */Oh, yes, numerator and denominator. I still remember it.*
Eva: *Aha, me too. I wouldn't remember it until now,* [ ] *if she* {*her primary school teacher*} *didn't teach it that way. She taught everything like that.*[6] (I1)

I: *Why some learn mathematics better, or why some do it better in the tests than the others?*
Eva: *They have a good memory.* [ ] *They have followed the lessons more carefully.* (I2)

Eva: *Weeeell you can write at least this: that I lost my nerves with those theoriical assumptotes. I am through with it. I want normal mathematics.*
I: *What is normal mathematics then?*
Eva: *So that first we do the new thing for the lesson. And that is asked from everyone in a row. Then you give pages and everyone counts and asks the teacher if something is wrong. Thus!* (I1)

Eva: *For me, you must begin like this: "Here is one, here is another one. In-between there is a plus. After the second one, there is an is-sign. Then you must add the two ones together."* (I2)

Maybe we shouldn't do everything so independently. (I1)

I: *Everyone [in your class] knows the basics so well./*
Eva: */* **I do not know!** {angry, banging the floor with her feet}
Anna: *Me neither. That's the point, that / we are going too fast.*
Eva: (? ?) */ I can't. You never tell that.. First when we get to a new matter, so you have never explained it on the board. That's annoying.*
Anna: *So one has to self* (?) *weather it was like this / or like that.*
Eva: */ Yes, and I hate that* (I2)

Her comments in the class cofirm her need for explicite instructions.
You don't teach anything on the blackboard (14.10.)
You must tell, what must be done.(10.1.)

She does however believe that to be good in mathematics includes something more.
So that she is smart, and has sort of own things and finds [ ] *own solutions.*(I2)

Her experiences with peers are of two kinds: support and disturbance.
I think that the nicest thing was when Ursula was at our home and we counted ourselves and I understood the whole thing and I did a whole page. (I1)

Julia: *I am teaching Eva now, don't come here.* (25.10)

Eva: You mean, that Anna behaves well, and I behave badly, as/
Anna: /But I didn't in the beginning behave well

Eva: I do know, that you behave well, and I think that that is good. But I just **can't** *help it. Something* **annoys** *me when it just annoys me.* (I2)

I try to [study well] at the beginning of every lesson, but then I am **disturbed** *so it annoys me. Well I don't know, 'cos at the beginning of every lesson I try. Today I went to sit alone. Then Paula comes next to me. And then Ursula comes there close and I can't be calm. I just can't be.*[7] (I2)

It annoys me, that when I got a seven in a math test, then Ursula looked [at me with a face], that "You are so bad at math". Really. (I2)

In that group I haven't got a right to say **anything**. *They think, that whatever I say, I'm wrong. Then in Julia's group I* **might** *be right. It* {laughter} *isn't necessary right, but I have a chance to say something...* (I2)

Her self-esteem in mathematics is not good.
I: *Do you think, that you are good at math?*
Eva: *No* {laughter}. *I don't know. It like mm... depends like what you are doing, kind of or hmm... At home, when I face a situation, or sort of ought to understand something of mathematics, so I do understand it. But then...*(I2)

Eva had a lot of difficulties today. She asked weather I think that she is stupid. I denied it. Later she commented: "*... even though someone thinks that I am stupid.*" (19.11.)

{*Another pupil gets help from her brother.*} *You see, I don't have brothers. I mean my mother doesn't...Mother doesn't... Well* **mother doesn't.** [ ] *Well if she starts to help me and I don't understand something, then [she would say] "Don't you now understand this!* {in Finnish the expression means, that you should by now} *Yak yak yak"* (I2)

Feelings
Eva: "*I HATE [this task]!*" (25.10.)
I would like to do only enjoyable tasks. (I2)

Discussion

This article was my first attempt to report my research in a qualitative way. I have learned something during the research process and I want to share my experiences with the reader. I hope that after reading these stories the reader will understand the processes in the pupils' minds somewhat better.

Anna made progress in her learning skills but Eva remained in the confronting position. They were different in many ways. Anna had an inner motivation to learn, while Eva rather "*tried to be really nice and all that.*" Eva was rather insecure and would have needed more positive feedback. The group dynamics with her peers were problematic and teacher invention would have been necessary. Anna seemed to be more ready to reflect on her actions and experiences while Eva was somewhat reserved.

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Gabriele Heintz

Research Questions on Private Tuition in Mathematics

Background

As external factors influencing pupils' environment, parents represent a significant component of the learning process. Within the framework of research on the theory of beliefs (cf. Pehkonen & Törner 1996), the role played by grammar/comprehensive school parents in private tuition for the subject of mathematics is to be more closely examined. Parents' role in the question of private tuition is an integral part of the web of relationships between parents, pupils, tutor and schoolteacher.

Research on private tuition with special regard to the role of parents is intended to clarify correlations and influencing factors and to determine changes due to social evolution in the results of previous work on such relationships.

The investigations will concentrate on individual aspects of the parents' role. To this end, video recordings of interviews are to be used. A pattern of questions for the interviews could be derived from the process of decision-taking experienced by parents when considering the need for private tuition. Integral parts of this process are their motivation and incentives, their conception and expectations of private tuition, their own support and the choice of tutor.

Motivation and incentives for private tuition

Foremost is the question of the criterion for deciding to take advantage of private tuition in mathematics. Are poor results or difficulties in learning a problem for parents? Oster (1997), with his problem-solving model designed from the point of view of the pupil, concluded that an increase in learning or achieving difficulties exercised a trigger function for taking action. Are marks a decisive factor for parents? How do parents notice any learning difficulties experienced by their children? Which problems in their children's learning process do parents notice at all? Which problems do they regard as important and therefore worthy of their attention? When do parents take action? For example, do the parent/teacher meetings that take place twice a year in the grammar schools provide the decisive "info-mart" and thus the trigger for introducing private tuition? Do other factors exist in the view of parents for initialising private tuition ? How is the decision reached? Who finally decides that private tuition is the right answer? Do the parents promise rewards for improved results? If so, are these effective? Previous research suggests that the choice of private tuition is somewhat embarrassing for the parents concerned. Whereas at the elementary school level the teacher is regarded rather as a partner and advisor, at higher levels the desire for evaluations becomes more important. The teaching, advising, helping and supporting functions of the educator retreat into the background, to be replaced by judging and examining functions, even the ability to determine chances in future occupations and living standards. The atmosphere between parents, pupil and specialist teacher can lead to secrecy, i.e. a decision for or against private tuition is taken without the knowledge of the teacher. A more complex analysis of the criteria for such decisions is not available in any research paper to date.

From the point of view of pupils, Oster (1996) decided that problems in mathematics are mainly regarded as being their own fault. Laziness and lack of interest are seen by pupils as the reason for low achievement levels, leading inevitably to gaps in their basic knowledge. How do parents judge weaknesses in the achievements of their children? In their opinion, what are the reasons for poor marks? How do they act as a result?

Parents' conception and expectations of private tuition

The role of parents in the relationships connected with private tuition is closely tied to the question of the parents' conception of mathematics teaching. What conception of specialised mathematics teaching can we anticipate when we examine the way in which they actually react? What notions of mathematics teaching come to light? Which of these opinions promote, which retard their children's progress?

One could consider the following possibilities:

- Mathematics is regarded as a "recipe book".
- Mathematics can be learned, i.e. one does not need to master mathematics, simply to know it.
- Mathematics teaching can be poor. The factors involved can be named.
- More time is needed for mathematics than for other subjects.
- Schools do not provide enough time for mathematics.
- Success in mathematics depends entirely on learning.
- Success in mathematics depends on learning methods.
- Mathematics plays a significant role in future vocational training.

What effect do such parental conceptions have on private tuition? Previous investigations (cf. Behr 1990) have invariably found that parents and pupils quote poor marks and school performance as reasons for choosing private tuition. Competitive advantages gained over their peers by these means are generally regarded as unjust with regard to equal educational opportunity.

To date, such surveys show that a decision, taken on the basis of poor performance, to pay for tuition leads in time to an evaluation of other criteria. This is confirmed by the fact that tuition continues even after a long period of unchanging results. More profound comments on the individual criteria are not made and must still be investigated.

Do parents notice a major change in the behaviour of their children during a period of private tuition? If so, where does this change occur according to the parents? Is the possible goal quoted by Oster (1997) from the point of view of the pupils, i.e. being able to learn without extra tuition, also the parents' goal, or is the tutor simply a welcome substitute for the parents' presence during homework? Do parents expect private tuition to bring about a change in the attitude of their children to learning? In such a case, how are these changes achieved? Does their own experience with private tuition have an effect on the tuition of their children? What is the parents' own conception of the private tuition? These are just some possible questions that could be part of the framework of interviewing documentation.

Support from parents

Another aspect of the investigation is concerned with the type and extent of any support the parents might provide before or during periods of private tuition.

In his investigations by means of questionnaires, Behr (1990) determined that more than 50% of parents have very little or no contact to the private tutor. What do parents regard as the reasons for this? The consequences of this lack of contact for the relationships between the pupil, tutor and specialist teacher with regard to content, method, didactic, education and communication will have to be more closely investigated.

One type of support provided by parents is the financing of tuition. Due to changes in the social structure - e.g. the high degree of unemployment - parents now seem to be keen to take advantage of private tuition to ensure that their children achieve satisfactory qualifications. The increasing number of private tuition centres could be regarded as proof of that. Questions then arise, however, that demand elucidating answers.

To what extent has the financing of private tuition changed over the last few years? To what extent is the pupil's motivation and willingness to work dependent on whether the parents or the pupils are footing the bill (the pupil, for example, from pocket-money or a part-time paid job)?

Examples of some general questions on this issue would be:

- How do parents support their children at all in learning mathematics?
- Does the nature of their support change during a period of private tuition?

Choice of tutor

The choice of tutor also poses many questions for parents with regard to the criteria involved. For example, the age, sex, and qualifications of the tutor are obviously important. To what extent do parents choose people from the pupil's immediate surroundings (relations, acquaintances)? Why are private institutes often preferred? The influence of the availability of individual or group teaching methods on the choice made and on the motivation and success of the pupil must also be examined more closely.

Outlook

The present state of research on private tuition with regard to the role played by parents therefore leaves open a wide field for the determination of interrelations and influencing factors. The use of video documentation in such work can comprehensively and authentically reflect the personal views of parents and has therefore been chosen as the research method in this case.

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Kirsti Hoskonen

Mathematical World Views of Some Seventh-Graders

Beginning of Research

Introduction

The mathematical word view of a pupil plays an important role when the pupil is learning and doing mathematics. The aim of my research is to find the mathematical word view of some pupils in one seventh grade just at the beginning of the lower secondary school and then to teach them three years. After that I will examine if the mathematical world view has changed during the three-year teaching period. Many kinds of research methods will be used during these three years. The research is a case study and the methods are qualitative: interviews, observation, written material like tests, tasks and so on, but there is also the same questionnaire to fill both at the beginning and at the end of the research. The teacher is the same person as the researcher. When I am teaching I try to organise my teaching so that the pupils can study and learn mathematics in a constructivist manner.

The pupils of this study are in the seventh grade of a Varkaus lower secondary school. The seventh-graders are thirteen years old. They come from four different primary schools. They have started their learning at the secondary school in August 1996. The number of the pupils is 18, of which 7 are girls and 11 are boys.

In Finland the National Board of Education has in 1996 started a new effort to raise mathematical and scientific knowledge and know-how in Finland to the international standard. This school in Varkaus, where I teach, is one of the schools that belongs to the development and information network. Therefore last year all the new pupils are asked if they are interested in this effort. So many pupils were interested in it that three classes consist of them. This class I teach is one of them. However this class consists of other pupils, too. This class consists of pupils, who came from the Swedish-language primary school and also those whose religion is orthodoxy. Later in autumn the pupils have told that in the primary school some teachers had said that it does not matter if they are interested or not. So some pupils were interested in order to be at the same class with his or her friend. I think that this class is a normal class in our school.

With these questions I try to find some parts of the world view:

- What is mathematics?
- What is the way of doing mathematics?
- What is the way of learning mathematics?
- What is the teacher's role?
- What is the pupil's role?

Mathematical world view of the teacher

The teacher is an important person, when the pupils form their opinions of the mathematics as a subject. She or he is not the only person, for the parents, the relatives, the friends etc. have an effect on them, too. However the teacher with his/her beliefs about teaching mathematics influences every lesson on the pupils. Traditionally the teacher gives the pupils a book and teaches a rule with some different examples and the pupils have to practice to get the routine. So the pupils have to calculate the tasks, first number one, then number two, etc. To get the right answer is the most important goal. Everybody must try to solve the tasks himself. If the pupils do not understand, they have to memorise the rules. Later they have to do a test so that the teacher can see, if the pupils have learned or not. Probably the pupils say that mathematics is calculation, mathematics is boring, mathematics is memorising and mathematics is difficult.

For some years ago I was the teacher with rules and routines. I tried to divide the concept into smaller parties so that the pupils could better understand it. My experiences of teaching were based solely on rules and routines. I have never seen how the teaching works with a constructivist way. I think it would be easier if I had been with in such a lesson. Now I am trying to find a new conception of teaching. It is not easy. I know that I myself must find out what I think mathematics is all about. I think that teaching should include rules and routines, discussion and games and also open-approach problems (Thompson 1991, Lindgren 1996) or mathematics could be a toolbox, system and process (Dionne 1984). What is the proportion between them? I think one difficulty is in it that I have only three lessons a week, together at most three times 38 lessons, in general three times 36 lessons a year. How can I plan my teaching so that there are routine tasks and all sorts of problems in a convenient proportion? I always realise that it takes much more time to get the pupils understand themselves the concepts I am teaching than to give them a rule and some tasks to practice the rule.

The pupils are not used to find out things themselves either. They say to me: "You always first give the problems and afterwards say to us, how to solve them. Why do you not teach them first and then give us the tasks?" Although the tasks are in such an order that first they should look at the task and its solution, then there is the beginning of another task and they have to continue, and at last they have to do the task completely themselves. However they say to me: "Why do you not teach the thing?" "I suppose the teacher is here in order to teach." "Do not say, think." They are used to that the teacher says what to do. They are used to that all the problems the teacher gives them to solve, could be solved with the given method. They do not have to use their brains.

I think that different pupils need to be taught in different ways. For example in our school in the eighth grade we have a habit of dividing the pupils of three classes into three new groups depending of their future plans and their earlier learning. If I have to teach a group, where the pupils are not interested in mathematics or interested in school at all, it is really difficult to get them to think themselves. On the contrary if all the pupils know that they need mathematics in their future studying or mathematics has been easy for them, it is more probable to get them together to think how to solve a problem without giving them a rule.

The pupils are used to that the teacher is the person, who tells whether the answer is right or wrong. It is not easy for me to remember that I cannot say to the pupils in the discussion if the answer is right or not. I ought to wait for the other pupils' reaction to the answer. Sometimes when the pupils have had the possibility to decide themselves about the correct answer the lesson has been very good. I have only directed the discussion to the right direction. Some were impatient and said: "Why can't you tell the right answer?" but some liked the discussion very much.

All my pupils sit in the class in groups of three or four pupils. When they solve problems they are allowed to discuss with each other. But it is not the problems all of them talk about. I think it is every possible thing except the problem some of them talk about. And then they say: "I do not know how to solve the problem" or "I do not understand these at all". But on the contrary there are groups, which discuss very eagerly about the problem and argue on behalf of their opinions. In such a group everybody tells the others, if he or she has not understood something. They really teach each others.

I am trying to tell the pupils that learning is using manipulatives, playing learning games, talking to each other, discussing other strategies to solve the problems, thinking and understanding, not memorising or learning by heart. Of course learning is practice, too. However I want to show that mathematics belongs to every day life and mathematics ought to be fun, too.

Beginning of the study

In autumn when I started with my class I asked them to write some sentences about a theme 'Mathematics is'. I asked them to tell, what they think mathematics is, what kinds of things they think belong to mathematics, to mathematics lessons, to learning mathematics or to teaching mathematics, where do they use mathematics.

The boys say that mathematics is calculating: adding, subtracting, multiplying, dividing and geometry. Difficult problems belong to mathematics. Eight of eleven boys say that it is boring. It is rather difficult, difficult or very difficult depending of the boy. It is complicated and a little hard but some of them say that it is useful, important for their future life and they will have benefit in it in jobs. One of the boys says that he do not like mathematics but nobody says that he likes it. Nobody is good at it but one says that he manages. Two of them say that mathematics is sometimes nice and sometimes difficult. One boy writes: "Mathematics is nice if you have easy problems or you understand something about it and it is boring if you have difficult problems and you do not understand anything". The other writes: "I personally do not like mathematics. But perhaps you find some positive things about mathematics, but you need to look for them".

One of the girls writes that "mathematics is calculating in different situations and it is needed in ordinary life". Girls say that mathematics is an important subject at school. Nobody is good at it but some of them like it. One girl "did not like mathematics at the beginning" but now she knows how understands that it is important to know it, because "if you cannot do mathematics you are in trouble". Some say that mathematics is rather difficult but no girl says that mathematics is boring. The girls think that "the teacher must teach well and take into account those pupils who have problems with mathematics". "When somebody do not understand she or he could take extra lessons."

After every lesson I have made notes about the lessons, what is the subject matter, what is the lesson like, some comments of the pupils. In autumn I tried to make a questionnaire to find out their mathematical world view. Some of the statements are my own, some are from different questionnaires. Many of the questionnaires are made for older students or teachers. They were not suitable as they are. I think that the language of the statements could not be too difficult so that the pupils could understand the statements without asking help.

In the questionnaire there is a part with 18 sentences (Dutton, 1988; Philippou & Christou, 1996):

1. Mathematics thrills me and I like it better than any other subject.
2. I never get tired working with mathematics
3. I enjoy working and thinking about math problems outside school.
4. I would like to spend more time at school working on mathematics.
5. I enjoy seeing how rapidly and accurately I can work on problems.
6. I like mathematics because it is practical. (1)
7. Sometimes I enjoy the challenge presented by a mathematics problem.
8. I enjoy doing problems when I know how to do them.
9. Mathematics is as important as any other subject. (1)
10. I like mathematics, but I like other subjects as well. (2)
11. I am not enthusiastic about mathematics, but I have no real dislike of it.(3)
12. I do not think mathematics is fun but I always want to do well in it. (2)
13. I do not feel sure of myself in mathematics. (2)
14. Mathematics is something you have to do even though it is not enjoyable. (3)
15. I have been always afraid of mathematics. (1)
16. I am afraid of doing word problems.
17. I have never liked mathematics.
18. I detest mathematics and avoid using them all the times. (1)

That sentence they think to be the best suitable for them, is asked to mark with number one, the second with number two and the third with number three. That sentence, which does not suit at all is asked to mark with a number nought. The sentences in the questionnaire are in this order. The most positive ones are the first and the most negative ones are the last. The 11th sentence is neutral. The number at the end of the sentence tells how many pupils have select this sentence to be the best suitable. The total number of the pupils is 16, because one girl has chosen many sentences and one boy did not answer the questionnaire.

Table 1 shows how the pupils have selected the sentences of the questionnaire. The numbers from 1 to 16 are the same as above. It is possible to see that only four pupils, two boys and two girls, have chosen a positive sentence to be the best suitable, three pupils have chosen the neutral sentence and the rest the negative one. One pupil, the boy 5 hates mathematics. Seven pupils find the most unsuitable the sentence 'Mathematics thrills me and I like it better than any other subject.' 'I detest mathematics and avoid using them all the times.' is the sentence three pupils think to be the most unsuitable. Clearly it turns out that the pupils do not like mathematics. Although the number of the positive sentences is greater than the number of the negative ones, the pupils have chosen positive sentences less than negative ones, even if you look at the first three choices. The most negative boy has chosen the sentence 18 as the most suitable and the sentence 1 as the most unsuitable, but the most positive boy has chosen the sentence 6 as the most suitable and the sentence 18 as the most unsuitable.

To find out the pupils with different mathematical world view I decided to choose one pupil of them, who had chosen the sentences 6, 9, 10, 11, 12, 13, 14, 15 and 18. So I had 9 pupils, who I will interview. The boy 1, the boy 6, the boy 3 and boy 5 were obvious, but the boy 5 answered that he will not be the interviewee. After that I had to choose those who had the other sentences as the most suitable. I have their opinions about mathematics in another part of the questionnaire. There are together 14 sentences. I used the opinions of mathematics

to select the ones, who have either the most positive or the most negative opinions or who did not say it at all. Together I have interviewed 8 pupils. Two of them wanted to be the interviewees.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Boy 1						1	2		3									0
Boy 2			0					3			1		2					
Boy 3														2	1			
Boy 4	0							3				1		2				
Boy 5	0													3			2	1
Boy 6							2		1						3	0		
Boy 7									0				1	2			3	
Boy 8	0						3		2					1				
Boy 9	0												2	1	3			
Boy 10	0											1	3	2				
Girl 2	0										1	3				2		
Girl 3	0								2	3				1				
Girl 4									3	1			2					0
Girl 5													1	2				
Girl 6					3			2		1								0
Girl 7		0							3		1		2					
0	7	1	1						1							1		3
1						1			1	2	3	2	2	3	1			1
2							2	1	2				4	5		1	1	
3					1		1	2	3	1		1	1	1	2		1	
	7	1	1	0	1	1	3	3	7	3	3	3	7	9	3	2	2	4

Table 1. Pupils as mathematics learners.

In the autumn, after they had learned geometry I asked them some questions like 'what is their goal, when they are learning mathematics', 'how well they have reached their goal', 'what they can do if they have not reached it', 'how do they do their homework'. In November they made the test, that belongs to the pilot study and they answered the questionnaire, too. In December and in January I interviewed some of the pupils. I asked the parents, where do they use mathematics in their jobs or in general. In February I asked the pupils to draw a mathematics lesson. Later in May I will ask them again 'What kinds of things belongs to mathematics?'.

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Sinikka Huhtala

Drug Calculation Ability of Practical Nurses

Introduction

'Practical nurse' is a new upper secondary vocational qualification (cf. Figures1 and 2) in social and health care in Finland. The first practical nurses (from two and a half year training) obtained their certificates in December 1995. The training is under continuous evaluation and development. One part of this kind of follow-up was a national test given to all qualifying practical nurses on 28th November, 1995.

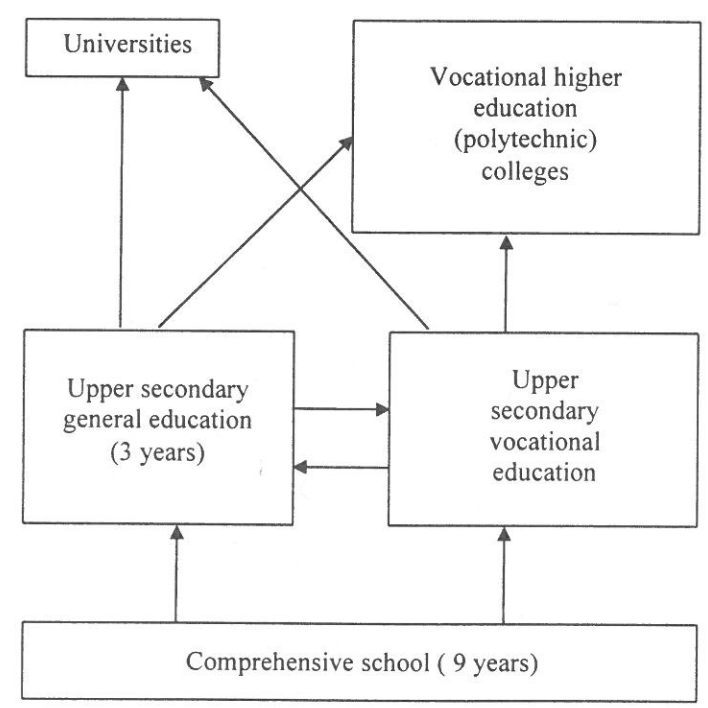


Figure 1. The Finnish education system.

SPECIAL MODULES	20 credits
Emergency care Oral hygiene Nursing and caring Social work among children, adolescents and families Care of the aged Care of the disabled Mental health care, social work with people in crises and with intoxicant abusers	
BASIC MODULES	50 credits
The support and guidance of growth Basic care and nursing Rehabilitation	
Studies common to all (includes 3 credits of mathematics)	20 credits
Elective studies	10 credits

Figure 2. Vocational qualification in social and health care, practical nurse (contents of the course: 100 credits)

The term "credit" refers to an average of 40-hour input of work by the student.

1. Purpose and methods of the study

The purpose of this study was to analyse the drug calculation ability of practical nurses.

Among other tasks there was one drug calculation exercise in the national test. In my study I examined one particular example of drug calculation in 3344 papers.

The task was as follows:
Mrs Malmi (aged 64) is to take 40 IU of insulin. The strength of insulin is 100 IU/ml. How much should the patient be taking?

The differences between the skills in different school types, different calculation styles and the special modules chosen by the students were examined.

53,6 % of the students managed to solve the calculation, the best results were attained in the schools of health care (67 %) and among the students who had chosen emergency care as their special module. Mathematical reasoning was the best method to calculate. In this study I have also analysed the errors made by the students.

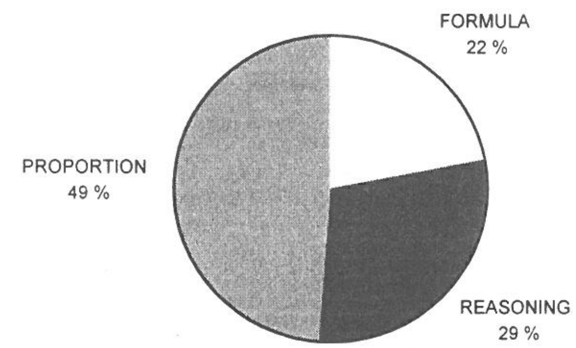


Figure 3. The calculation method used.

The three methods which the students can use to solve dosage calculations are: formula, mathematical reasoning and proportion. In this test proportion was the most popular method (Figure 3.) However, mathematical reasoning was the best method to calculate (see Figure 4.)

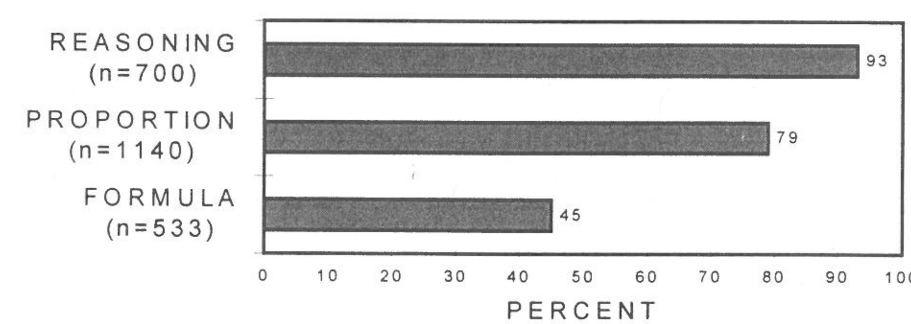
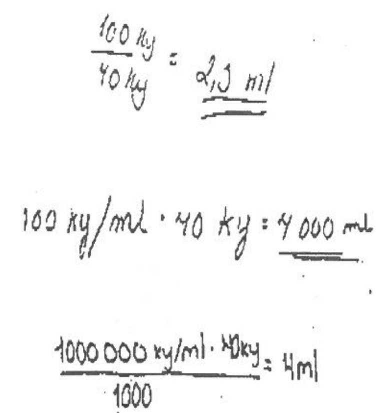


Figure 4. Correct answers / different calculation method.

Here are some examples of the errors made by the students:

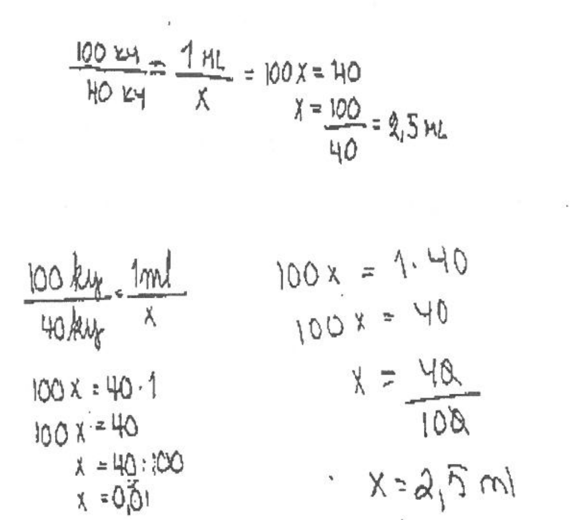
1. The method used: ***Formula***

$$\text{Dose} = \frac{\text{Ordered quantity of the active ingredient of a drug}}{\text{Strength of a drug}}$$



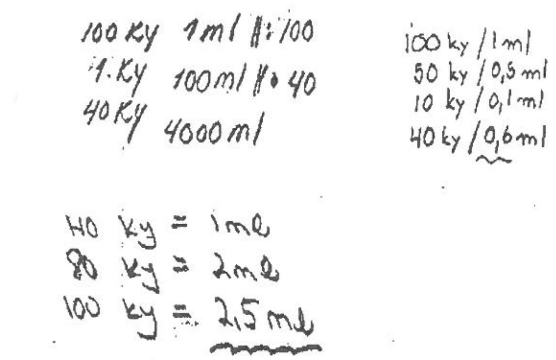
When using the formula as a method to calculate there is a risk of remembering it the wrong way. Many students also mixed percentages and millions (in antibiotics there are millions of international units) in this calculation.

2. The method used: ***Proportion***



The explanations for these errors are on the one hand that some students think that in division you must always divide the bigger number by the smaller one. On the other hand, the students make many mistakes when they must calculate without a calculator.

3. The method used: ***Reasoning***



In mathematical reasoning the errors can also be conceptual or computational.

3. Further Research

Error analysis as a tool in learning drug calculations; the idea is that errors provide a window to a student's internal thinking processes and help the teacher to understand better why students make errors in drug calculations and why some students consider them difficult. The aim is to improve the learning and instruction of drug calculations.

Example: I asked my student to convert 2,5 g to mg. She told me that it is 2500 mg, and I asked her to tell me how she knew that. She said that when you convert grams to milligrams the answer always has four numbers. Later she converted 0,5 g to mg and got the following answer: 0,005 mg (four numbers!). She knew that it couldn't be 5000 mg and 500 mg only has three numbers.

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Erika Kuendiger & Siegbert Schmidt

What Does It Take To Be Successful in Mathematics?

An Empirical Study Investigating Pre-service Teachers' Attributions of Mathematical and Language Arts Achievement, and Their Confidence in Teaching.

1. Introduction

The importance of causal attribution for students' motivation to learn is well known. Already in 1974 Heckhausen's process model of motivation was published in the widely read book: Funk-Kolleg Pädagogische Psychologie. Kuendiger (1982) applied the model to mathematics learning. Both publications stress the importance of attributions not only for students' motivation but also for cognitive learning. Borkowski et al. (1990) summarizes recent research studies which show the connection between attributional beliefs, knowledge about learning strategies and the development of knowledge, as well as the relationship between attributional beliefs, self-efficacy, self-esteem, affect and metacognition and academic performance. An overview about different theoretical approaches and corresponding studies that link attributions with achievement is provided in Heckhausen (1989). With regard to mathematics education, until recently attribution theory was used nearly exclusively to explain difference in motivation only, in particular to explain gender related differences in students' motivation to learn mathematics (Leder 1992, McLeod 1992).

To date the majority of studies dealing with teachers' mathematics related beliefs, investigates beliefs on the nature of mathematics, e.g., mathematics as process/product. A recent summary by Pehkonen (1994) shows that only few of these studies focus on pre-service teachers (see also Törner & Pehkonen, 1996). In addition, there is a lack of studies on the beliefs pre-service teachers have about themselves (Pintrich, 1990). The research study presented here focuses on the latter aspect. It investigated the attributional beliefs pre-service teachers had established about themselves as former learners of mathematics and language arts before enrolling in a teacher training program, and how these perceptions influenced their confidence in teaching the two subject area.

2. The concept "learning history"

Motivation theory based on attribution has been well researched within the context of student achievement. Although different models have been used in research studies, the following aspects, which are relevant for this study, have been generally accepted:

Cumulative achievement experiences in a subject matter, e.g., mathematics, lead to the development of a subject matter related self-concept which includes perceptions about once performance standard and its causal attributions. The latter are referred to as attributional styles which are relative stable over time as they are developed based on cumulative past experiences. The perceived level of performance and its attributions serve as a motivational framework for the next achievement situation related to the subject matter for which they were developed (Heckhausen 1989, Chap.14&15) and thus influence -besides other variables - students' confidence in their cognitive skills to perform the succeeding task. These self-efficacy beliefs (Bandura 1986) are relatively domain-specific and have to be distinguished from students' outcome expectancies related to a specific task. A student may belief that s/he will be successful in a mathematics class, because s/he is very good at mathematics but nevertheless expects a poor grade in the test just finished, because s/he did not prepare her/himself.

On entering a teacher training program, pre-service teachers generally do not have any previous experiences in teaching mathematics, which would allow them to establish perceptions about their ability to teach the subject. Yet, they do have established precise perceptions about themselves as former learners of mathematics, in particular, about their mathematical achievement level and about the causes they consider being relevant to explain this achievement. A series of research studies investigated this motivational system pre-service teachers bring to the profession, in particular if it provides the basis on which prospective teachers judge perceptions they have about themselves as future teachers. The above considerations hold not only for mathematics but for language arts as well.

Based on several pilot studies (Kuendiger 1987, Kuendiger 1989) Kuendiger (1990) provide the empirical evidence on the link between pre-service teachers' perceived former mathematical achievement and its attributions. Teachers had to indicate the applicability of each of the following attributions:

- ability, lack of ability
- effort, lack of effort
- good, bad luck
- easiness, difficulty of mathematics
- good, poor teachers' explanation
- help, lack of help by others.

Based on their perceived former mathematical achievement two groups of primary/junior teachers were formed: one group consisted of pre-service teachers who perceived their former performance as above average, the other group perceived it as average or below average. The two groups of teachers used distinctively different attributions to explain their mathematical achievement. Significant differences ($p<0.01$)were found for ability and lack of ability, lack of effort, easiness and difficulty of mathematics, poor teachers' explanation, and lack of help by others. The differences were in the expected direction: primary/junior teachers with an above average perceived performance decisively attributed their achievement to ability and not to lack of ability nor to lack of effort, poor teachers' explanation or lack of help by others. Moreover, this group considered the attribution "math is easy" as more applicable and the attribution "math is difficult" as less applicable than the group with the lower past performance. Both groups considered effort as equally applicable. In summary, teachers who judged their former achievement as high used a favorable attributional style, those with lower achievement used an unfavorable attribution style. Most importantly, prospective teachers who judged their past performance as high, and thus indicating that they think of themselves as having been successful in mathematics, attributed this success clearly to their ability, a stable, uncontrollable, internal attribution, and to effort an internal, controllable attribution. Prospective teachers,

who think of themselves as having been less success in mathematics, had also put forward effort but as the achievement outcome consistently was not very high, they concluded that lack of ability was at least a partial factor. These attributional differences are in line with research conducted within the area of learned helplessness - mastery learning (Heckhausen 1989 pp.423 ff.). Low ability attribution is often referred to as dysfunctional attribution as it is likely to lower an individual's potential to achieve in the succeeding task.

In the research study of Kuendiger (1990) participants were asked to indicate cumulative past performance level. The connection between perceived former performance and its attributions was found to be stable over time, i.e., samples investigated in two consecutive years yielded the same results based on an overall sample size of more than 300. Moreover, the average Gamma coefficient for the test-retest reliability for all items related to this set of variable was more than 0.80. Thus, it was concluded that perceived former performance together with its attribution constitutes a well established motivational system which was called "mathematical learning history" and that the variable "perceived former performance" can be used to identify groups of individuals with different learning histories.

On entering a teacher training program prospective teachers see themselves confronted with a novel task, i.e., teaching. With regard to teaching mathematics their mathematical learning history is the motivational system closest related to the new task. The above study found that primary/junior teachers with a less favorable mathematical learning history not only were less confident to teach mathematics, but also considered their personal insufficiency as a more relevant reason to explain students' lack of progress in mathematics. Thus, it was concluded that the mathematical learning history constitutes a motivational framework which influences prospective teachers' perceptions of themselves as future mathematics teachers. Moreover, it was found that those individuals who strove for teaching in kindergarten to grade 6 had a less favorable mathematical learning history than those who strove for teaching in grade 4 to 10. Both groups of teachers became more confident in teaching mathematics during the course of a one year teacher training program, yet the program did not succeed in overriding the impact of the mathematical learning history.

Kellenberger and Kuendiger (1993) applied the concept of learning history not only to mathematics but also to language arts. They investigated the relationship between subject-related learning history and self-efficacy of prospective primary/junior teachers (N=194). In this study the learning history was measured in a slightly different way. The attributions "interest" and "lack of interest" were added to allow for a more differentiated comparison between subject areas. Moreover, the format of the questionnaire was altered. Participants no longer had to evaluate each reason, but were asked to name those attributions which were most applicable and somewhat applicable. Concurrent with the results found by Kuendiger (1990), pre-service teachers used causal attributions which corresponded with their perceived former achievement level. Comparison between subject areas turned out to be limited as there were hardly any participants who judged their past performance in language arts as average or below average.

The relationship between learning history and self-efficacy was most apparent for pre-service teachers with a less favorable learning history in mathematics and a more favorable learning history in language arts (Low-High group). Teachers in this group judged themselves as significantly less able to influence students' effort, interest, achievement in mathematics than in language arts. Moreover, when the Low-High group was compared with a group of pre-service teachers which had a favorable learning history in both subjects (High-High group), the Low-High group judged themselves as significantly less able to influence students' mathematical achievement than the latter group. In summary, differences in pre-service teachers' learning history lead to differences in their perception of self-efficacy.

In addition to the above studies the concept of learning history was also applied to computer education. Kellenberger (1994) found the subject-matter related learning history to have a significant effect on Canadian pre-service teachers' self-efficacy with regards to the use of computers in teaching.

In a comparison study between pre-service primary/junior teachers in Ontario and Quebec (Kuendiger, Gaulin & Kellenberger 1992, Kuendiger, Gaulin & Kellenberger 1993) the learning history of prospective teachers and the attributions they used to explain achievement of a high- and low-achieving students were measured. The questionnaire was comparable to the one used in Kellenberger & Kuendiger (1993). Prospective teachers from both provinces (N=441) had again a more positive learning history related to language arts than the one related to mathematics. Overall, the comparison of perceptions of these two groups of prospective teachers showed astonishing resemblances: they did not only use similar patterns for attributing their own achievement in the two subject areas but also when they were asked to attribute high and low student performance. Moreover, when asked to rank student effort, interest, and achievement with regard to the influence they, as future teachers, would have, pre-service teachers from both provinces ranked achievement the highest and interest the lowest. This was true for mathematics as well as language arts. Despite these similarities significant differences were found with regard to the relative importance of some of the attributions as well as one aspect of perceived future self- efficacy; pre-service teachers in Ontario felt significantly less confident to influence their students' interest. The reported differences are substantial enough to suggest caution when generalizing results from one cultural setting to another.

3. Objectives of the study

The above studies demonstrate that the concept of "learning history" is an important motivational framework which influences the initial perceptions pre-service teachers develop about themselves as teachers. So far all above mentioned research studies have been carried out in Canada. Results presented below are part of a larger study whose main purpose was to investigate the relevance of the concept "learning history" for German pre-service teachers as it relates to mathematics and language arts (German). First results are published in Kuendiger, Schmidt & Kellenberger (1997) and Kuendiger & Schmidt (1997). The questions addressed in this paper relate to two areas:

First, it was investigated if prospective German primary teachers with different perceived past performance call upon distinctively different attributions to explain this performance, that justify a classification along favorable-unfavorable learning history for the subject areas mathematics and German (language arts). Morever, the question was addressed, if the same attributions are considered being relevant for achievement in mathematics and language arts.

Second, this study was to investigate the relationship between learning history and confidence in teaching in more detail. Earlier studies that investigated the learning history used the variable "perceived former performance" to identify individuals with a favorable/unfavorable learning history. Although the results obtained by this method proved to be consistent and meaningful (Kellenberger & Kuendiger 1993, Kuendiger 1990, Kuendiger, Gaulin & Kellenberger 1993) one could argue that the variable perceived former performance is a rather quick and dirty method to identify individuals with different motivational systems when relationships between learning history and teaching related variables were investigated. Thus, in this

study the relationship between learning history and confidence in teaching was investigated by relating attributions and confidence in teaching. Accordingly the following questions were investigated: Is there a relationship between perceived past performance and confidence in teaching, and is the use or not-use of a particular attribution related to differences in the variable "confidence in teaching"? In particular, is the relationship different for mathematics and German?

4. Data gathering and analysis

Data were gathered via a questionnaire. The learning history in both subject areas was measured similar to Kellenberger & Kuendiger (1993). Participants were asked to recall their own learning and to indicate their perceived former achievement level on a five-point Likert scale ranging from excellent to poor. In addition, they chose from a list of provided attributions, those that were most applicable to explain their past achievement. Compared to earlier studies, the attributions "general intelligence" and "lack of general intelligence" were added. The full list of provided attributions is listed on the tables and figures below.

Moreover, participants were asked to indicate their confidence in teaching either subject area in the future on a five-point Likert scale reaching from very confident to not at all confident. In addition, the questionnaire gathered demographic information and other variables not considered here.

The participants of this study were pre-service teachers (17% male, 83% female) enrolled at the University of Hamburg in their second year and pre-service teachers (5% male, 95% female) enrolled at the University of Köln in their first year (see Table 1 for sample sizes). Subjects ranged in age from 19 to 38, the mean age in Hamburg was MHamburg = 23.3, the one for Köln was MKöln = 22.2. The questionnaire was distributed in both universities at the end of course.

The pre-service programs in both universities prepare students to teach within elementary schools. Although the programs are structured differently (for details see Kuendiger, Schmidt, Kellenberger (1997)), one can identify two groups of students which are comparable between the universities. These are those who specialized in one of the subject areas and those who did not.

According to the non-interval nature of the data non-parametric tests were used. To test the impact of achievement and place of study on the use of a specific attribution log-linear models (e.g., Darlington 1990) were used. The results of the log-linear analyses can be interpreted similar to those of analyses of variances (see Table 3). A significance level of 1% was used throughout the study.

5. Results

5.1 Achievement and confidence in teaching

Tables 1 and 2 provide descriptive information on how pre-service teachers judged their past performance in mathematics and in German, and how confident they were teaching, respectively. It has to be noted that only one student had mathematics as well as German as teachable subject. As a general trend one can observe, that pre-service teachers who specialized in a subject area, judged their past performance as higher and were more confident teaching this area (for further discussion on the differences between universities see Kuendiger & Schmidt 1997).

	in Mathematics					
	specialized in mathematics	below average (1 or 2)	average (3)	above average (4 or 5)	total	N
Hamburg	no	36.4	45.8	17.8	100	107
	yes	7.9	31.6	60.5	100	38
Köln	no	15.1	42.9	42.0	100	119
	yes	not consequential				1

	in German (language arts)s					
	specialized in German	not confident (1 or 2)	neutral (3)	confident (4 or 5)	total	N
Hamburg	no	9.9	56.8	33.3	100	111
	yes	0.0	32.4	67.6	100	34
Köln	no	5.5	63.7	30.8	100	91
	yes	6.9	41.4	51.7	100	29

Table 1. Perceived former achievement (achievement in %).

	Mathematics					
	specialized in mathematics	not confident (1 or 2)	neutral (3)	confident (4 or 5)	total	N
Hamburg	no	37.8	28.9	33.3	100	90
	yes	16.1	25.8	58.1	100	31
Köln	no	9.6	20.9	69.9	100	115
	yes	not consequential				1

	German (language arts)					
	specialized in German	not confident (1 or 2)	neutral (3)	confident (4 or 5)	total	N
Hamburg	no	22.2	32.2	45.6	100	90
	yes	6.5	29.0	64.5	100	31
Köln	no	14.8	23.9	61.4	100	88
	yes	3.6	17.9	78.6	100	28

Table 2. Confidence in teaching (confidence in %).

5.2 Learning history

In the log-linear analyses a specific attribution is looked upon as a function of past achievement and place of study (see Table 3). The results are based on those future teachers who did not specialized in math or German respectively. Thus, pre-service teachers with comparable background in mathematics and in German were investigated.

	for mathematics					
	achievement		city		achievement	
attribution	*Chi (df=4)*	*p*	*Chi (df=1)*	*p*	*Chi (df=4)*	*p*
intelligence	35.11	*	0.65		3.28	
math ability	48.71	*	0.66		1.90	
effort	11.37		2.33		3.77	
interest	39.97	*	0.18		2.88	
easy subject	3.05		2.15		1.30	
teacher's ability	45.65	*	3.34		2.84	
help from others	5.31		0.96		1.72	
lack of intelligence	7.59		3.49		0.00	
lack of math ability	53.86	*	0.01		0.11	
lack of effort	30.98	*	1.69		2.57	
lack of interest	29.42	*	1.90		8.92	
difficult subject	15.21	*	6.09		1.58	
lack of teacher's ability	35.3	*	1.23		3.02	
lack of help from others	9.42		1.25		4.25	

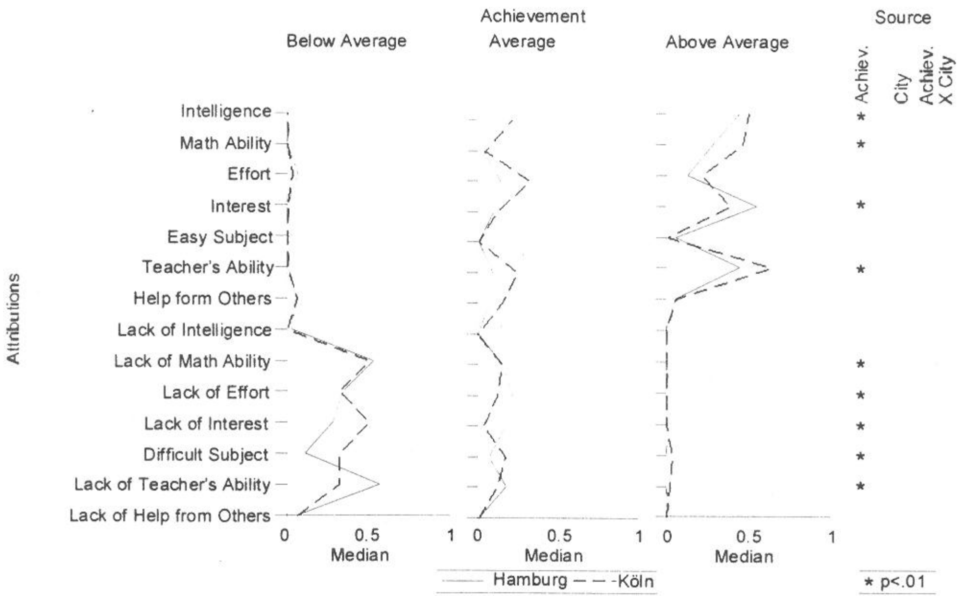
* p<0.01

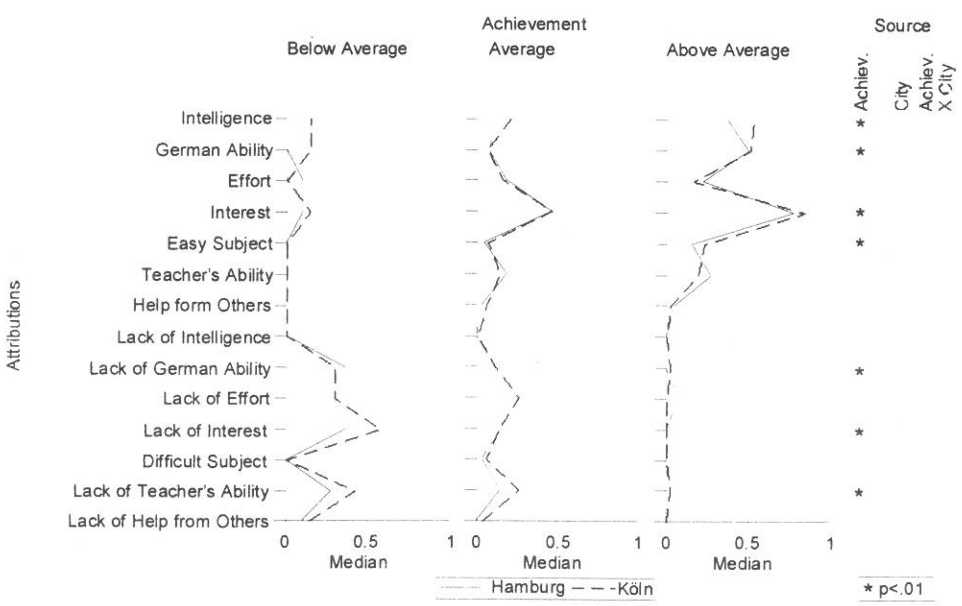
	for German (language arts)					
	achievement		city		achievement	
attribution	*Chi (df=4)*	*p*	*Chi (df=1)*	*p*	*Chi (df=4)*	*p*
intelligence	26.24	*	1.88		5.13	
math ability	60.55	*	0.02		0.06	
effort	2.62		2.41		1.43	
interest	36.00	*	0.15		0.74	
easy subject	19.02	*	1.81		0.05	
teacher's ability	10.67		1.51		4.76	
help from others	2.74		0.29		1.73	
lack of intelligence	0.91		1.48		0.00	
lack of math ability	22.44	*	0.09		0.10	
lack of effort	12.05		0.01		6.34	
lack of interest	32.94	*	0.17		2.65	
difficult subject	2.52		0.02		2.50	
lack of teacher's ability	20.70	*	3.12		2.23	
lack of help from others	3.37		0.74		6.38	

* p<0.01

Table 3. Summary for the different attributions as a function of achievement and city. (Log-linear analysis for non-specialists)

Table 3 shows that for mathematics as well as German the use of a particular attribution did not depend on the place of study (city). Moreover, no significant interactions between "city" and "achievement" were found for any of the attributions. The use or not use of an attribution was influenced only by the level of past achievement. For mathematics 9 significant differences were found, for German 7 were found. To facilitate the interpretation of these differences the results of Table 3 were presented graphically by forming three achievement groups (see Figure 1a & b) and by plotting the medians for each attribution (0 = attribution not used, 1 = attribution used).





Figures 1a & b.

For both subject areas the relationship between level of achievement and use of attribution was meaningful: Higher achievement was associated with favorable attributions, i.e., "intelligence", "math ability", "interest" and "teacher's ability" for mathematics and "intelligence", "German ability", "interest" and "easy subject" for German. Lower achievement was associated with unfavorable attributions, i.e., "lack of math ability", "lack of effort", "lack of interest", "difficult subject" and "lack of teacher's ability" for mathematics and "lack of German ability", "lack of interest", and "lack of teacher's ability" for German.

In summary, the relationship between perceived past performance and use of attributions which was found in earlier studies was confirmed. For both subject areas pre-service teachers, who perceived their past performance as high, had developed a favorable attribution pattern and thus, can be described as having a favorable learning history, whereas those who perceived their past performance as low can be described as having an unfavorable learning history in mathematics and German, respectively.

Keeping in mind that the pre-service teachers explained cumulative past achievement, the called upon attributions can be looked upon as generalized reasons. When those prospective teachers who considered themselves as having been successful (above average achievement) in mathematics were compared with those who considered themselves as not successful (below average achievement) the corresponding attribution pattern can be interpreted as follows (see Figure 1a): Successful pre-service teachers thought of themselves to be intelligent and able in mathematics, and were interested in the subject area. Besides these internal reasons they attributed their success to good teachers. Those, who judged their past mathematical achievement as low, came to the conclusion that they lack mathematical ability and at the same time admit that they lacked effort and interest. Besides these internal reasons mathematics was blamed to be too difficult and the teacher was blamed for not having done a good teaching job. The position of the three graphs in Figure 1a shows that - although there is some variation - the relative importance of the above-mentioned attribution for high/low achievement is about the same. This is confirmed in Table 4a, where the frequencies with which an attribution is chosen, alone and together with other attributions, are given based on the information obtained from all participants, specialists and non- specialists.

When those prospective teachers who considered themselves as having been successful (above average achievement) in German (language arts) were compared with those who considered themselves as not successful (below average achievement) the corresponding attribution pattern can be interpreted as follows (see Figure 1b): Successful pre-service teachers thought of themselves to be intelligent and able in German, and were interested in the subject area. Besides these internal reasons they attributed their success to good teachers and easiness of the subject area. Those, who judge their past achievement as low, came to the conclusion that they lack German ability and at the same time admitted that they lacked interest. Besides these internal reasons the teacher was blamed for not having done a good teaching job. Looking at Figure 1b with regard to the relative importance of the above attributions, "interest" stands out as particularly important for high achievement in German. "Interest" is not only the attribution mentioned the most to explain high achievement in German but it is also mentioned more often than any attribution related to achievement in mathematics. The importance of "interest" for German is confirmed in Table 4b, which shows that it was - based on the responses of all participants - by far the most frequently named reason.

When one compares the two subject areas mathematics and German, one can derive that in the mind of these pre-service teachers, general and subject area specific ability as well as interest is important to achieve in either subject area, with interest playing a particular important role for achievement in German. Failure does not indicate lack of general ability but is due to a lack of subject area specific ability. Thus, failure in one subject area does not necessarily imply that one cannot be success in other areas. An additional cause for failure is lack of interest. For mathematics lack of effort has to be added. Moreover, the teacher is blamed for low achievement of his/her students. Yet, in mathematics the teacher is also given credit for high achievement. Thus, the teacher seems to play a stronger role regarding achievement in mathematics compared to the one in German. Moreover, the differences between the attributions used to explain achievement in mathematics and German showed, that mathematics is looked upon as difficult (a reason for failure) and German as easy (a reason for success). More significant differences were found for mathematics achievement than for achievement in German. This indicates that the motivational framework related to mathematics is more pronounced.

5.3 Confidence in teaching and attributions

The proceeding results show that all differences in attribution found between pre-service teachers, who did not specialize in a subject area, can be explained by differences in their perceived former achievement. The same holds for pre-service teachers who specialized in a subject area (Kuendiger, Schmidt & Kellenberger 1997, Kuendiger & Schmidt 1997). Thus, all participants were included in the analysis that compared the confidence in teaching of pre-service teachers, who named a particular attribution, with those, who did not.

Table 4a shows that those pre-service teachers, who attributed their achievement in mathematics to their mathematical ability, were more confident in teaching the subject than those, who did not call upon this reason. The same was true for those who were interested in mathematics. Thus, of the favorable attribution only those are relevant for expected future success in teaching, that are internal and related to mathematics. Pre-service teachers who chose the attribution "lack of ability", "lack of effort" and/or "lack of interest" were less confident in teaching then those who did not. In addition, the attribution "lack of help from others" resulted in a significant difference. Yet, only 16 of 264 pre-service teachers chose this attribution, alone or with other attributions. Therefore, the following interpretation seems warranted: if a person really feels the need to blame others for his/her low achievement, then it does not surprise when the same person is not very confident teaching mathematics either.

Disregarding the latter result, because it affected only very few participants, one can summarize: attributions relevant for teaching were a subset of those linked with a favorable/unfavorable learning history. They are internal to the prospective teacher and are related to mathematics. "Intelligence", "teacher's ability" and "lack of teacher's ability", which were also relevant for explaining achievement do not carry over into perceptions related to future teaching. These results make intuitive sense.

The attributions related to confidence in teaching German were "ability in German", "interest" and "lack of interest" (see Table 4b). These differences were in the expected direction corresponding to those found for mathematics. For German there were only 3 attributions related to future teaching compared to 5 (6 respectively) in mathematics. When the attribution pattern associated with achievement were discussed, more significant differences were found for mathematics than for German, 9 compared to 7 (see Table 3). Therefor, in line with the more differentiated motivational framework related to mathematical achievement, more attributions are related to teaching mathematics than to teaching German.

Looking at the motivational aspects of learning how to teach mathematics, the above results suggest that those attributional causes which are related to confidence in teaching will be transferred onto corresponding attributions, when beginning teachers evaluate their first teaching experiences. If one admits that the evaluation of a mathematics lesson with regard to

being successful or not successful, is more subjective than the result of a mathematical problem, the initial attributional style gains additional importance.

	Mann-WhitneyU test		**Frequency with which an attribution was chosen**	
attribution	$z^{+,-}$	*p*	*alone*	*with others*
intelligence	+1.71		5	66
math ability	+4.10	*	2	51
effort	+0.22		5	61
interest	+2.62	*	6	66
easy subject	+2.45		0	12
teacher's ability	+0.87		3	75
help from others	-0.27		0	37
lack of intelligence	-0.20		0	3
lack of math ability	-3.59	*	3	51
lack of effort	-3.52	*	7	47
lack of interest	-3.20	*	2	39
difficult subject	-0.53		1	39
lack of teacher's ability	-1.93		4	55
lack of help from others	-2.68	*	5	11

* p < .01, N = 264.

Table 4a: Confidence in teaching mathematics.

	Mann-Whitney U test		**Frequency with which an attribution was chosen**	
attribution	$z^{+,-}$	*p*	*alone*	*with others*
intelligence	+1.13		5	68
ability in German	+4.12	*	2	63
effort	+0.71		1	45
interest	+4.14	*	30	151
easy subject	-0.11		2	28
teacher's ability	+1.79		2	48
help from others	-0.49		0	10
lack of intelligence	-0.78		1	1
lack of ability in German	-2.30		7	23
lack of effort	+2.70		8	40
lack of interest	-3.84	*	6	30
difficult subject	-1.83		2	9
lack of teacher's ability	-1.46		3	37
lack of help from others	-1.82		0	7

* p < .01, N = 265.

Table 4b: Confidence in teaching German

(Comparison of pre-service teachers, who chose a specific attribution, with those who did not choose this attribution to explain their former achievment.)
+ positive z-values mean that pre-service teachers who chose this attribution, are more confident in teaching German compared to those who did not choose this attribution.
- negative z-values mean the opposite.

Beginning teachers, who enter the profession already with less confidence in their ability to teach mathematics, are more likely to focus their self-evaluation on those parts of their teaching that are -objectively or subjectively- less successful and are likely to call upon attributional causes equivalent to those that were found to be related to lower confidence, i.e., lack of ability to teach mathematics, lack of effort and interest, and - for some - lack of help by others. In short, they enter a negative motivational cycle according to Heckhausen's process model. Following the same line of reasoning, beginning teachers who are very confident in their initial ability and interest to teach mathematics, are likely to enter a positive motivational cycle.

Looking at the motivational aspects of learning how to teach German, the same two favorable attributions linked to teaching mathematics are also linked to teaching German. These are: "subject specific ability" and "interest". Thus, one can infer that pre-service teachers who use these attributions for their own achievement in German are also more likely to enter a positive motivational cycle when they evaluate their first teaching experience. With regard to the unfavorable attributions, the link between attribution and confidence in teaching is less pronounced for German than for mathematics. The only attribution that showed a significant difference for German is "lack of interest". If the above line of reasoning is followed, i.e., that perceived failure in teaching German is explained by lack of interest, then this attribution would not influence the beginning teacher's self-concept of his/her ability to teach the subject area. "Interest" is an internal reason that potentially can change, if the teacher decides to become interested.

The above described differences between mathematics and German with regard to unfavorable attributions linked to lower confidence in teaching, may be explained, if one accepts the assumptions that - although "failure/success in teaching mathematics" can be determined less precisely than low/high mathematical achievement – "failure/success in teaching mathematics" can be determined more "objectively" than "failure/success in teaching German" when looking through the eyes of prospective teachers:

The teaching mode "teacher question - student answer" is considerably more frequent in mathematics than in language arts. Due to the nature of mathematics and the way it is taught, there is generally only one correct answer to each of these teacher questions. Whether the student's answer is correct or not, can be determined without any room for negotiation. On the one hand, student failure is more likely to occur in mathematics, simply because there are more questions, and "failure" is more clearly defined, on the other hand, it is accepted that the teacher is more important in learning mathematics (see Table 3). Thus, student failure and the teacher's lack of ability to teach are more strongly linked for mathematics than for language arts.

The above offered explanation is obviously highly speculative and does not explain the similarity between mathematics and German with regard to the favorable attributions. Further research is needed to clarify the above results.

6. Summary

The above described study first investigated the concept "learning history" for a sample of German pre-service primary teachers. The relationship found between perceived past achievement and the attributions used to explain this achievement, confirmed the results from earlier studies: the above variables can be summaries in the concept "learning history" and pre-service teachers can be classified with regard to possessing a more or less favorable learning history in mathematics as well as language arts (German). The two subject areas differed in part with regard to the attributions related to achievement and inferences were made on what is relevant for success in either subject area.

Second, the relationship between learning history and confidence in teaching was investigated by linking the use/none-use of a specific attribution with the level of confidence in teaching. For mathematics the results show that the concept "learning history" can be looked upon as a motivational framework that sets the stage for pre-service teachers evaluation of their initial teaching experiences. Implications of these results for teacher education are discussed in Kuendiger, Schmidt & Kellenberger (1997) as well as in Schmidt (1992) .

For language arts (German) the results were in part different from those for mathematics. An explanation of these differences was offered by discussing the relative ambiguity of determining failure in teaching language arts compared to teaching mathematics.

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Pekka Kupari

Teachers Mathematical Beliefs - Four Teacher Cases

Background

In the second MAVI Workshop in March 1996 I discussed in my paper (see Kupari 1996) about the changes in teachers' beliefs of mathematics teaching and learning. The analysis was based on the questionnaire data which was collected from the Finnish comprehensive schools in 1990 and 1995. In both phases the belief inventory was presented to the teachers. It was important that there were 15 same teachers in these study phases because it made possible to analyse more closely the changes in beliefs taken place during five years.

In the spring 1996 I interviewed four of these 15 teachers to receive deeper and more personal information on teachers' beliefs and thoughts. I chose the teachers so that they represented two different belief types and furthermore in the both types the other teacher was female and the other was male. This paper continues the analysis of the former article and it focuses on teachers' mathematical beliefs and also their relationships to teaching practices on the basis of the interview data.

About the method

In analysing teachers' mathematical beliefs and changes in them I have used the so called *belief profile method* (cf. Middleton et al. 1990). For all 68 teachers in the 1995 study mathematical belief profiles were drawn by using the questionnaire data of that study. After that two criterion profiles (teacher types) were selected for the purposes of analysing and comparing teachers' profiles. These types are called the traditional type and the innovative type (cf. Neyland 1995). The traditional teacher type could be said to represent the behaviourist learning and teaching tradition. The content has broken down to a sequence of tasks to be mastered and facts to be learned. The focus tends to be on what students can do, rather than on what understandings and meanings have been achieved. Students are not really worried about their understanding - they believe that if they can get right answers, then they understand. For the innovative teacher type it would be typical the orientation towards the alternative approach of teaching, i.e. the social constructivism. The teacher has the responsibility of aiding students' reconstructive process, which involves learning the concepts, orientations, values and processes of the expert community.

For those 15 teachers which were same both in 1990 and 1995 we could draw two belief profiles. Altogether, by using these belief profiles we could then compare teachers' mathematical beliefs in two ways. First, we could find the places where there were clear deviations (changes) in one teacher's (expressed) beliefs. Second, we could make comparisons with the criterion profiles and look at 'the distance' between some teacher's belief profile and the criterion profile.

In the interviews two of the four teachers represented traditional teachers and the other two innovative teachers. The interview method was thematic interview and the interview themes were prepared on the basis of the questionnaire. One interview lasted about one and half hours. Afterwards the interviews were transcribed.

Now it is especially interesting to examine the relationship between the two types of information: the questionnaire data and interview data. How well do they match? In this paper, the following two questions are discussed:

- Do the interview data support the questionnaire results or are they contradictory in some respect?
- In what way does the interview data complement the questionnaire results?

Teacher cases: results and interpretations

In this part I present some first results of teachers' interviews. I describe here just two teacher cases in a detailed way but the conclusions at the end of this paper are based on all the interviews.

Case 1 (Teacher L: The female teacher, teaching experience 31 years)

Below (see Diagram 1) I have presented teacher L's belief profiles. In my classification (the traditional type - the innovative type) teacher L could be located quite near the traditional type. We can see in the diagram that this teacher's mathematical beliefs has been quite similar in 1990 and 1995. The following quotations will describe some aspects of her beliefs (two statements:V23 and V12).

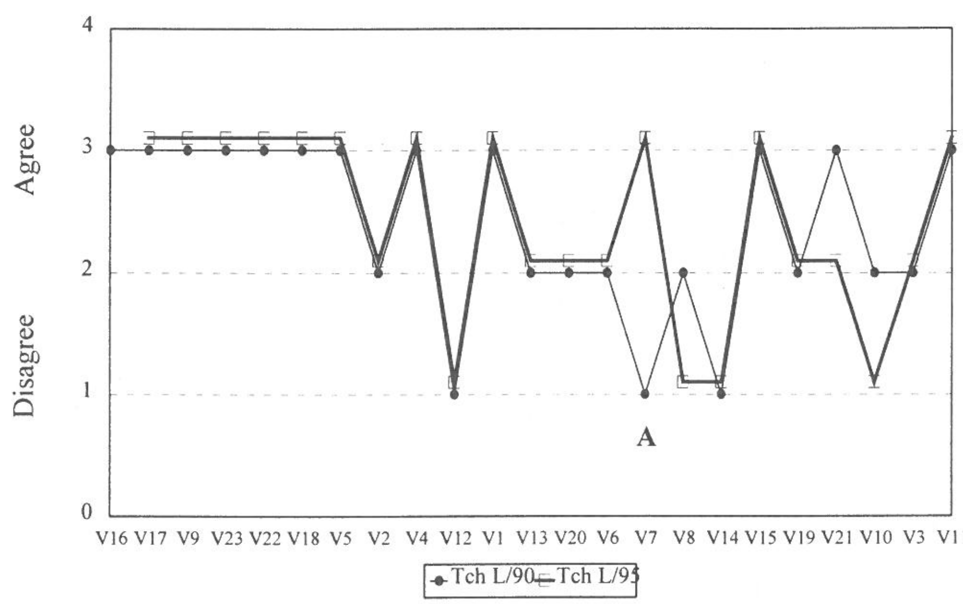

Diagram 1. The belief profiles of teacher L in 1990 and 1995.

V23: In the teaching one should clearly point out that mathematics is an essential part in our culture.

I think that parents appreciate mathematics...A great part of parents, I think, has the opinion that mathematics is needed and it should be studied well. Of course, there should be in school much more all kind of connections to the society. We have included just this in the curricula. Within this one lesson we try to go into the issues of the day, it means sales, elections, competitions, new European money... So that they would see where it is needed...

V12: The student need not necessarily understand every argument and procedure.
I am so old-fashioned that I think that a student himself is not able to (study)... The question must be discussed together in some phase. 'Why do you think in this way' and then to find the mistakes in thinking...

According to the first quotation teacher L seems to understand that the important part of the cultural nature of mathematics is to include a lot of social connections and everyday situations in teaching. The second quotation reveals that teacher L finds it essential that there are room in teaching to discuss and to make students to think and to understand.

In Diagram 1 there was just one point (A) where the belief profiles of teacher L were clearly different. That statement concerned working with concrete material in mathematics teaching and the following quotation expresses teacher L's opinion in 1995. Perhaps teacher L has found some new possibilities (in curricular issues or in teaching arrangements) to implement her instruction and this has also affected to her beliefs and attitude towards concrete material

If I draw it so of course he/she will understand it... A great part of problems can be solved by drawing. Then we have tried all kinds of measuring, sometimes constructing objects...They see the result and make a mistake in it, 'oh I forgot the deck of the box'... I think it is of some use to them.

The next diagram (Diagram 2) shows what are the differences in belief profiles between teacher L and the criterion teacher. We can see that there are three clear deviations (A, B and C) and some smaller differences in the profiles. The following quotations are consistent with teacher L's answers of the belief inventory and they tell that teacher L emphasises students' understanding in teaching, that the way how the textbook is dividing the mathematical content into parts is good for her and that she does not emphasise routine problems in her teaching.

A (Understanding of arguments and actions)
If (a student) does not understand in a certain way then I find quite easily an other way to show on the object or the figure... I think I have the mathematical ability that I can imagine it to some drawing or figure... When (a student) says something wrong, so why. I am asking and sometimes we consciously go to quite a wrong direction. I continue asking. The best thing was when some other student says that he does not understand. I asked further on. Then they themselves came to a dead end. Then I laughed in my turn that I knew this but it is good that things are clear now.

B (Facilitating learning by dividing the content into small parts)
Any one of teachers (in this school) does not follow his/her own style. I think that everybody has this kind of style. We have liked this textbook... I suppose that most of us go it through in some way...

C (Emphasising routine problems)
When they get calculators so I have used quite much the practice that I give them problems concerning the school environment. They start to calculate prices of books and new curtains for the class. They make the measurements themselves. Then on the 9th grade when they already know the trigonometric functions I try to find something, for instance a ramp for wheelchairs. It cannot be steeper than 20 degrees. They go and measure the ramps there... You may link it somehow to the society.

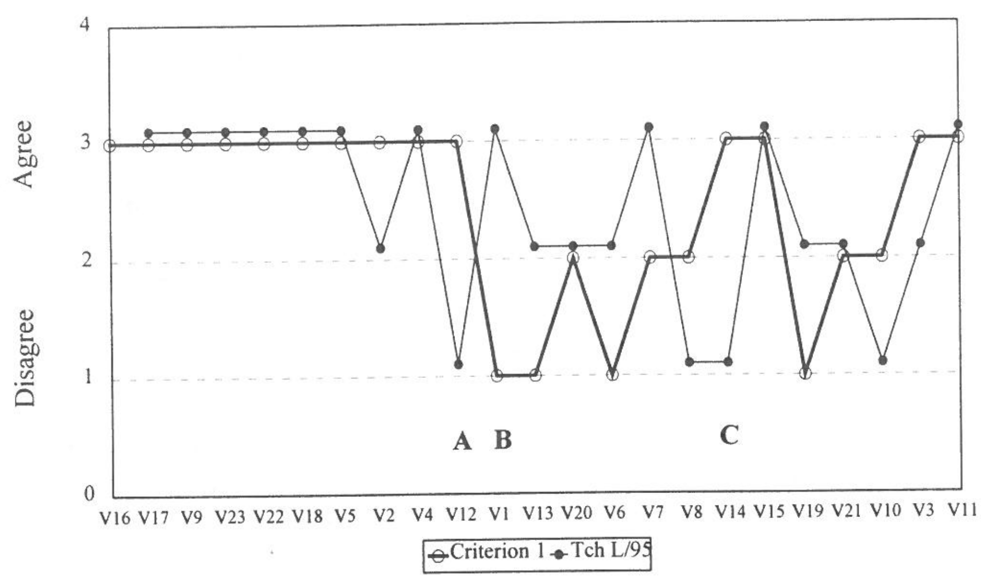

Diagram 2. The belief profiles of teacher L and the criterion teacher (the traditional type).

Summing up the results of teacher L, I could say that the picture of her mathematical beliefs was not so traditional as I saw it through the questionnaire data. Teacher L's beliefs were a mixture of different beliefs - as they usually are - and there were also innovative features in her beliefs. Her descriptions about the approaches of teaching also supported this.

Case 2 (Teacher N: The male teacher, teacher experience 24 years)

In Diagram 3 you can see the belief profiles of teacher N. In contrast to teacher L, teacher N could be located nearer the innovative type on the basis of his mathematical beliefs. The diagram reveals very well that also teacher N's belief profiles were very similar in 1990 and in 1995. The first quotation will give the reader some impression of the nature of his beliefs (the statement V16).

V16: During mathematics lessons one should emphasise the importance of thinking.
As is well known, the use of brains is exhausting and hard. How do you get the students to adopt an attitude that it is nice to solve problems. They would have some joy when they solve problems. The biggest problem is that you get them to think at all... students are a little bit shortsighted... Well, I would be very careful with the fact that if there were more time (in mathematics) then the content would be expanded. I would not see any sense in this, really. But on the contrary, it is

mathematical thinking that could be mainly improved and developed... perhaps to encourage more to continue on the fields or branches where mathematics is needed.

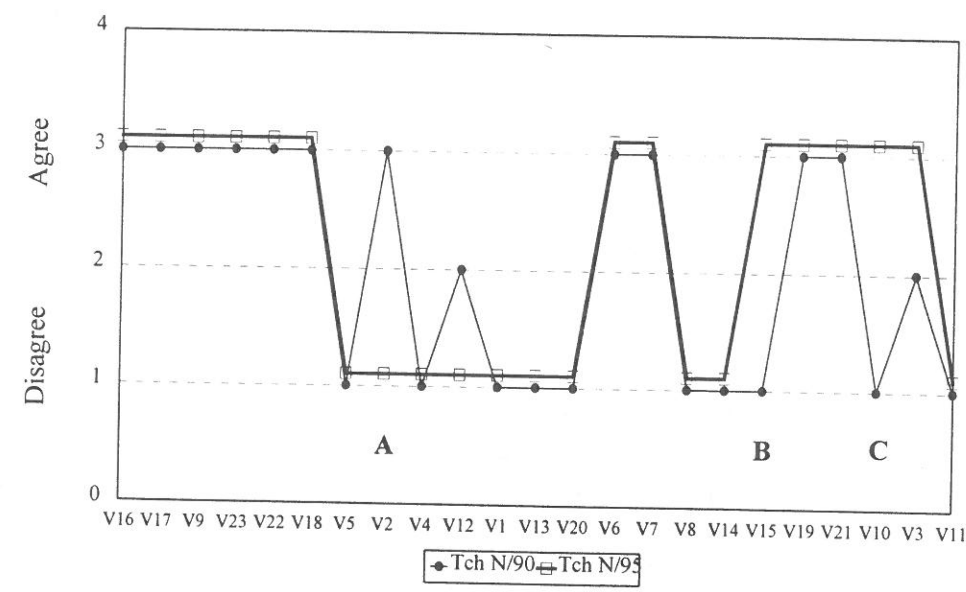

Diagram 3. The belief profiles of teacher N in 1990 and 1995.

On the basis of the belief profiles some changes had possibly taken place in teacher N's mathematical beliefs during 1990-95. Especially there were three point (A, B and C) where the differences between profiles were clear. In the point A there was a question about the importance of the exact and mathematically correct language in teaching but in the interviews no signs concerning this issue could be found. The following quotations give some reasons or explanations for the changes in the points B and C.

B (The frequent use of applications in teaching)
I would like to go further to the direction that the teaching would really move towards the problem-oriented approach. The problems in the textbooks would be developed so that unnecessary parts could be cut out. It would be also good to think more carefully what is it... what is really helpful for the development of mathematical thinking, for the skills of practical mathematics and for the needs of everyday life.

C (The importance of keeping order on the mathematics lesson)
I understand mathematics so that there should be about 10-15 minutes of the lesson rather quiet time. I have this kind..., I know that all teachers don't undersign this... I have this kind of habit that it is a quiet moment when everyone can think... Then we think.

Usually the strict order could be regarded as characteristic to the traditional type of lesson where the teacher is the only authority. The teacher is always explaining the content and telling what and how to do. I understand that teacher N does not see the keeping order in the same way. For him 15 minutes quiet time means the time for students' own thinking and concentration.

In the last figure (Diagram 4) we can see that the belief profiles of teacher N and the criterion teacher (the innovative type) are very similar. However there are three places where the differences seem to be clear. These statement deal with the importance of keeping order on the lesson, the use of learning games in teaching and the role of solution processes in assessment. Through the interview data these differences can be seen in the new light, and in fact the mathematical beliefs of teacher N are more innovative than the beliefs of the criterion teacher.

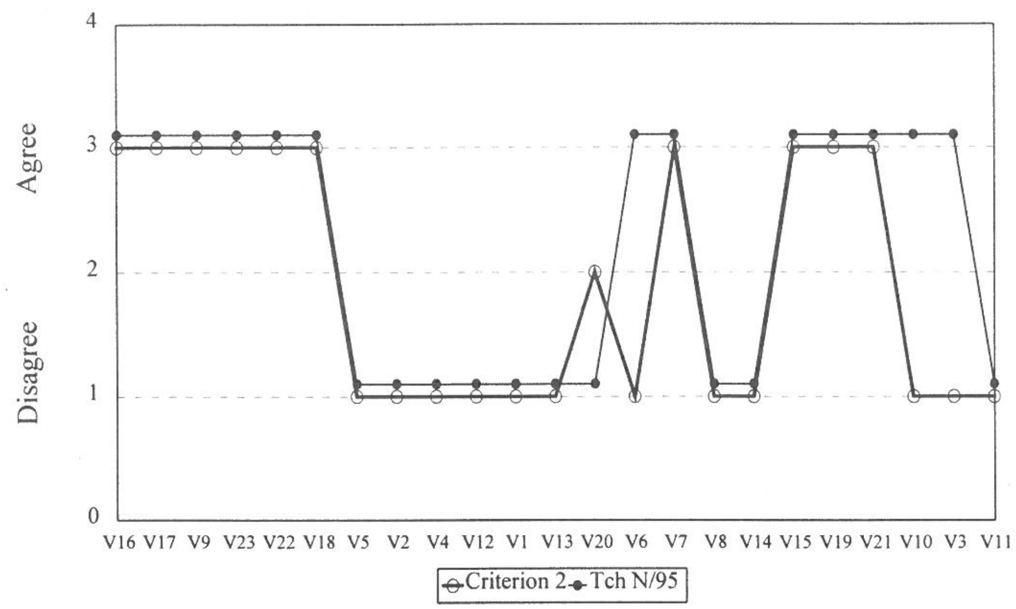

Diagram 4. The belief profiles of teacher N and the criterion teacher (the innovative type).

Conclusions

The analysis of these four teacher cases was very useful and rewarding experience for the author. By combining the questionnaire results and the interviews the following conclusions could be made.

First, the results support the multidimensional and flexible nature of teachers' beliefs. This means that there are quite different (traditional, formalist, 'socioconstructivist') features in all teachers' beliefs. Probably, the clustering nature of beliefs (cf. Green 1971) makes it possible to hold also conflicting sets of beliefs and this can explain some of the inconsistencies among the beliefs. Through the interviews the questionnaire results can be better understood (the context-dependence of beliefs).

Second, the questionnaire results indicated that the more traditional view was not denied by the responses of teachers, but they placed less emphasis on this view than on those that emphasized mathematics as a way of thinking and as problem-solving. On the contrary, in the interviews the teachers were very modest about their own ideas and rather emphasized the traditional features of their teaching. Perhaps it is so that traditionality is some kind of 'virtue' in mathematics (for the older generation) while introducing innovative thoughts is not so desirable. The social environment of the school or teacher unit might act as a filter.

Third, some changes in teachers' belief profiles may be the result of changes in the curricula or in teaching arrangements.

Fourth, the interview data was important when relationships between beliefs and teaching practices were assessed. For example, the practices which teacher L used were more versatile than what the teacher questionnaire could reveal. In the author's opinion these four teachers had their belief systems quite well in balance with their teaching practices. They were very experienced and now they had self-confidence and courage to do things partly in their own way. They had freedom in their teaching.

Fifth, the different methods complement each other. On the basis of one method you can often make too simple and too strong conclusions. The questionnaire data makes it easier to do 'erroneous' interpretations at least for two reasons. The first is the fact that teachers want to give desirable answers, if somebody outside the community asks the questions. Also the point of view can be different. In the interview you can also get important information which you have not asked at all. This is not possible when using just the questionnaires.

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Sinikka Lindgren

Prospective Teachers' Math View and Instructional Practices

An Analysis of Four Cases

Background

The study at hand is a longitudinal research effort. The main target group is the prospective elementary school teachers at the Department of Teacher Education of the University of Tampere. My intention has been to follow the target group through the phase of teacher education, during lessons, exercises, and teaching practicum, and during their first year of teaching on the field after graduation from university.

I understand an individual's mathematical *beliefs* to be composed of his or her subjective implicit knowledge on mathematics and its teaching and learning. *Conceptions* are conscious beliefs. The beliefs - conscious and unconscious - can be seen as a *belief system.* (Törner & Pehkonen 1996, Lindgren 1995.) In an effort to achieve school reform we need to focus our attention on the belief systems of teachers and teacher candidates. It is also important to understand the structure of beliefs and their birth and development.

In my earlier reports I have reported theory and data concerning the structure of beliefs and conceptions on teaching mathematics. I have been using data gathered in Tokio in fall 1991 (n=101) and data from my own students at the University of Tampere (n=163). I have used both a quantitative and a qualitative approach. My study has been principally based on the theories of Thomas Green, Alba Thompson, Paul Ernest, Alan Schoenfeld, and Kuhn & Ball. (See Lindgren 1995, and 1996.)

Method and procedure

In the task to assess the prospective teachers level of development on beliefs about teaching mathematics I have been using the theory of Thompson. Her framework consisted of three levels. At each level specific features can be distinguished in relation to the following questions (Thompson 1991):

1. What is mathematics?
2. What does it mean to learn mathematics?
3. What does one teach when teaching mathematics?
4. What should the roles of the teacher and the student be?
5. What constitutes evidence of student knowledge and criteria for judging correctness, accuracy, or acceptability of mathematical results and conclusions?

In my factoral analyses I found to some extent corresponding dimensions with Thompson's levels 0, 1, and 2. In accordance with the items which formed these factors I named these dimensions as new variables: level RR (Rules and Rutines), level DG (Discussions and Games), and level OA (Open Approach). In the questionnaire each item was scaled from 1 to 5, where 5 meant full agreement with the proposal of the item. For each student the values for the levels RR, DG, and OA were obtained by counting algebraically the means for the items that formed the certain level. The values of these means were then standardized.

On the basis of frequency analyses of these three variables from the whole target group the level DG could be seen as divided to three parts: levels GR, GRO, and GO. The part GR (Games and Rules) overlaps with level RR, the part GO (Games and Openness) with Level OA, and GRO is their conjoint area. Thus GRO stands for a situation where the teacher simultaneously supports the methods of games, rules and routines, and open approach. Thus there are five categories. (See illustration in Lindgren 1996.)

In this part of my study I want to find information on the question: What is the relation between the prospective teachers' professed beliefs and observational data as the students encounter teaching practicum?

In order to answer this question, 12 students were selected from my first target group of 72 students for a closer follow-up. I had been teaching these 72 during their first year in teacher education, both basic courses in mathematics and mathematics pedagogy. They had completed the Likert type questionnaire twice: at the beginning of September, 1993 and at the end of April, 1994. The 12 were selected on the basis of the results of the answering of the questionnaire and a math exam during their first academic year. In the second year of the teacher education program these 12 had been interviewed with special attention to their educational memories (See Lindgren 1996). Lessons including their teaching practicum had been videotaped. On the ground of the values of the new variables (RR, DG, and OA) and on the interviews and my evaluations of the practicum lessons 11 of these 12 students could clearly be grouped into one of the five categories named above (in one case all of the three variables were clearly under the mean). In the following this indicator is presented in parenthesis for the chosen cases.

Only six of the 12 held mathematics lessons during their fourth academic year in their final teaching practicum. Again they completed the questionnaire, now including four further open questions. One of the three or four math lessons which I have been listening has also been videotaped. Their supervising class teachers were asked to fill in a questionnaire. When I constructed this questionnaire I added to the above five questions by Thompson the sixth question: What is problem solving? This question is also included in the table about the development of beliefs about teaching mathematics designed by Erkki Pehkonen. (See Pehkonen 1994, 64.) Each of the six units thus obtained included six statements referring to Thompson's levels 0, 1, and 2 (marked with 1, 2 or 3 stars respectively in the table). The teacher was asked to choose two of these statements which best fitted to the practicing student's view of mathematics and teaching mathematics. The best choice was asked to be marked with number 1 and the second best with number 2. At the end there was an open question regarding the supervisor's general view of the student. When the sum of the attained points from each unit were divided by two I obtained a measures for the openness of the prospective teacher's instructional practices, IP. In the following tables the means of IP for the six units for each case is presented.

A QUESTIONNAIRE TO THE SUPERVISING CLASS TEACHER
OF THE PRACTICING PROSPECTIVE TEACHER

For each unit mark the best choice with number 1 and the second best with number 2.

1. MATHEMATICS AS A SUBJECT
__ PT appreciates "everyday math" , i.e. MA is useful knowledge in practical activities.(**)
__ MA is an interesting and a challenging subject.(***)
__ Rules and routines are essential in MA.(*)
__ Ma is processes, generalizations, and perception.(***)
__ MA is learning and understanding of concepts.(**)
__ MA is finding the right answer.(*)

2. MATHEMATICS TEACHING
__ PT uses manipulatives him/herself and lets the pupils sometimes use them.(**)
__ PT allows every pupil to take part in the development and formalizing of MA.(***)
__ PT places emphasis on the learning by heart of rules and routines.(*)
__ PT encourages the pupils to find their own solutions and to debate on the different solutions.(***)
__ PT concentrates on covering the contents of the book in order.(*)
__ PT wants to make the study of MA fun by using games, a variety of manipulatives and other materials.(**)

3. LEARNING OF MATHEMATICS
__ Learning MA is understanding MA.(**)
__ Learning MA is the solving of MA problems neatly.(*)
__ Learning MA is to invent and assess mathematical processes and generalizations.(***)
__ Learning of MA manifests itself in the quick finding of the correct solution.(*)
__ Learning of MA is to memorize rules and routines.(*)
__ Learning of MA is the pupil's autonomous thinking and assessing.(***)

4. TEACHER'S ROLE
__ Maintainer of discipline and good order.(*)
__ Supportive supervisor of pupils' autonomous thinking.(***)
__ Manager of pupils' work.(**)
__ Listener to pupils' ideas/proposals.(***)
__ Interpreter of rules.(**)
__ Demonstrator of the right method to solve a problem.(*)

5. EVALUATION OF THE LEARNING PROCESS AND LEARNING OUTCOMES
__ The teacher tells/shows the correct answer.(*)
__ The pupils are responsible for the accuracy of the solutions.(**)
__ The pupils are allowed to discuss what is correct or incorrect in MA.(***)
__ PT always checks the pupils' work.(*)
__ PT appreciates more a good process than a correct answer.(***)
__ The teaching has been successful if the pupils have understood the matters in hand.(**)

6. WHAT IS "PROBLEM SOLVING".
__ Doing story problems.(*)
__ Going through problem cards or duplicated copies.(**)
__ Almost all work in the math class.(***)
__ Finding of right strategies or algorithms.(*)
__ A method by which the pupil learns new mathematical contents.(***)
__ Sometimes the main object of the lesson, and therefore concrete problems are used.(**)

THE GRADING USED IN THIS EVALUATION SCALE:
(*) => 0 p, (**) 1. => 3p, 2.=> 2 p, (***) 1. => 6 p, 2. => 4 p.

Table 1. A scale for evaluation of instructional practices in mathematics.

Before I looked at the supervising teachers' evaluations I carefully studied the videofilm of each student. To get a more precise view of the student teacher's behavior and professed beliefs I had prepared an assessment scale for evaluation of a math lesson. This had the same units as the questionnaire for the supervising teachers. For each unit the six items - somewhat worked up - formed three bipolar axes. When I studied the videofilm I drew lines on the axes referring to what happened, how intensively and how often. The line was scaled from 1 to 5. Through counting the means for each unit and the whole scale I got a measure for the student's instructional practice. This value of IP is marked in the following tables in parentheses. It was quite astonishing how close these values came to the IP values attained from the supervising teachers' evaluations.

Results

In this short presentation I restrict my attention to four cases. They are not particular compared with the other students. As all the 12 have been given the questionnaire for the third time I give a more complete analysis of the changes in their evaluated levels on teaching mathematics in a later presentation.

I begin the portrayals with short descriptions of some educational memories of the math teachers of the case. After each name in parenthesis there are the z-values of the variables OA, DG and RR from the second measuring (1994). The following tables present the raw values of the same variables. Thus it is possible both to get an idea of where the student stands in relation to the whole target group and to assess the development of these variables during the four years of teacher education.

The last math grade in upper secondary school is also included in the parenthesis. The abbreviation Ex. means that the case completed the extensive course in mathematics. The last letters stand for the evaluated level of the development of beliefs and conceptions on teaching mathematics as it was evaluated in fall 1994. As can be seen in the tables since that year there have been changes: a clear decrease in the agreement of the RR method and for Eva and Greetta an increase in the appraisal of the OA method.

In the last column of the tables there are the evaluations of the instructional practices, IP, both the one made by the supervising teacher, and the one I attained through the videotape analysis of one lesson. After the tables there will be the answers obtained from the questionnaires of 1997 to the following two open questions:

1. How do you know that your lesson has been a good one?
2. What does it mean to "teach mathematics"?

Then there are some ***reflections*** which can be found in the student teacher's portfolio in their evaluation of their final practicum. Finally follow my own ***comments***.

1. Eva (-.68, .63, .15, Ex. 7), GR

In the primary school we only calculated, and calculated the problems from the book. Then for a period I felt math was really difficult. At the lower secondary school I liked better math.

1) Pupils have been working eagerly and actively getting into the matter. The objective of the lesson has been learned well on the average.

2) It is getting into the matter together with the pupils taking into account their level of attainment and their needs.

Eva	*OA*	*DG*	*RR*	*IP*
1993	4.00	4.25	3.17	3.3
1994	4.13	4.00	3.00	(3.2)
1997	4.50	3.50	2.83	

Table 2. The instructional practices in 1997 and the development of Eva's beliefs.

Reflections:

I wanted to make the math period a comprehensible whole where children learn things through their own concrete working. ... I tried to get the things close to everyday life, so that the children would get an idea of why the learning of these things is important.

Comments:

Eva's latest OA value was quite high and the RR value low. The DG value decreased during the four years. It seems obvious that she has risen to the level OA. Her mathematics knowledge might not be as good as for the other female teachers presented here. It seems to affect her instructional practices.

2. Greetta (.40, .63, .50, Ex. 8), GRO

My primary school teacher was really frightening. He was an old man who did not spare us any punishments. We had traditional math. We did routine problems, nothing was demonstrated. The whole elementary time was such that we were afraid of him. ...The lower secondary school was little better. The teacher was really strict and all problems had to be done. Some kids were afraid for him. In the upper secondary school we had really wonderful math lessons.

Greetta	*OA*	*DG*	*RR*	*IP*
1993	3.88	3.75	3.50	4.0
1994	4.25	4.00	3.17	(3.8)
1997	4.50	3.50	2.83	

Table 3. The instructional practices in 1997 and the development of Greetta's beliefs.

1) The pupils are contented, eager, and somebody wants to do extra. Comments "*This is fun*" have been heard.

2) It is helping the pupils. It is to make easier/complicate the problems in such a way that all pupils get fitting challenging tasks.

Reflections:

Math is a fairly clear subject to teach. To hold a routine lesson is quite easy. I want to learn more what Sinikka calls "Guided discovery". How can I give hints that lead to perception in the weakest pupils? I have learned very much. What's most important is that I have learned to differentiate my teaching to individuals.

Comments:

Greetta held very good lessons. It is obvious that she has grown to the level OA. The increase in her OA value was very high and the IP measure given by the supervising teacher was one of the highest. She was very enthusiastic about teaching mathematics.

3. Ian (.95, .20, 2.28, Ex. 6), GRO

My primary school teacher was the headmaster, and a very busy man, he gave us the problems, and then came to check if they were done. We proceeded precisely according to the text of the book. My lower secondary school teacher was more demanding. I think his discipline was too strict, and thus embittered the pupils' attitudes towards math.

Ian	*OA*	*DG*	*RR*	*IP*
1993	4.63	3.75	4.00	2.8
1994	4.88	3.75	4.00	(2.7)
1997	4.13	3.00	3.17	

Table 4. The instructional practices in 1997 and the development of Ian's beliefs.

1) The pupils are concentrated on getting into the matter and the lesson does seem to end all too soon. The pupils will come back later to the themes dealt with, e.g. during the recess.

2) To stimulate the pupils' brains to work on mathematical problems and solve them with good reason and perseveringly, alone and together.

Reflections:

I found the lesson successful when the orientation for the objective was easy to give and to the point. ...I tried to use teacher-centered methods, but they proved really paralyzing as far as I myself and the pupils were concerned. After this, based on my own experience and the feed-back I received, I directed my efforts toward more pupil-centered methods. The independent work of the pupils was amazing in the sense of the intensity with which they concentrated on their work.

Comments:

The class where Ian was teaching was quite a difficult one. Ian clearly had some discipline problems. But he was very enthusiastic about teaching mathematics and always wanted to do his best. I think that the decrease in the OA value depends on these problems. While the later OA value is below the mean and the RR value quite high (over the mean in respect of the second measuring) I conclude that he is still on the level GRO. The IP value given by the supervising teacher is one of the lowest.

4. Laura (.13, -1.5, -.21, Ex. 10), OA

From the primary school I have only good memories. I had several teachers - an old lady, a young female prospective teacher, a preoccupied man - the best memory is that I once was allowed to teach the rest of the class. In the upper secondary school I had a very good teacher. His personality was great, and he used all kinds of experiments.

1) The objectives set for the lesson were attained: the pupils had learned what they were supposed to and they had experienced "the joy of learning" as they had been able to solve problems that were an appropriate challenge for them

Laura	*OA*	*DG*	*RR*	*IP*
1993	4.88	3.25	2.83	3.5
1994	4.63	2.75	2.83	(4.4)
1997	4.50	4.25	3.17	

Table 5. The instructional practices in 1997 and the development of Laura's beliefs.

2) The teacher helps the pupils to realize, to learn and to apply various mathematical matters bringing the subject matter as close to the pupils' own experiences as possible.

Reflections:

I set as my goal that I could incorporate and apply the principles of constructivism along with my teaching. By constructivism I understand that teaching is based on the pupils' earlier knowledge and that it is as close to the pupils' own world of experience as possible. ... I hope to find out what is the relationship between, on the one hand, practicing with the help of games and playing, and, on the other hand, mechanical practicing, in cases where learning is most effective and motivated.

Comments:

Mostly Laura's lessons were very good. Sometimes she had discipline problems. She was teaching a very difficult multi-grade class. Anyway her enthusiasm for teaching mathematics was great and her reflections about her teaching were most profound and through-going. The increase in the RR value can be understood by the problems she had with the class. It also explains the fact that she did not get the highest IP values.

Summary

According to Paul Ernest there are three key elements that influence the practice of mathematics teaching:

- The teachers' mental contents or schemes, particularly the system of beliefs concerning mathematics and its teaching and learning
- the social context of the teaching situation, particularly the constraints and opportunities it provides
- the teacher's level of thought processes and reflection.

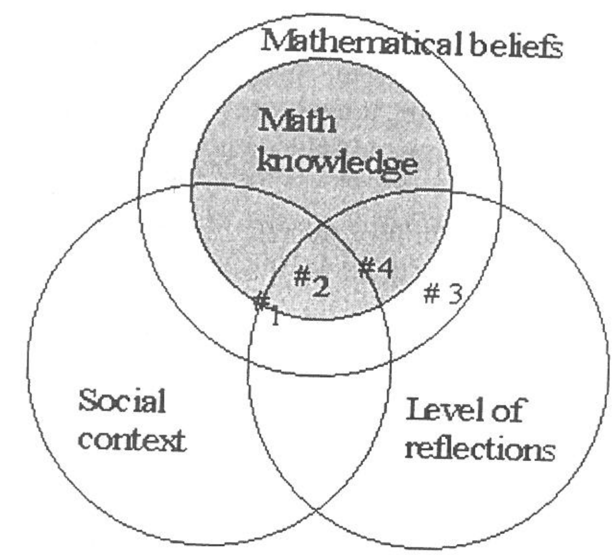

Figure 1. A summary of the instructional practices of the presented cases.

According to him these factors determine the autonomy of the mathematics teacher and the outcome of his or her teaching. (Ernest 1989.) While following the intructional practices of above presented prospective teachers I could mirror the relevance of these Ernest's factors. In order to get a clearer view of the positions concerning the beliefs and instructional practices of the cases I have illustrated the above Ernest's key elements as three partly overlapping circles. As one of the individual's mental schema is his or her knowledge of mathematics, I have depicted it over the circle of mathematical beliefs.

Using the values of the variables OA, DG, RR, IP (Tables 2-5) and the reflections by the prospective teachers I evaluated the location of the cases as shown in the following design. The area where all the circles overlap illustrates the most desirable situation in teaching mathematics. In the drawing the circumferences of the circles illustrate the evaluated mean values of the dimensions. The areas inside the circumferences represent values higher than the means and areas outside the circumferences values less than the means. Of course this is a very imprecise picture of the situation, but I think it helps to outline or assess how different factors have influenced the observed instructional practices.

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Christoph Oster

Problem Solving Situations in Private Lessons

A Phase-Model

Introduction

In addition to my last presentation on the MAVI-workshop 3 (Oster 1996) this study refers to connections between the reasons for being bad at mathematics and the aims of private lessons. This connection must be regarded within the context of the process of problem solving situations, and this is one of the most important aspect in understanding students´ behavior in the situation of being bad.

In the interviews I made the students of the German Gymnasium talk about their experiences, their activities and reasons for these activities. While talking nets of copies of experiences, based on students' mathematical beliefs (see Pehkonen & Törner 1995, p.1) and subjective theories (see Groeben 1988, p. 19), were reconstructed in their minds and reproduced (see von Glasersfeld 1997, p. 162, p. 203, p. 221) in the linear way of talking. This process is reproduced in the mind of the person evaluating these individual stories about being bad and coaching. Students´ aims in taking private lessons, the causes why they needed them and how they used them are important parts of these reconstructions.

An aspect of a case-study analysis: aims in private lessons

The following quotations taken from an interview with a basic-course student of grade 12 are arranged in three different categories. While categories 1 and 2 are directly taken from the interview, the third one represents the researchers point of view and refers to additional aspects to describe causes of increasing problems the student had had before.

Quotations from an interview with a student of a basic course, grade 12

<u>1. Quotations referring to the reasons for ordering private lessons</u>

a) In grade 9 I either did not understand much or I did not work enough. When I got mark 5 and my mother and I did not see any possibility for me to become better, we decided to order private lessons.

b) When I got marks between 4 and 5 in grade 11.2, I at once ordered private lessons, because I did not see any possibility of being successful on my own.

c) Private lessons aimed at getting better marks. - The aims were better marks in written mathematical tests.

The two factors influencing the decision to order private lessons are *helplessness* and *bad marks*.

2. Quotations referring to deficiency in understanding

a) The problems are my gaps, a result from school lessons, where I did not get everything or did not understand much.

b) From the beginning I did not understand differentiation.

c) I needed help at home, because I did not learn successfully at school.

In the student's opinion the causes of bad marks are the result of *unsuccessful learning* and *increasing gaps.*

3. Quotations referring to causes of unsuccessful learning

a) A problem was the exchange of the teacher. The new teacher explained in a different way.
Student´s comments: Teachers´ explanations are often difficult to understand. Explanations can be simpler. Students explain in a more relaxed way and I am able to understand better.

b) I am not attentive all the time, because the group is too big and
- in class it is often loud.
- in class we sit quite close together.
- the teacher is not able to care for everyone.
- The teacher cannot repeat explanations ten times.

c) further aspects:
- I have little mathematical talent, so mathematics has been no fun for me since grade 6.
- I have less practice.
- It is difficult for me to work continuously.

Several kinds of *disturbances in the process of learning* seemed to be the causes of deficiencies in mathematics.

From this point of view there are four individual sub-categories of disturbances experienced during the process of learning:

- linguistic (way of explanation at school)
- communicational (in-class conditions and in-class behavior)
- affective disturbances
- personal deficiencies (poor conditions)

Obviously it is possible to transform the hierarchy of causes - as mentioned above - into an individual model of aims in private lessons divided into 3 levels:

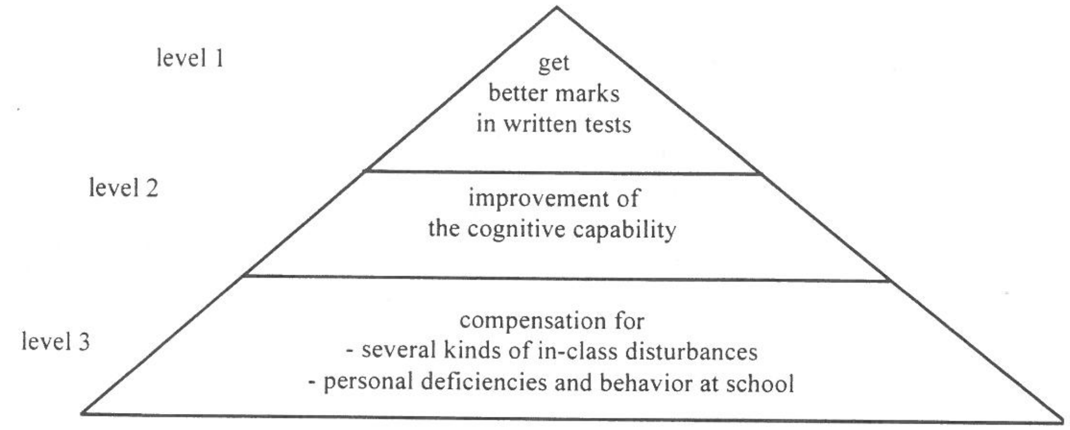

Figure 1: Individual model of aims in private lessons.

A problem solving model[1] to the process coaching

The three phases in the problem-solving process of coaching - see Figure 2 - are connected by the system of aims as mentioned above. While the aims are determined by experiences in phase 1 and the wished conditions of phase 3 (arrows A and B), the concept itself influences students' behavior in phase 2 (arrow C).

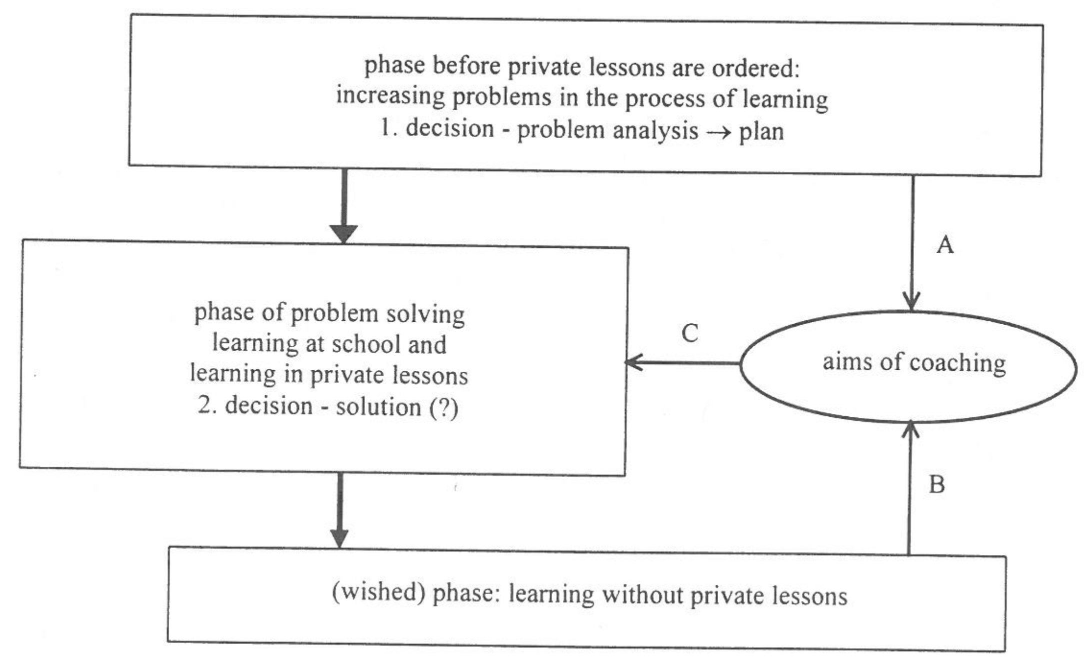

Figure 2: Problem solving model about coaching.

[1] The German version was presented at the 31. Tagung für Didaktik der Mathematik, Leipzig 1997 (to appear).

This model of problem solving is suitable to embed all other case-studies I have just made about bad students' behavior in connection with coaching.

Summary

The student I regarded here had quite a differentiated concept of his situation of learning mathematics and its context. Based on the view of himself the student felt imprisoned in a „chain" (orig.: eine Kette, die alles mit sich führt) that drags everything away with it. Regarding the interview as a whole it is possible to reconstruct this chain step by step:

1. lack of understanding at school
2. problems with homework
3. bad marks in written tests
4. ordering private lessons.

The problem situation is regarded as fateful. The student felt inable to escape from this fate. Moreover, the chain itself is the metaphorical description of his fate and part of his mathematical belief system.

In his problem solving process to become better (see Figure 2) coaching is regarded as the suitable tool to destroy the chain, and this conception is the response to the helplessness experienced in mathematical learning (see quotations above), which is the prevalent reason for the necessity of coaching.

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Erkki Pehkonen

Beliefs of Mathematics Professors on Teaching Mathematics in School

1. Background

In order to understand teachers' performance – also university professors' performance, it is helpful to know their beliefs. An individual's beliefs conduct his performance and filters his observations. Therefore, beliefs are the paramount interesting research object. In order to develop teacher training, one should find out the belief systems of teacher educators.

1.1 Definitions for some concepts

Here, I will use those definitions for the basic concepts which are dealt with in detail in our earlier published papers (Pehkonen 1995, or Pehkonen & Törner 1996): I understand *beliefs* as one's stable subjective knowledge (which also includes his feelings) of a certain object or concern to which tenable grounds may not always be found in objective considerations. Then I will explain *conceptions* as conscious beliefs, i.e. I understand conceptions as a subset of beliefs. An individual's *view of mathematics* is a wide compound of his beliefs and conceptions.

In some cases, persons may not be able to describe their view of mathematics or some parts of it. But they might have some kind of preconceptions or images on the topic under discussion. The object of an image can be a thing or a person or both. For example, Clandinin (1986) defines it, as follows: "Image is a way of organizing and reorganizing past experience, both in reflection and as the image finds expression in practice and as a perspective from which new experience is taken. *Image* is a personal, meta-level, organizing concept in personal practical knowledge in that it embodies a person's experience; finds expression in practice; and is the perspective from which new experience is taken."

2. Realization of research

This is a part of a larger research enterprise "Pupils' mathematical beliefs" supported by the Finnish Academy which enterprise aims to improvement of mathematics teaching in the upper level of the comprehensive school (13–15 years-old pupils). In the research enterprise in question, we will restrict ourselves in improving mathematics teaching within the framework of beliefs. An individual's mathematical beliefs forms a regulating system for his mathematical knowledge structure. In order to be able to improve teaching, it is of paramount importance to inquire information from this regulating system and its function. Only thus we might be able to understand the determinants of mathematics teaching in school.

2.1 Research questions

The purpose of this research is to find out what kind of mathematical beliefs are submitted to teacher students during their university studies at mathematics departments. The teachers of mathematics courses are naturally in the key position. Since it is not possible to investigate mathematics views of them all, we decided to restrict here on leading mathematics professors.

Thus, we wanted to clarify:

What kind of beliefs do mathematics professors have *on mathematics* and its learning and teaching?

What kind of beliefs do mathematics professors have *on teachers' view of mathematics* and its improvement?

What kind of beliefs do mathematics professors have *on student teachers' view of mathematics* and its improvement?

Since this paper is the very first one from the research project, we will take only one research question under considerations. We will restrict ourselves to the research question (B).

2.2 Practical realization

In May 1996 seven mathematics professors from five Finnish universities were interviewed with the method of theme interview. These professors are in their departments responsible for education of mathematics teachers. The mode of interview was the so-called theme interview (e.g. Hirsjärvi & Hurme 1995), in order to get the most free style of narration. When using a narrative style of interview, the outcome of beliefs was more probable.

The length of each interview was about sixty minutes. All interviews were recorded on a video tape. To the interviewed professors, the themes (the main questions) of the interview were told in advance in phone, in order they were able to accommodate themselves to the forth-coming interview, as well as in the beginning of the interview.

For each research question, I have generated three interview themes. For the research question (B), i.e. *teachers' view of mathematics* to which we will here restrict ourselves, the following three themes were attached:

What kind of view on mathematics and its learning and teaching do mathematics teachers have?

What kind of view on mathematics should they have, in order it would be optimal from the view point of school teaching?

How could one promote adapting of such a view of mathematics during university studies?

2.3 Methodology used

In the qualitative research, one possible approach is the so-called naturalistic paradigm (Lincoln & Guba 1985, Guba 1990, Najee-ullah 1991). Here we try to follow the leading idea of an inductive data analysis (Lincoln & Guba 1985) according to which theory comes from the data, the so-called grounded theory. The interview method used, the so-called theme interview belongs to semistructured interviews (Hirsjärvi & Hurme 1995).

The data analysis. The steps of phenomenological analysis of interview data suggested by Hycner (1985) have served as a model for the data analysis: The procedure used was, as follows:

The video tapes were transcripted and copied on paper.

The written interviews were read several times, and such segments in the text were underlined which consisted a clear idea unit answering the research question.

The clear idea units found which are called here research units and which answered the research question were underlined.

These research units with the code in brackets of the person stated it were collected to form a new file,

The research units were printed, cut into pieces and physically classified into groups.

Through ordering the research units within each interview question, the groups for the ideas behind the research units were found. Thus, the "messages" of the interviews emerged.

3. Research results

In Table 1, there is the distribution of the research units according to the interview questions of research question (B).

	teachers' view of mathematics		
interview question	(4)	(5)	(6)
frequency of research units	36	30	22

Table 1. The distribution of research units according to the interview questions (4)–(6) within the research question (B).

Through classifying research units within each interview question I was able to find groups into which the ideas (research units) were clustered. Thus I could reveal some answers to the research question. In the interviews, the real (4) and ideal (5) view of mathematics were tried to differ from each other; and then ask for methods of improvement (6). For the lack of space, I am compelled here to restrict my considerations to the first of the professors' views of mathematics, i.e. to the interview question (4).

3.1 Real view of mathematics

The central ideas (36 research units) of the responses given in the interviews by the mathematics professors to the question "What kind of view on mathematics and its learning and teaching do mathematics teachers have?" might be easily arranged into the five following groups (the response frequency in brackets):

teachers' knowledge is poor (15),
considerations on teachers' personality (9),
teachers' working environment should be improved (6),
professors' task (5),
positive in teachers (1).

In the following, the interviewed professors are numbered from 1 to 7, and there are used abreviations, e.g. P1.

Teachers' knowledge is poor. In these statements, teachers' mathematics view was critized very strongly in six research units. It was stated that school mathematics, and also mathematics teachers are *"rather far on the level of 4000 years old Babylonian mathematics"* (P6), i.e. *"mathematics is [for them] a rather concrete matter"* (P6). The professors emphasized that in schools *"also teaching is some kind of rigid"* (P3), that also in upper secondary schools *"teachers do not seem to have security and courage to shorten the courses"* (P7) and that there are teachers who must *"work hard in order to make those tasks which they give to their pupils"* (P1).

During the interviews, explanations to the mentioned lacks were given in the following four ideas. On the one hand, it was stated that *"teachers' mathematics learning has been stopped or gone backwards"* (P6) and that *"all the time it [learning] will get worser"* (P3). And on the other hand, it was speculated that *"teachers ... will support, as a rule, to that book-based knowledge"* (P4) and that *"of course, [teachers] have not reached that level"* (P3). At the same time, it was emphasized that *"the level of teachers might still be improved ... we should have such teachers for whom mathematics is more natural"* (P1).

Furthermore, the worry for primary teachers' mathematics skills and attitudes were put forward in two research units. These were: *"[teachers] might have at the primary level ... also in knowledge a few lacks"* (P3), and *"at the primary level, there are too many such teachers who do have no interest in mathematics"* (P7).

N.B. These statements were given by five professors.

Considerations on teachers' personality. In this group, there were gathered such thoughts which were not directly connected with teachers' skills, but more with their background. In three ideas, the meaning of a teacher's personality and personal view for his teaching was taken into consideration: *"it depends very much from personality"* (P7), *"if he [a teacher] doesn't have ... a view of mathematics ... his possibilities to survive may be difficult"* (P7) and *"seeing of connections stays mainly on a teacher's responsibility"* (P3). The view of mathematics possessed by upper secondary school teachers was prized in two research units, as *"at the upper secondary school, mathematics is presented more many-sided"* (P3) and *"a upper secondary school teacher is, however, nearer ... good mathematics"* (P1). In addition, it was delightful for school reform that *"in each school, there are also teachers who are ready to improve themselves"* (P4).

N.B. These statements were given by four professors.

Teachers' working environment should be improved. When pondering teachers' view of mathematics, it came also into light a teacher's poor working environment. In these ideas, the hard points of teaching, and especially at the upper level of the comprehensive school (13–15 year-old), were put forward, for example *"the work [teaching at school] is apparently very tedious and unrewarding"* (P1).

The question of working discipline was taken as the next one: *"the share of mathematics teaching have sometimes the second place instead of keeping discipline in class"* (P7) and *"school teaching should be got in some sense clearer job that you are not compelled to be at the same time a policeman and a father and mother and a teacher"* (P1). The other point to be stressed was the heterogeneousness of teaching groups: *"if one has a heterogeneous group ... it is much more complicated to teach"* (P7) and *"pupils in advanced course are also rather heterogeneous"* (P1). The third aspect to be discussed was the hurry in teaching situation, as *"mathematics courses are packed too full"* (P1), and the lack of degrees of freedom, as *"a mathematics teacher is rather much knotted with the concrete schedule ... he has been left very little time to ripe these proper ideas"* (P1).

N.B. These statements were given by two professors.

Professors' task. Some of the thoughts were concentrated around that what at the university should / should not be done. One professor pondered school mathematics and his own relationship to it. We are *"perhaps too self-sure about that this is school mathematics and it has always been taught like that"* (P6). The other one looked for a reason to the situation from the power of analysis: *"a problem in Finland ... is the very strong school of classical mathematical analysis"* (P4). The third professor considered the situation from his own view point: *"at the university ... we should make the teacher program somehow more appealing"* (P1).

N.B. These statements were given by three professors.

4. Discussion

In the interviews, the professors were asked to describe their conceptions on mathematics teachers' view of mathematics. During the interviews, it came clear that some professors admitted that they don't know the school reality, and therefore were more or less explaining their images of teachers' view of mathematics. Especially, their knowledge of comprehensive school teachers situation seemed to be questionable, not to mention primary school teachers.

Summarizing the results into a few statements, we might put forward the following ones: Professors considered teachers' knowledge base as poor and old-fashioned which needs to be improved. As another aspect, they emphasized the meaning of a teacher's personality and his own view of mathematics for his teaching. They blamed teachers' working environment as very poor one, and stressed that it should be improved.

This research work shows very clearly that the professors agree with the situation that teachers' knowledge base is poor. And they have a common idea that a teacher's personality plays a role in teaching. But many of the (interviewed) mathematics professors don't seem to have a good picture about school mathematics and on the teaching conditions at school. For example, professors P3 and P6 blamed very much teachers' poor mathematical knowledge, but during the interviews they didn't refer to the school reality, as e.g. professors P1 and P7. Therefore, it might be that they were stating only their images of school mathematics and mathematics teachers – which they also very openly admitted. A couple of professors (P2 and P5) presented only their ideas about teachers' ideal view of mathematics. Summarizing we might conclude that most of mathematics professors involved in teacher education don't have a clear picture of what is happening in school, and therefore, they don't understand what are the needs for a modern mathematics teacher.

In order to improve mathematics teacher education, it is very clear that one of prerequisites for improvement is to help the mathematics professors involved to develop a proper and truthful picture of the modern school reality and its requirements. Therefore, they should become aware of teachers' requirements which contains i.a. curricular boundary conditions and the nature and learning conditions of our children.

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Martin Risnes

On Self-Efficacy and Mathematical Beliefs

A Structural Modeling Approach

This paper reports an attempt to examine and describe students' beliefs about mathematics at undergraduate level. Our findings indicate interest, anxiety and self-efficacy beliefs as important variables influencing student behavior in mathematics.

Introduction

Mathematics constitute a central part in study programs at university level. It is a common observation that many students find mathematics to be a difficult subject. Students going to a business college, often have a rather weak background in mathematics. In addition many students have in their former mathematics classes, had quite frustrating experiences in learning the subject. In the project we are studying the interrelationships between students prior exposure to mathematics, their beliefs about mathematics and their achievements at college level. This paper will report preliminary results from a study to examine and describe students mathematical belief structures.

Theoretical framework

During the last ten years researchers have stressed the important role of students' beliefs in the teaching and learning of mathematics. This is part of a renewed interest in the affective aspects of mathematical education, as summarized by works in McLeod and Adams (1989), Schoenfeld (1992), Pehkonen and Torner (1996), McLeod (1992). McLeod describes four categories of beliefs studied in the literature, beliefs about mathematics, beliefs about oneself as a learner, beliefs about mathematics teaching and beliefs about the social context. The beliefs students bring into the learning situation, determine both how they approach mathematics and the mathematical skills they are motivated to acquire and use. Carr (1996) brings together researchers from different perspectives to treat the issue of motivation in mathematics. A common theme in the papers in Carr is a need for further studies on the relationship between motivation and metacognition as they affect students mathematical strategy use and achievement.

We will mainly base our presentation on social cognitive theories of motivation like expectancy-value theory and self-efficacy theory, Meece et al (1990), Wigfield (1994), Bandura (1995). In expectancy-value theory the focus is on expectancy judgments concerning anticipated outcomes and consequences of action, treating the perceived value as a strong predictor of achievement. In self-efficacy theory the emphasis is on efficacy judgments concerning ones ability to successfully perform the action, stressing the impact of efficacy beliefs. Following Bandura we could say that how people behave can often be better predicted by their beliefs about their capabilities than by what they are actually capable of accomplishing, for these beliefs determine what individuals do with the knowledge and skills they have.

Purpose of the study

The purpose of this study is to examine and describe student beliefs about oneself as learners of mathematics and beliefs about mathematics. Following social cognitive theories we hypothesize that beliefs related to self-efficacy and self-regulated learning will play a mediating role in the influence of mathematical experiences on motivational beliefs and achievement.

Beliefs about mathematics

Beliefs about mathematics are closely related to motivational issues. Pintrich and De Groot (1990) present a conceptualization of student motivation based on a general expectancy-value model with three motivational components: a) an expectancy component which includes students beliefs about their ability to perform the task, b) a value component which includes students' goals and beliefs about the importance and interest of the task and c) an affective component which includes students' emotional reactions to the task. The value component and the affective component in this model is closely related to achievement value in the expectancy-value model due to Eccles et al, defining the achievement value part as composed of intrinsic value, utility value, attainment value and cost, (Wigfield 1994).

The utility value may be considered analog to the construct perceived usefulness of mathematics as used to study gender differences and math performance. The intrinsic or interest value is related to the enjoyment in performing the activity and the interest in pursuing the task. The cost value may be compared to negative emotional aspects like anxiety relating to engaging in the task. Constructs of anxiety has been widely used in studies of attitudes and beliefs even though its theoretical foundation is rather vague and the results are often somewhat inconclusive. We also include in our analysis a construct relating to the belief that understanding concepts is important in mathematics.

Beliefs about oneself as learner

The other main component of beliefs relates to beliefs about oneself as a learner of mathematics. In the literature a number of reports are studying the relation between achievement and constructs like self-concept or self-confidence. These terms have mostly been used to assess students confidence in their ability in mathematics or their general academic self-concept.

The term self-efficacy is used to describe how a person think he is able to accomplish on a given task , Bandura (1995). The term self-concept may be seen as a global estimate of efficacy, compared to self-efficacy giving a more domain and task-specific measure of efficacy. The relationship between self-concept and self-efficacy and their influence on math performance, have been studied by Meece et al (1990), Pajares and Miller (1994).

The two self-beliefs about mathematics, self-concept and self-efficacy, may be seen as constructs related to how students regulate their achievement outcomes. Relationships between motivational beliefs and self-regulated learning strategies have been studied by Pintrich and De Groot (1990). Based on social cognitive theories, Zimmerman has worked on the self-regulatory processes that govern human development and adaption. We include in our study

Banduras scale of self-efficacy for self regulated learning, adapted from Zimmerman et al (1992).

Methodology

Sample

The target group for this investigation was students starting a study program in economics and business administration at a college in Norway. As part of their program this group has to take a compulsory course in calculus emphasizing applications in economics. The course is taught in a traditional large lecture group. At the start of the fall semester of 1996 the students were asked in an ordinary class session to fill in a self-report questionnaire. Of the 290 students present in class that day, 266 completed the form.

Instruments

The items in the questionnaire were adapted from Schoenfeld (1989), Pintrich and De Groot (1990). Zimmerman et al (1992), Kloosterman and Stage (1992), Wigfield (1994), Skaalvik and Rankin (1995). The items were organized to take advantage from results on different factors used in earlier studies. The students were asked to indicate if they agreed or disagreed to the statements on a four point Likert scale with response alternatives strongly agree, somewhat agree, somewhat disagree, strongly disagree. On the items of self-regulation we used a seven point scale with answers from not well at all to very well.

Data analysis and findings

Variables for beliefs

An exploratory factor analysis based on principal components with varimax orthogonal rotation was conducted to study the main factors describing the structure of students beliefs. Some of the 50 items in the questionnaire were not directly related to constructs used in this paper and were omitted in the analysis. After excluding some ill-fitted items with low factor loadings or loadings on several factors, we identified the following 8 factors based on 25 indicators:

REG1 Self-regulation1. Sample item: "How well can you concentrate on school subjects?" This variable measures part of self-efficacy for self-regulated learning adapted from Zimmerman, Bandura and Martinez-Pons (1992).

REG2 Self-regulation2. Sample item: "How well can you organize your schoolwork?" The construct self-efficacy for self-regulated learning in Zimmerman et al (1992) split in our analysis into two variables.

MOT Self-efficacy of motivation. Sample item: "I'm certain I can understand the ideas taught in this course". This variable includes three items from self-efficacy as part of motivational beliefs in Pintrich and De Groot (1990).

ABIL Ability. Sample item: "I can learn mathematics if I work hard". This variable is analog to 'self-concept of mathematics ability' in Pokay (1996). The items are identical to items in the construct 'self-perceived ability to learn' in Skaalvik and Rankin (1995).

INT Interest. Sample item: "I like mathematics". The variable for mathematics as an interesting and enjoyable subject is analog to 'intrinsic motivation' in Skaalvik and Rankin (1995).

ANX Anxiety . Sample item: "I feel anxious at mathematics tests". Variable in the tradition of studies on mathematics test anxiety.

USE Useful. Sample item: "Mathematics is a worthwhile and useful subject". Variable for mathematics as a useful subject stressing the utility aspect of mathematics as in the Fennema and Sherman Scales. In some studies this variable is related to intrinsic value.

UND Understand. Sample item: "Getting a correct answer is more important than understanding why the answer works". This variable relates to the importance of understanding concepts in mathematics and to give reason for your approach when solving a mathematical problem, Kloosterman and Stage (1992).

Variables for prior exposure to mathematics.

Students prior background in mathematics are measured by year giving the number of years (1,2 or 3) following a mathematics course in upper secondary school and grade giving the grade (2 lowest,3,4,5,6 highest) at the final math exam. This information was provided by the students as part of the questionnaire.

Variable for assessing basic mathematical knowledge.

In the second part of the questionnaire following the section on mathematical beliefs, we presented a test on basic math knowledge from comprehensive school at agelevel 16-17. This test was developed by The Norwegian Mathematical Council in the 1980ties and was in 1991 used in the Nordic countries to assess mathematical skills in freshmen university and college students,(NMR1991). The 38 items are covering the areas: arithmetic T1, everyday life T2, algebra T3, problem solving T4, geometry T5.

Structural Models

To study the relationships between variables, we are applying a structural equation model (SEM) approach with latent variables, commonly known as the "LISREL model". For a general discussion on structural modeling see Bollen (1989) . Structural models related to our models can be found in Stage and Kloosterman (1995) and in Pajares and Miller (1994). We used the implementation LISREL8.14 developed by Joreskog and Sorbom (1996). The results are based on studying the covariance matrix for the indicators involved, treating the indicators as continuos variables. In later studies we will reexamine our models by using polychoric coefficients for studying ordinal variables. For estimation of the models we use estimation by the method of maximum likelihood (ML). Due to missing values the number of cases in this study is 246. We present one measurement model and two causal models.

Model 1 is a congeneric measurement model for the 8 latent variables identified by the previous factor analysis. Table 1 gives the standardized path coefficients for the factor loading on each of the 25 indicators in the model after estimation by ML and 10 iterations.

The hypothesized Model 1 is evaluated by the chi-square goodness-of-fit test of our specified model versus the alternative that the data are from a multivariate normal distribution with unconstrained covariance matrix. Our model has a chi-square of 396 with 247 degrees of freedom compared to the unconstrained model with a chi-square of 3444 and 300 degrees of freedom. The ratio of chi-square to degrees of freedom in model 1 is less than 2 and seems to be acceptable. The Root Mean Square Error of Approximation (RMSEA) measure used to assess the degree of lack of fit of the model is 0.050. Browne and Cudeck (1992) suggest that RMSEA values less than 0.08 indicates an acceptable fit and values less than 0.05 indicates a good fit. The root mean square residual (RMR) of 0.051 is also indicative of a good representation of the data, even though the index AGFI=0.86 is below the recommended level of 0.90. Based on these indicators we conclude that our measurement model gives an acceptable fit to the data.

REG1	0.81 (14.09), 0.82 (14.21), 0.72 (12.10)
REG2	0.89 (15.85), 0.94 (16.96)
MOT	0.78 (14.07), 0.84 (15.45), 0.81 (14.65), 0.76 (13.49)
ABIL	0.70 (11.45), 0.79 (13.17), 0.67 (10.83)
INT	0.87 (16.83), 0.75 (13.41), 0.86 (16.52), 0.93 (18.82), 0.70 (12.29)
ANX	0.79 (12.29), 0.74 (11.96), 0.70 (11.29)
USE	0.57 (7.35), 0.95 (9.84)
UND	0.64 (10.30), 0.85 (13.93), 0.79 (12.84)

The number of coefficients give the number of indicators for each variable.

Table 1 Standardized factor loading for each of the latent variables with (t-value) in Model 1.

A closer study of modification indices for model 1, indicate that it would be possible to get a better fit by letting some of the indicators load on two different latent variables. Given the reasonable low value of RMSEA, we prefer to keep the simple measurement model with pure components. An alternative version of Model 1 treating REG1 and REG2 as one construct for self-efficacy of self regulated learning like in Zimmerman et al (1992), gives a poorer fit and this model is rejected. The correlations among the latent variables are presented in table 2.

	REG1	*REG2*	*MOT*	*ABIL*	*INT*	*ANX*	*USE*
REG2	0.61						
MOT	-0.12	-0.13					
ABIL	-0.03	-0.00	0.67 *				
INT	-0.18 *	-0.05	0.64 *	0.55 *			
ANX	-0.20 *	-0.18 *	-0.39 *	-0.52 *	-0.34 *		
USE	0.02	0.06	0.31 *	0.37 *	0.45 *	-0.11	
UND	0.04	-0.02	-0.19 *	-0.26 *	-0.29 *	0.07	-0.19

* t-value> 2

Table 2 Estimated correlation coefficients between latent variables in Model 1.

Model 2 To study the relation of the beliefs variables and the test score, we introduce a structural model with the 8 latent variables from the measurement model 1 as independent variables influencing the dependent latent variable TEST with the 5 indicators T1-T5. This model may be considered as a multivariate regression model taking advantage of the SEM capabilities of handling latent variables with several indicators and measurement errors. A modified model obtained by fixing a zero path coefficients from REG2 to TEST and from UND to TEST, gives a chi-square of 562 with 371 degrees of freedom and RMSEA=0.046. Based on the chi-square difference test we conclude that this modified Model 2 is a preferable model. This model gives a squared multiple correlation of 0.34 (34% explained varians), in the TEST variable.

Model 3 To study how the variables in Model 2 relates to students prior exposure in mathematics, we can introduce a model giving causal paths from the independent latent variable SCHOOL with indicators year and grade to the dependent latent variables for beliefs and allowing for paths from the beliefs variables to the TEST variable. Based on the findings in Model 1 and Model 2, we decided to exclude the latent variables REG2 and UND from this analysis. Following self-efficacy theory, we estimated different models by introducing paths from the self-efficacy beliefs REG1, ABIL and MOT to the beliefs ANX, INT and USE. By making use of the modification index strategy in LISREL, we present a modified model Model 3 with structural part as shown in figure 3. The variable SCHOOL has nonzero paths to MOT, ABIL and INT and USE. To test for the mediating role of the beliefs variables, we decided to not include a path from SCHOOL to TEST. In this model we have included a correlation between the error terms for the test indicators T1 and T2, as both are measuring number skills.

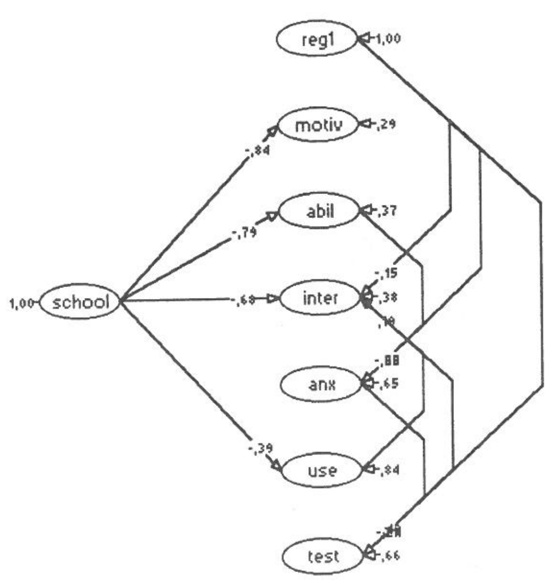

Figure 3. Paths between latent variables in Model 3

Estimation of model 3 took 20 iterations. Goodness of fit statistics give chi-square of 490 with 312 degrees of freedom, RMSEA=0.048, RMR=0.064 ,AGFI=0.85. We conclude that Model 3 gives a good fit to our data.

The structural part of Model 3 is shown in figure 3. The path coefficients from SCHOOL to MOT (-0.84), ABIL (-0.79) and INT(-0.68) are high. This is indicating that students with more years of math and higher grades, have more favorable beliefs about mathematics. For the paths between the belief variables, we notice that the variable REG1 has paths to INT(-0.15) and ANX (-0.22), ABIL has a path to ANX(-0.55) and USE has a path to INT(0.19).

The model includes paths to TEST from REG1(0.21), INT(0.43) and ANX(-0.24). All the paths have a t-value >2. The signs of the path coefficients are as would be expected by motivational theories. The squared multiple correlation for TEST is 0.34, for MOT, ABIL, INT and ANX the numbers are 0.71, 0.63, 0.62, 0.35 respectively. The estimated correlations between the latent variables are given in Table 4.

	REG1	*MOTIV*	*ABIL*	*INT*	*ANX*	*USE*	*TEST*
MOTIV	--						
ABIL	--	0.67					
INT	-0.15	0.64	0.60				
ANX	-0.22	-0.37	-0.55	-0.30			
USE	--	0.33	0.31	0.46	-0.17		
TEST	0.20	0.36	0.39	0.47	-0.41	0.24	
SCHOOL	--	-0.84	-0.79	-0.75	0.44	-0.39	-0.43

Table 4. Estimated correlation coefficients between latent variables in Model 3.

Model 3 could be compared to a regression model with simultanous regression of all the dependant beliefs variables and TEST on the independant variable SCHOOL. This regression model has a chi-square of 545 with 317 degrees of freedom and RMSEA=0.054. In this model the squared multiple correlation for TEST is 0.32 as compared to 0.34 in model 3.

Discussion

The results from the confirmatory factor analysis in Model 1 indicate that our 8 constructs may be seen to give an adequate description of students beliefs variables. The signs of the correlations are as would be expected. The correlationvalues in our study are somewhat higher than often reported in the literature, Malmiviuori and Pehkonen (1996). The variable MOT for self-efficacy of motivation from Pintrich and de Groot (1990), has a nonsignificant correlation with the measure REG for self-efficacy for selfregulated learning in Zimmerman et al (1992), and we can treat these variables to be rather independents traits of self-efficacy. In this study the indicators for self-efficacy for self-regulated learning split into two separate components with a nonsignificant correlation. The variable ABIL for ability (self-concept) correlates strongly (0.67) with MOT self-efficacy of motivation. The variable INT for interest correlates strongly positive with ability and self-efficacy for motivation, somewhat lower for useful and negatively to anxiety. The variable for anxiety correlate negatively with ability and self-efficacy of motivation. The variable for understand correlates only modestly with the other variables. Based on our findings the variable understand seems to be a rather poor construct of little significance in our study. This gives support to Kloosterman (1991) who didn't find any important relations between understand an other variables.

Model 2 indicates that our measures of students beliefs are related to the performance on the test, explaining 34% of the varians in test score. The findings in model 3 illustrate the mediating role of the self-efficacy beliefs on the motivational beliefs and on the test score. Our study indicate self-regulation, ability, interest and anxiety as particularly important belief variables influencing student learning in mathematics.

The study used an achievement variable testing on math skills from comprehensive school. In later studies we will include performance indicators relating to achievement at college level. We also plan to look closer into the question of gender differences.

We have in this paper studied some structural models giving paths between beliefs variables. It would however be premature to draw any conclusions about causal relations between our variables. It is possible to present a number of structural models all giving good statistical fit to the data. Inferences about causal relations can only be justified based on a strong theoretical basis. It seems important to continue the work to further develop the theoretical basis for the study of mathematical belief structures.

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Günter Törner

Methodological Considerations on Belief Research and Some Observations

1. Introduction

It is the purpose of this paper to present a short discussion on a few methodological concepts on belief research. We would especially like to point out the meaning of metaphors as linguistic variables.

Beliefs, belief systems and mathematical world views respectively will be understood as attitude structures (see [17]). In this case of identifying beliefs one could actually fall back on the corresponding methods used in psychology. The analogue transferral of statements from the research into attitudes establishes itself only as being conditionally suitable (see the discussion in [9]), because teaching mathematics must be understood as an integral, multi-layered process. In this respect one will want to develop specific methodologies in researching mathematical beliefs.

A widely accepted definition states that, attitudes consist of coherent, cognitive, emotional and action-relevant components. So far, the question arises to which extent the methodological concepts, namely identifying beliefs, carry the responsibility for the parallelism among structures. If one transfers the elements from BLANEY's theories (see [6]) onto the research into beliefs, it must be assumed, that the cognitive net of mathematical contents overlaps the emotional net. We can also assume that single, mathematical objects, i.e. the theorems, the methods, the formulas, the terms etc. have an emotional loading. The spectrum of it all reaches past the every-day observances from the negative estimations through the indifferent ones right up to the positive estimations

2. Evaluational approaches

The reform of teaching mathematics during the 1960s assumed unexplicitly a monolithic view of mathematics which was, however, internally specified in the sense of BOURBAKI's axiomatics. The disallusioning questions along with the reform among others have made clear that one can not be fair with this image of mathematics in relevance to the school classroom. It is surprising, how qualifying changes in the estimations of the philosophy of mathematics coincide with the observation that individual views of mathematics must be assigned to a psychologically greater meaning in the learning and teaching processes. With this idea the research into beliefs and world views, respectfully, won tremendous notoriety.

In the meantime the tool used by DIONNE to evaluate perceptions ([5]) is recognized as classical, namely to coordinatize belief structures by vectorial distributions of weights with respect to some basis components. It was the idea of DIONNE that beliefs constitute a mixture of (basic) components.

For his research, DIONNE [5] used the following three perspectives of mathematics:

(A) Mathematics is seen as a set of skills (*traditional* perspective): Doing mathematics is doing calculations, using rules, using procedures and using formulas.

(B) Mathematics is seen as logic and rigor (*formalist* perspective): Doing mathematics is writing rigorous proofs, using a precise and rigorous language and using unifying concepts.

(C) Mathematics is seen as a constructive process (*constructivist* perspective): Doing mathematics is developing thinking processes, building rules and formulas from experiences on reality and finding relations between different notions.

In his book, ERNEST [7] describes three views of mathematics: instrumentist, platonist and problem solving. These correspond more or less to the three perspectives of Dionne (1984) mentioned above. Furthermore, the same three-component model was rediscovered ten years later by TÖRNER / GRIGUTSCH [27] calling (A) the *toolbox aspect*, (B) the *system aspect* and (C) the *process aspect*. In order to determine a person's mathematical beliefs, this person must be asked to distribute a total of 30 points to these components.

The methodical problems appearing as a result are three-fold in nature. Firstly, the problem is the prior identification of the assumed dimensions. Regarding the above mentioned basis categories it can be accepted that these factorally analytical examinations have pointed themselves out to be quite stable although they seem to be in need of completion. Recent research shows that there are at least four basic components in an individual's view of mathematics: the fourth component seems to be application (GRIGUTSCH / RAATZ / TÖRNER 1997).

Secondly, even more difficult is ascertaining the vectoral data from the test persons in question. Almost no considerations of plausibility are available to the test subjects. Therefore, his appraisals should not be over interpreted.

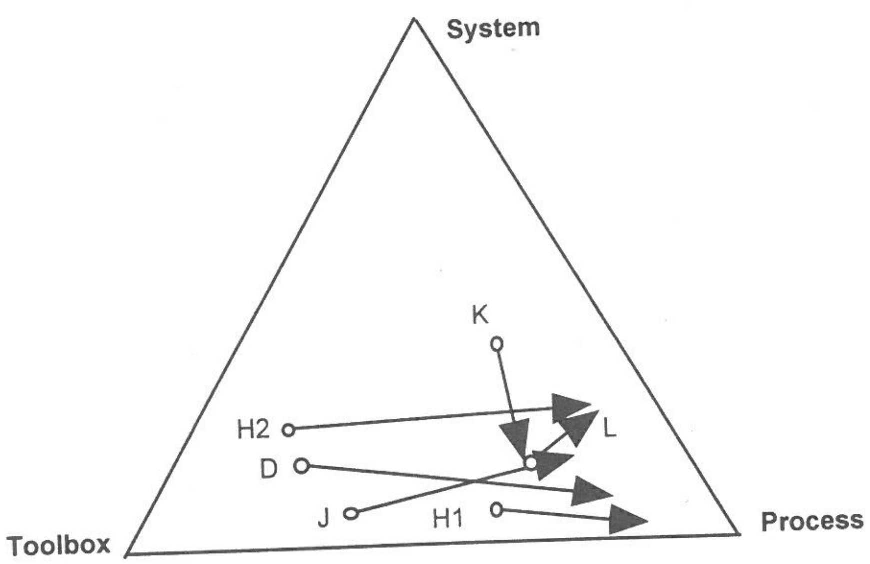

Figure 1: The self-estimation of six teachers in the aspects of toolbox, process and system in the view of real and ideal teaching of mathematics, respectfully (the arrow shows the tendency of change from the real to the ideal teaching situation).

Finally the basic question is presented to which extent the ascertained point distribution can justify the statement: the theoretical construction of 'beliefs.' In other words, the problem presents itself to which extent to the cognitive, emotional and action-related components can be understood with a numerical list.

In each case the Dionnian statement must be classified as coarse. It should however be noted as positive that this entirely questionable method is accessible only on the elementary level. It proves itself in practice to be robust and convincingly provides the proof that there are various, subjective estimations of the raised attitude objects.

PEHKONEN / TÖRNER [18] have recently refined the basic DIONNE fundamentals into a graphic illustration of components which is more than just a complementary for of presentation. Test persons were asked to graphically mark their self-estimations on an equilateral triangle in regard to the practical and the ideal teaching situation of mathematics. We are referring to the work appearing subsequently. The result is that such a graphical evaluation of data is not redundant across a clear listing of the data, but rather produces a complementary character throughout. The following diagram from [18] contains the self-estimation value of the test persons through which we, then, clarified the tendencies of change (real to ideal teaching situation) by means of vectors.

This method allows in principle only two free variables. In practice one can assume that three variables in their relationship to one another can be described, and a normalization can be assumed.

These complementary statements are to be recognized as advantages:

- the method combines the advantages of DIONNE's approach
- it serves as an additional data source
- vectorial data are represented graphically
- it assists the interviewed person to generate data
- different data can be visualized in one figure
- tendencies can be made apparent

3. Procedure for factor analysis

The prevailing number of quantitative works under consideration about the beliefs of teachers and students (compare with the literature [28]) serves as methods by which certain factors are analyzed. Beside the fundamental problem of quantitative procedures the statements from the questionnaires must be kept out of our sight in this context and classified. The expense on the part of the test persons is not insignificant, especially if going on the assumption that the simultaneous reaction to analogue, linguistic charms is necessary for considering an attitude. To this extent each component must be represented repeatedly in the questionnaires. As a result, the number of the testable items is inevitably limited. Because the discussion about the consistency-theorem has not found a satisfying conclusion until now, the consistency between the cognitive and emotional net must be postulated *a priori*.

In contrast to statements up until now GRIGUTSCH, RAATZ and TÖRNER have (compare also with [29] and the dissertation by GRIGUTSCH [11]) constructed graphs on the basis of the partial correlations. The vertices are the factors whereas the edges of the graph are defined through the significant correlations. It is obvious that these structures have to be interpreted. The produced structure will be labelled by us as a *factor analytical model of a belief system*. This net structure clarifies especially the relative stability of mathematical views of the world. From [10] will quote the following diagram which touches on a partial correlation (n = 253) of a factor analysis with the four factors: formalism (F), application (A), process (P) and system (S).

It must, once again, be established that this methodological statement doesn't provide an explicit differentiation between the cognitive aspects and the emotional loadings. Formalism is, for example, for the individual in question neither *a priori* positively nor negatively loaded.

	F	A	P	S
F	1,000	,042	-,127 *	,364 ***
A	,042	1,000	,127 *	,087
P	-,127 *	,127 *	1,000	-,146 *
S	,364 ***	,087	-,146 *	1,000

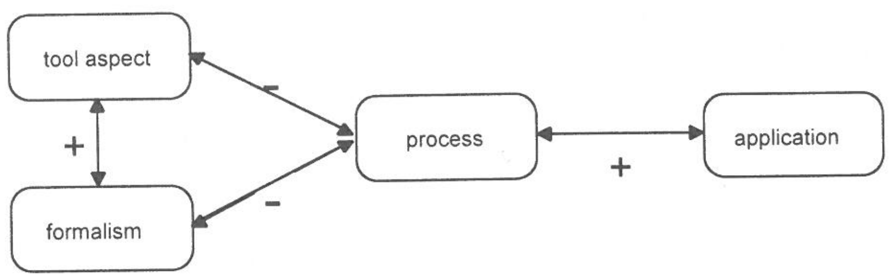

Figure 2: Intercorrelative relationships between the scales.

The method portrayed in Section 3 is trivially statistical in nature and can state nothing specific in regard to the individual case. It only makes clear that belief structures are complex and must be classified as stable. It does not make any reference, however, to how these structures could possibly be changed. Thus, it is quite natural to ask for more individual, quantitative indicator.

4. Beliefs as linguistic variables

4.1 The role of metaphors

According to MAIER [15, p. 119] language (also in the mathematics education) is the most important medium which is readily available to the teacher. Language accompanies and even initiates each constructivistic learning process. Also, the choice of language medium in the classroom is of extreme importance as in almost no other field regarding technical language, idioms and the actual language competence of the students.

In addition, the translation process, the process of modelling using mathematics, is too a significant theme of the lesson. The process of translation (everyday language to mathematical terminology) gains further degrees of freedom as well increases in complexity if internal language levels attracts some attention through the use of figures of speech.

It is primarily through the work of LAKOFF [12] (see also [1] and [15]) to bring attention to the meaning of figures of speech in teaching and learning processes, in particular in mathematics. These figures of speech are not simply a beautified decoration in the process of communication but rather specific elements which are brought about, formed and presumably initiated.

It was BAUERSFELD and ZAWADOWSKI [1] who, already in 1981, emphasized the meaning of metaphors for the classroom:

> We want to mention a further figure of speech. ... Metaphors, quite in contrast to this, are nearly always formed deliberately and with intention and sometimes with much intellectual effort. In order to the use of metonymies we produce metaphors in cases when we want to evoke a certain understanding, we want accentuate an aspect, or to lay emphasis on certaon properties, and yet we are in lack of common words for it with an established meaning.
>
> In this sense a metaphor is like the square root of 2 expressed within the rationals. Such situations occur very often not only in living speech but also in mathematics teaching, where they seem to build the rule rather than rare exceptions.

In this regard we refer to the chapter entitled 'Mathematik und Rhetorik' in [3] in which the meaning of rhetorical elements by means of mathematics is enlightened.

(1) Some cognition scientists see a close connection between the learning and the use of metaphors in the following way:

> The metaphor is one of the central tools in overcoming the epistemological gap that exists between the old and the new knowledge. (Petri)... Simplifying thinks a little bit, one may say that, in the first case, the metaphor serves understanding, and, in the second, that it serves explanation. (compare wih [19], pp. 92)

(2) Memory psychologists favour the coding of stored information using pictures. Here the metaphors play the role of basis macros:

> The role of metaphor for organizing and communicating thoughts about one's personal reality is central to a constructivist's approach to language which views individual constructions of personalized realities as limited by individual knowledge and language. (cf. [8]. p. 104)

(3) For constructivists metaphors are powerful aids:

> We must give students "tools to think with" - and these are not merely formulas and algorithms. They include concepts and powerful metaphors... What kinds of experience does school need to provide to children? ... we might list four ... (2) deliberately created "assimilation paradigms" - that is to say, carefully designed metaphors that correctly mirror the structural features of various pieces of mathematics, and which therefore give the student a basis for powerful mental representations... (cf. [4], pp. 188)

4.2 Metaphors and beliefs

It is probably more than just coincidence that the platonian view of mathematics, hence the classical prototype of a mathematical world view, was clarified by PLATO using metaphors as a medium:

> ... hunters and fishermen hand over their catch to the cooks... but the surveyor, arithmetician and the astronomer are also hunters, because they do not create their figures and rows of numbers, rather these already exist and one only comes across them as they already are and hand them over to the dialecticians (transitional dialogue EUTHYDEM 290 B.C.).

It was TOBIN especially[17a] who brought attention to the role of metaphors by identifying mathematical beliefs and who was supported by his empirical results.

But why can we expect that metaphors illuminate mathematical beliefs? In order to understand teaching and learning processes in mathematics, an analysis of the different underlying relations is specifically required. There is the teacher; there is the student and his class, and their social interrelations. Then, we have the fixed curriculum on the other side the mathematical content. In short, there are many highly woven complex relationships among different subjects and objects which have to be modelled and partly personally evaluated.

Metaphors are of relational character.

In a sense, teaching and learning within a class is like playing ball. There is the ball (= material), there are players and so on. It is obvious that illuminating metaphors produce a corresponding, relational structure. We can also speak of a 'salesman' and 'clients' and the 'ball' is the product which has to be sold or bought respectively.

LEINO, A.-L / DRAKENBERG, M. [15] approach this theme from the other side and name as the core of suited metaphors the categories of schools, up-bringing and curriculum. Previous research (TOBIN [20]; TOBIN / LAMASTER [25]) suggested metaphors for teaching typically describe three distinct roles of teachers: teaching, assessing, and classroom management. It has the appearance that the relevant, fundamental categories, which have the character role of the teacher, have not yet been definitively nor thoroughly discussed.

Starting points suited to particular professions seem to be metaphors in that they concern the interaction between student, teacher and material because these produce similiar relations in regard to the clients (customers) of the time and the products in question. Certain occupations produce moreover the advantage that their characteristics can be knowingly assumed as being sufficient.

Without continuing the discussion , we will mention at this time some professions which are named in the literature in the context of mathematics lesson: gardener, gas station attendant, construction worker, guide (BERRY / SAHLBERG [2]); intimidator (TOBIN / GALLAGHER [22]); preacher (TOBIN / ESPINET [19]); policeman, mother hen?, entertainer (TOBIN / JAKUBOWSKI [23]); captain (TOBIN / KAHLE / FRASER [24]); saintly facilitator, manager, assessor, comedian (TOBIN / ULERICK [26]); comedian, miser, social director, researcher, mentor, coach, city-planner, telephone-operator, mother, magician. (FLEENER ET AL. [8]).

Already at first glance this list makes clear that these professions display typical characteristics of certain attitudes and are always in a context associated with feelings for the world as if they played particular roles. Also the dual relationship between teacher and student is modelled as typical of the profession. To a certain extent the relative diversity of the belief constructions carries the responsibility.

In this lies the second advantage for the use of metaphors:

Figures of speech, e.g. the metaphors, carry subjective emotional loadings.

Our basic concern is, as mentioned above, the description of beliefs by the utilization of adequate metaphors whereby the aspect of the roles and the material must carry the main responsibility.

4.3 Metaphors as methodical tools in the research of beliefs

In addition to the fundamental roles described above in context with beliefs figures of speech can be seen as a methodological tool. The following categories appear to be meaningful, which are already mentioned in various papers.

(a) Metaphors help to make abstract belief constructs concrete.

Refering to well-known relational figures, it is possible to generate belief constructs:

(b) Metaphors help to conceptualize beliefs.

Naturally the test person does not want to inevitably be identified with an analoging metaphor situation. This separation from a declination up until an agreement must be valued as a terrible process of self-reflection which can be described as a conceptualization of beliefs.

(c) Metaphors help to group beliefs as belief systems.

Because the figures of speech in question display, on the most part, many relative "lefts" and are associated also with emotional components (e.g. orders from a captain) not only are the single beliefs addressed but entire belief systems are grouped.

(d) Through metaphors beliefs become conscious and are open for reflections.

Just as it was addressed in (b), the self-reflection leads to an awareness of possible unconsciously functioning beliefs.

(e) Metaphors help to verbalize and represent beliefs.

A critical consideration of related figures of speech support a verbalization of beliefs. At the same time a representation independent of language is made possible as the paper by BERRY / SAHLBERG [2] shows. Here, metaphors are being represented through comics.

(f) Through metaphors comparisons of beliefs become possible.

On the same note, the paper of BERRY / SAHLBERG [2] shows that beliefs represented by metaphors are accessible to an international comparison independent of language. A recent paper of LERMAN [15] shows that

(g) Through metaphors different psychological theories in mathematics education can be made apparent.

Admittedly the statements sketched here nothing more than plausible and conclusive hypotheses of this paper which allow themselves to be justified by their subsequent productivity. In this context LAKOFF's work (see [13]) raised the question: *Invariance Hypothesis: Do Metaphors Preserve Cognitive Topology?* we may ask the following questions:

- Do metaphors preserve affective contexts of beliefs?
- Do metaphors preserve behavorial aspects of beliefs?
- Do metaphors preserve cognitive structures of beliefs?

Here, certainly many questions still remain unanswered.

4.4 Metaphors as Methodical Tools in the Influence of Beliefs

At this point it should be briefly mentioned that Tobin stresses the fact that the reflecting of the teacher's behavior on the basis of metaphors can, as a master switch, ultimately lead to a change in the teacher's own individual behavior.

4.5 Using metaphors as linguistic variable to measure beliefs

It has become clear that metaphors can be understood as linguistic variables. A similar statement is known as the Fuzzy Control Theory. In this case linguistic variables model physical conditions, whereby the coordinates are not of acute but rather linguistical value, which are coupled over the function of membership (value distribution between 0 and 1) with the acute values. The linguistical variable "age" can be understood and compared with the conditions "young", "middle-aged", "old" and "very old", whereby it requires a subjective interpretation in order to characterize, for example, a person 50 years of age as being 80% "middle-aged" and 15% "old". The value of the function of membership for "young" can be shown as 0% for a person 80 years of age.

We will apply this statement by concerning ourselves with the linguistic variable "distance" in a teacher/student relationship. Apparently this proximity is within the parameters of the teaching/learning process and overpowers varying estimations. This lingustic variable "proximity" is coordinatize by the metaphors "entertainer" and "captain" which play the role of functions of membership. The question about beliefs on teaching mathematics would at the same time be seen as the question, to which extent does the teacher play the role more or less of the "entertainer" or the "captain". Here the person in question can graphically integrate him/herself. A corresponding picture reveals this point as the following:

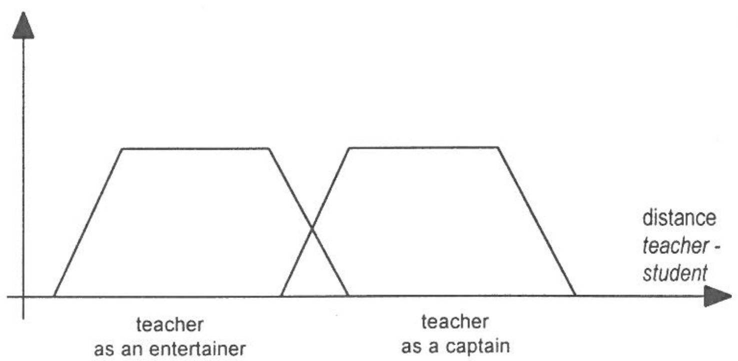

Figure 3: Membership functions of metaphors with respect to the linguistic variable 'distance'.

Apparently more variables play a role in the teaching / learning process; therefore, not only the management and entertainment aspects, respectfully, but also the aspects of training, of being an expert, etc. play roles as well. On the other hand, these linguistic variables are roughly typified by the use metaphors which then take on the role of function of membership.

5. Summary

On this note, the statements of this paper should be portrayed as an overview of an understanding of belief structures. In addition to the classical, quantitative analysis, the qualitative and apparently also the linguistic statements seem to be promising. Not much can be addressed in further detail at this point concerning the application of the teacher interviews

which were conducted, but these connections will be addressed and presented in a further detailed paper.

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Bernd Zimmermann

On a Study of Teacher Conceptions of Mathematics Instruction and Some Relations to TIMSS

Questionnaires on mathematical beliefs were administered to teachers of mathematics and students – grade 7 mainly – from different types of school in the area of Hamburg in 1989. Statements of 107 teachers and 2658 students could be analyzed. There will be a report on some of the findings of the teacher study which will be related to some results from the Third International Mathematics and Science Study (TIMSS).

1. Some reasons for and goals of studying teachers conception of mathematics

Empirical reasons: Some time ago several trends in mathematics education were found by analyzing the literature.[1] It should be checked empirically, to what extent such theoretical trends of mathematics education could also be determined in the conceptions of teachers of mathematics. This was the startingpoint of our study.

Pragmatic reasons: One has to determine, *understand and respect the "mental home"* of teachers before reaching out for change - normally understood as "improvement" - and new common goals. Knowledge of *individual differences and different profiles of beliefs* might be especially useful to find individualized approaches for further developments. In this respect comparative studies are also of great value (as carried out and proved, e. g., by E. Pehkonen in cooperation with colleagues from many other countries).

Results of such research might help teachers to become aware of and reflect on their beliefs about learning and teaching of mathematics. Such metacognitive activities might reinforce the striving for change and improvement.[2]

Theoretical reasons: Conceptions of mathematics instruction might guide problem solving processes on *three levels*:

- the mathematical problem solving processes of the *student*,
- the teaching processes of the *teachers* and
- the research processes of the *math's educators*.

These activities as well as the guiding conceptions are mutual related, whether they are conscious or not. They are to be considered as important parts in the construction process of an encompassing theory of mathematics education (cf. our startingpoint, mentioned above).

[1] Cf. ZIMMERMANN 1979, 1981, 1983, 1987.
[2] Cf., e. g., PEHKONEN 1994.

2. Design and methods of the study

Our questionnaire for teachers was developed by

- using the trendanalysis in mathematics education, which had been mentioned above,
- using some results from beliefs-research[1],
- comprehensive discussions with colleagues from schools and universities.

The questionnaire had three parts:

The first one was a technical part, including questions to get specific data from the teacher (like age, teaching-subjects, gender, teaching experience, type and size of school etc.).

The second part included 55 statements about mathematics as well as about its teaching and learning, which the teachers were asked to respond on a 5-step rating scale from 1 ("I agree completely") to 5 ("I disagree completely"). The teachers were asked to give their responds by assuming the ideal situation that they can organize their lessons completely in a way they want.

The third part included some questions about their personal priorities for teaching and possible obstacles to teach in a way they want. Furthermore, the teachers had the opportunity to present some wishes.

We refer here to the second part of the *teacher questionnaire* only, which was structured as follows (some key words are written in bold type to which we will refer later when presenting some results):

1. One should emphasize **formal structures**. *Examples*: Sets, groups, fields, functions as sets of ordered pairs etc.
2. **Intuition** in mathematics should be stressed.
3. The teacher should **break up** the teaching matter **into small parts**, so that the students can comprehend better.
   Example: $(a+b)^2=(a+b)(a+b)=(a+b)a+(a+b)b=a^2+ba+ab+b^2=a^2+2ab+b^2$.
4. Correct use of **precise language** should be emphasized. *Examples*: to discriminate consequently between angel and size of an angle, area and size of an area.
5. Different areas of mathematics should be **connected** and relations to other disciplines should be drawn as often as possible.
6. There should be more time for **practicing** than for all other activities in the mathematics classes.
7. Mathematics should be taught as an **open system**, which develops by conjectures and mistakes in different directions.
8. The teachers should give the opportunity to the students to cope with and make precise **proofs** as often as possible.
9. **Project oriented instruction** should be carried out as often as possible.
   Examples: setting up of an aquarium, preparation and organization of an excursion or a students journey, elections, speedlimit.
10. One should prove the following theorems:
    a. **Pythagoras**
    b. side splitter theorems (**similarity** ; "Strahlensätze")
    c. **irrationality** of $\sqrt{2}$
    d. $(a+b)(a-b)=a^2 - b^2$ (***binom. form.***)

[1] Cf., e. g., THOMPSON 1985, FRANK 1985, 1988.

    e. Prove: If a line meets two parallel lines and is perpendicular to the first one than it is also perpendicular to the second one. (***geom. proof***).
11. One should place emphasis on **playing** with mathematics (e. g., by playing mathematical games).
12. Mathematical procedures should be treated in connection with their reverse procedures (***operatoric principal***; *cf. Piaget*). *Examples*: Addition and subtraction, multiplication and division as well as composing and dissecting geometrical figures should be treated in connection.
13. Students should work as often as reasonable with **manipulatives** (e. g. models made or to be made in wrapper).
14. The use of **mathematical symbols** should be practiced intensively.
15. **Social learning** (e.g. helping one another) should be emphasized.
16. **Systematics** is especially important in instruction.
17. Teacher should care for a **tight instruction**.
18. Whenever possible, the teacher should **visualize** his teaching matter. *Example*: Visualizing $(a+b)^2 = a^2 + 2ab + b^2$ by the areas of squares and rectangles.
19. The teacher has to care for **discipline** in his class.
20. One should make clear, that truth, precision and **rigor** are properties of mathematical statements, which can always be verified objectively.
21. The teacher should care for the **state of development** of the students and respond to their specific needs mainly.
22. There should be main emphasis on learning (may be by heart) **basic techniques** (e. g. computational methods, to handle formulae).
23. The **esthetic aspect** of mathematics should be stressed.
24. Examples from **history of mathematics** should be treated as often as possible.
25. One should teach mathematics as a mainly **closed** and **clear system** of definitions, theorems and procedures.
26. The teacher should strive for and guide an **intensive discussion** in his classes.
27. One should especially care for **lower achievers**.
28. One should concentrate on **solving routine tasks** of such type, that the correct application of a well known scheme leads to a result safely.
29. **Applications** should be treated mainly.
30. Practice in formalizing and **abstracting** should be emphasized.
31. Working **logically** should be emphasized.
32. **Group work** should be initiated as often as possible.
33. It should be stressed that mathematics is a **useful tool** (e. g. in many professions).
34. The teacher should **explain** as often as possible the subject matter to the students.
35. As often as possible students should make the experience that for many mathematical tasks there are many **different ways** to come to the same result.
36. One should prefer to cover **less content more carefully** than more themes more on a surface level (as far as it is permitted by the school board).
37. The students should have the opportunity to pose and pursue their **own tasks** and **questions** as often as possible.
38. It should be made clear that **mathematics** is **not perfect** either and that there is no absolute security and truth even here.
39. When **grading** one should concentrate on the **results of the tests**.
40. When **grading tests** one should concentrate on the **solution paths**.
41. When **grading tests** one should concentrate on the final **results**.

42. When **grading oral and other contributions** of the student one should concentrate on the **correct or wrong answer**.
43. When **grading** one should concentrate on **oral and other contributions** of the students but tests.
44. When **grading oral and other contributions** of the student one should concentrate on possible **solution ideas** to given tasks as well as contribution to further questions.
45. One has to care especially for the needs of mathematically **gifted students**.
46. A teacher should take care that **one student could explain** as often as possible **to other students** mathematical questions and difficulties.
47. As often as possible students should cope with **problems** where they have to think and knowledge of mere routine techniques does not suffice to come to solutions.
48. One should think more about the possibility to use **computers** in mathematics classes.
49. It should be clarified, that **mathematics**, too, is a domain which had been **created by men**.
50. One should support **single working**.
51. **Carefully selected examples** should be in the center of mathematics instruction.
52. Many students try to generate the illusion of cooperation and understanding by fitting to the terminology of their teachers. Such superficial **"language games"** should be discussed explicitly in the class.
53. One should make clear that mathematics is an important part of our **culture**.
54. One should emphasize dangers of a possible **misuse of mathematics** (e. g., by applying statistical methods in a way which cannot be taken seriously).
55. One should stress **heuristical methods** (for solving problems) by making them conscious for the students. Examples of heuristical methods are: drawing a figure, examine special cases, dividing a problem into smaller problems, using a related problem.

Statistical methods (SPSS; nonparametric tests mainly, esp. "Quickcluster") were applied as heuristical tools mainly to get information on possible preferences, individual differences (with respect, e. g., to gender and type of school) and different profiles (clusters) of teachers' beliefs. 107 questionnaires were analyzed, including 43 from female teachers. 31 teachers came from upper secondary schools (Gymansium), 35 from lower (Hauptschule) or middle secondary (Realschule) and 41 from comprehensive schools.

3. Some results from our study

In Table 1 (column 3) the items of the questionnaire (column 2) are ordered by the degree of consent. "Visualizing in mathematics instruction" is ranking top (mean of all judgments $\overline{x}_{total}$=1,346), whereas "grading the solutions in tests only" is - on an average - most strongly rejected ($\overline{x}_{total}$=3,792). The first column contains mainly abbreviations for encompassing trends in mathematics education to which most of the items correspond, characterizing and operationalizing the respective trends more clearly. Of course, many items can be contributed to more than one trend. The ranking of the items in column 3 makes clear that many "problemoriented" items are valued very high. Statements which could be attributed especially to "New Math" (e.g., which refer to logic, systematics, precise language) seems to be also broadly, but less accepted. The "elementarizing principle" (item 3) is beyond all items defined to be "conventional" that one with the highest score ($\overline{x}_{total}$=1,962).

Trends	Items	$\overline{x}_{total}$	STD
PROBLEMORIENTATION	VISUALIZE (18)	1,346	0,534
PROBLEMORIENTATION	HEURISTICAL METHODS (55)	1,596	0,807
SPECIAL PROOF	PYTHAGORAS PROOF (10A)	1,615	0,862
PROBLEMORIENTATION	CONNECTIONS/RELATIONS (5)	1,636	0,806
STUDENT ORIENTATION	STATE OF DEVELOPME. OF STUD. (21)	1,667	0,762
PROBLEMORIENTATION	STUDENT TASKS/QUESTIONS (37)	1,714	0,756
	OPERATORIC PRINCIPLE (12)	1,738	0,805
STUDENT ORIENTATION	SOCIAL LEARNING (15)	1,757	0,811
PROBLEMORIENTATION	GRADING/ORAL/IDEAS (44)	1,792	0,658
PROBLEMORIENTATION	DIFFERENT WAYS (35)	1,810	0,775
PROBLEMORIENTATION	GRADING/TESTS/IDEAS (40)	1,877	0,752
	LESS MORE CAREFUL (36)	1,896	0,904
NEW MATH	LOGIC (31)	1,905	0,714
NEW MATH	SYSTEMATICS (16)	1,916	0,912
NEW MATH	PRECISE LANGUAGE (4)	1,953	1,076
CONVENTIONAL	BRAKING UP INTO SMALL PARTS (3)	1,962	0,955
CONVENTIONAL	DISCIPLINE (19)	1,972	0,941
SPECIAL PROOF	BINOM. PROOF (10B)	2,000	1,128
PROBLEMORIENTATION	MANIPULATIVES (13)	2,009	1,005
STUDENT ORIENTATION	STUDENTS' EXPLANATION (46)	2,019	0,793
CONVENTIONAL	BASIC TECHNIQUES (22)	2,038	0,904
PROBLEMORIENTATION	PROBLEMSOLVING (47)	2,075	0,801
STUDENT ORIENTATION	LOWER ACHIEVERS (27)	2,075	0,825
CONVENTIONAL	INTENSIVE DISCUSSION (26)	2,170	0,910
SPECIAL PROOF	SIDE SPLITTER THEOREM (10B)	2,178	1,090
	EXAMPLES (51)	2,226	0,843
CONVENTIONAL	PRACTICING (6)	2,280	0,909
CONVENTIONAL	TEACHER EXPLANATIONS (34)	2,280	0,954
APPLICATIONS	UTILITY (33)	2,283	0,923
STUDENT ORIENTATION	GIFTED STUDENTS (45)	2,295	0,909
PROBLEMORIENTATION	PLAYFULNESS (11)	2,330	0,953
NEW MATH	TRUTH, RIGOR (20)	2,349	1,171
CONVENTIONAL	TIGHT INSTRUCTIONSTYLE (17)	2,381	0,984
	GRADING-ORAL (43)	2,385	0,851
NEW MATH	SYMBOLS (14)	2,393	0,939
APPLICATIONS	APPLICATIONS (29)	2,406	0,814
	CULTURE (53)	2,528	1,131
SPECIAL PROOF	√2/PROOF (10C)	2,550	1,493
	MATH CREATED BY MAN (49)	2,558	1,173
	MISUSE OF MATHEMATICS (54)	2,563	1,177
PROBLEMORIENTATION	GROUP WORK (32)	2,575	0,985
NEW MATH	PROOFS (8)	2,743	0,991
	GRADING-TESTS (40)	2,781	0,899
PROBLEMORIENTATION	OPEN SYSTEM (7)	2,819	1,125
PROBLEMORIENTATION	INTUITION (2)	2,867	1,038
	MATHEMATICS NOT PERFECT (38)	2,863	1,126
APPLICATIONS	PROJECTORIENTATION (9)	2,869	1,182
SPECIAL PROOF	GEOMETR. PROOF (10e)	2,980	1,166
NEW MATH	STRUCTURES (1)	2,981	0,945
NEW MATH	CLOSED SYSTEM (25)	3,087	1,030
	LANGUAGEGAMES (52)	3,127	1,078
CONVENTIONAL	ROUTINE TASKS/SCHEMES (28)	3,144	1,074
	SINGLE WORK (50)	3,179	0,984
NEW MATH	PRACTICE ABSTRACTION (30)	3,190	0,942
	GRADING/ORAL/RESULTS (42)	3,240	0,940
UNEXPECTED LOW ESTIMATION	COMPUTER (48)	3,250	1,104
UNEXPECTED LOW ESTIMATION	ESTHETIC ASPECT (23)	3,343	1,254
UNEXPECTED LOW ESTIMATION	HISTORY OF MATHEMATICS (24)	3,443	1,070
CONVENTIONAL	GRADING/TESTS/RESULTS (41)	3,792	0,943

Table 1

Two groups of items are emphasized separately: there are five special types of proof, from which the proof of the theorem Pythagoras is taken to be most important. Computers, esthetics and history of mathematics were less emphasized as we expected, the last item perhaps because of lack of knowledge of the teachers (as some of them said in interviews, we conducted additionally).

Calculating the average of the scores of all items belonging to one of those trends proves "problemorientation" and "student orientation" to be the favorite guidelines of our teachers:

Trend	Mean
PROBLEMORIENTATION	1,902
STUDENT ORIENTATION	1,963
CONVENTIONAL	2,45
NEW MATH	2,502
APPLICATION	2,52

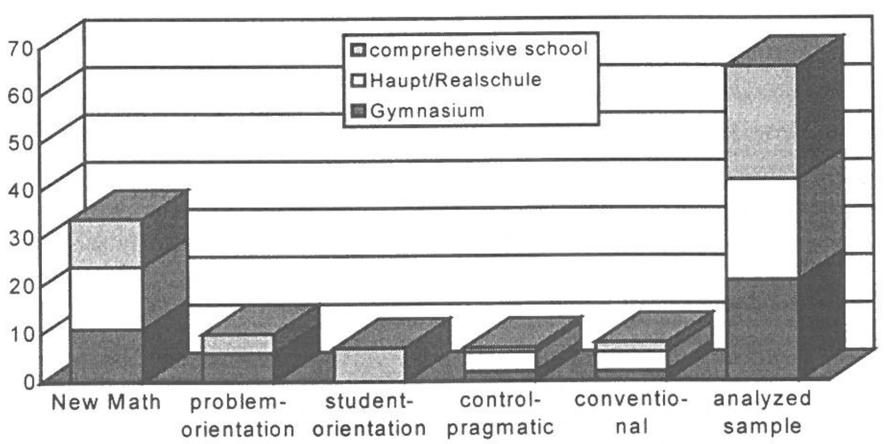
Table 2

There is a tendency towards "problem orientation", as it is in present national and international discussions about mathematics education. The following facts are to be considered:

Our sample is not representative. Probably, it represents teachers who are motivated more than the average, demonstrated, e. g., by their participation in this research study.

We did not study the real instruction praxis of our teachers. Their might be major differences between their leading ideas and their teaching practise, caused by many reasons and different boundary conditions.

Clusteranalysis (qickcluster; SPSS) lead to the following five clusters of teachers' conceptions of mathematics:



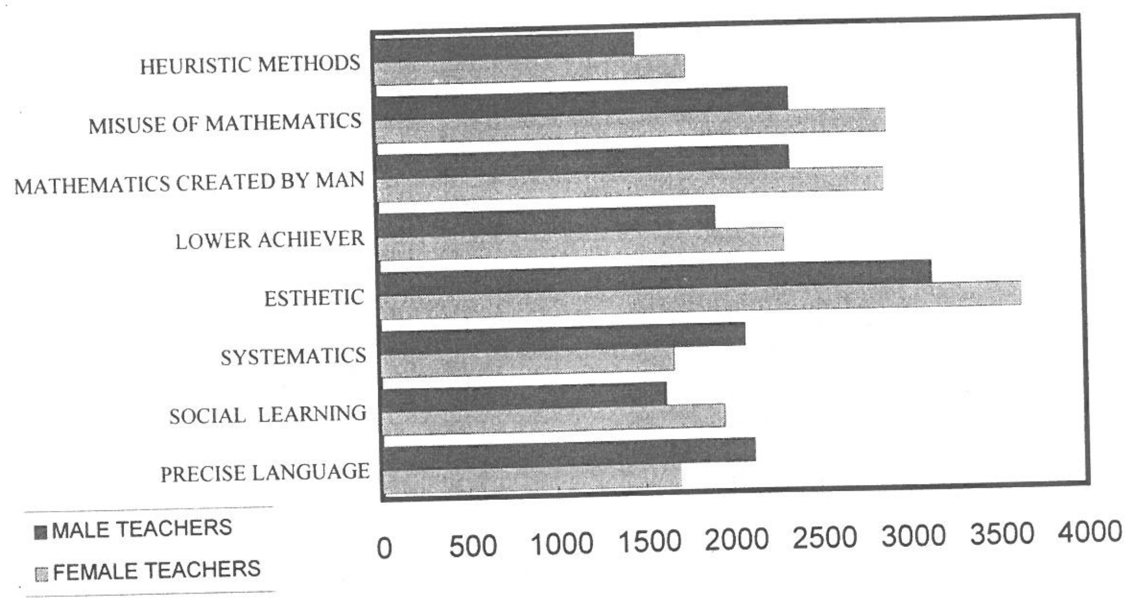
Two clusters should be presented in more detail: The *"New Math"-cluster* (n=34) is defined by the following dominating items (the mean of agreement outperforms all other means of all other four clusters, except that one for item 40, where we have the minimum for all clusters):

Item	x_{total}	STD	x_{NM}
PYTHAGORAS PROOF (10A)	1.615	.862	1.4412
PRECISE LANGUAGE (4)	1.953	1.076	1.5
SYSTEMATICS (16)	1.916	0.912	1.59
LOGIC (31)	1.905	0.714	1.62
SIDE SPLITTER THEOREM: (10b)	2.178	1.090	1.70
BINOM. PROOF (10d)	2.000	1.128	1.7647
TRUTH, RIGOR (20)	2.349	1.171	1.88
INTENSIVE DISCUSSION (26)	2.170	0.910	1.9118
STUDENTS EXPLANATION (46)	2.019	0.793	1.9706
SYMBOLS (14)	2.393	0.939	1.9706
GIFTED STUDENTS (45)	2.295	0.909	2.0588
UTILITY (33)	2.283	0.923	2.0588
GRADING/TESTS/IDEAS (40)	*1.877*	*0.752*	*2.1471*
PROOFS (8)	2.743	0.991	2.3235
GEOMETRIC PROOF (10e)	2.980	1.166	2.47
CLOSED SYSTEM (25)	3.087	1.030	2.68
PRACTICE ABSTRACT. (30)	3.190	0.942	2.7353
STRUCTURES	2.981	0.945	2.7647
SINGLE WORK (50)	3.179	0.984	2.8529
GRADE/TESTS/RESULTS (41)	3.792	0.943	3.23

The smaller *"problem oriented" cluster* (n=10) has the following profile:

Item	x_{total}	STD	x_{PO}
Agreement above mean:			
HEURISTIC METHODS (55)	1.596	0.807	1.10
SOCIAL LEARNING (15)	1.757	0.811	1.30
GRADING/TESTS/IDEAS (40)	1.877	0.752	1.50
CONNECTIONS/RELATIONS (5)	1.636	0.806	1.60
GRADING/ORAL /IDEAS (44)	1.792	0.658	1.60
MATHEMAT. CREATED BY MAN (49)	2.558	1.173	1.60
MISUSE OF MATHEMATICS (54)	2.563	1.177	1.70
CULTURE (53)	2.528	1.131	1.70
PROBLEMSOLVING (47)	2.075	0.801	1.70
$\sqrt{2}$ PROOF (10c)	2.550	1.493	1.80
OPEN SYSTEM (7)	2.819	1.125	2.30
INTUITION (2)	2.867	1.038	2.30
ESTHETIC (23)	3.343	1.254	2.60
LANGUAGEGAMES (52)	3.127	1.078	2.80
HISTORY (24)	3.443	1.070	3.10
Agreement below mean:			
OPERATOR. PRINCIPLE (12)	*1.738*	*0.805*	*2.50*
APPLICATIONS (29)	*2.406*	*0.814*	*2.60*
UTILITY (33)	*2.283*	*0.923*	*2.60*
BRAKING UP INTO SMALL PARTS (3)	*1.962*	*0.955*	*2.80*
BASIC TECHNIQUES (22)	*2.038*	*0.904*	*3.10*
PRACTICING (6)	*2.280*	*0.909*	*3.20*
SYSTEMATICS (16)	*1.916*	*0.912*	*3.30*
PRECISE LANGUAGE (4)	*1.953*	*1.076*	*3.30*
TEACHER EXPLANATIONS (34)	*2.280*	*0.954*	*3.30*
SINGLE WORK (50)	*3.179*	*0.984*	*3.60*
GRADING/ORAL/RESULTS	*3.240*	*0.940*	*3.80*
GEOM. PROOF (10e)	*2.980*	*1.166*	*4.00*
ROUTINE TASKS, SCHEMES (28)	*3.144*	*1.074*	*4.40*

There are also some *"significant" differences (t-test, $p<0.05$) between* some judgements of *male and female teachers* (1000: complete agreement, 5000: complete rejection):



4. Some results from the TIMSS-study

Comparison of the Steps Typical of Eighth-Grade Mathematics Lessons in Japan, the US, and Germany[1]

The emphasis on understanding is evident in the steps typical of Japanese eighth-grade mathematics lessons:

- Teacher poses a complex thought-provoking problem.
- Students struggle with the problem.
- Various students present ideas or solutions to the class.
- Class discusses the various solution methods.
- The teacher summarizes the class' conclusions.
- Students practice similar problems.

In contrast, the emphasis on skill acquisition is evident in the steps common to most US and German math lessons:

- Teacher instructs students in a concept or skill.
- Teacher solves example problems with class.
- Students practice on their own while the teacher assists individual students."

These results might lead to the following impression: Japanese school-practise resemble the conception of "*problem orientation*", which seems to be esteemed high by our teachers[1].

[1] Source: TIMSS; *Pursuing Excellence*, NCES 97-198, Videotape Classroom Study, UCLA, 1996.

On the other hand, (average!) German school-practise could be described as "*conventional*" mainly, which was esteemed less by our teachers (cf. Table 1 and Table 2).

There might be possible obstacles, which might prevent many German teachers from implementing often well known and highly esteemed teaching principles in their practical teaching.

Furthermore, in TIMSS there was also an analysis of "teachers perceptions about mathematics"[2]. There seems to be *no relations between the success of the students and some beliefs of their teachers*: e. g. many teachers of very successful students as from Singapore (number 1 in this study!) as well as many teachers of less successful students as from Iran agree (more than 80%) that "Mathematics is primarily a formal way of representing the real world". But less than 40% of the teachers from the students of the Czech Republic (no. 6) agreed to this statement. A similar impression can be received by reactions of teachers to the statement "If students are having difficulty, an effective approach is to give them more practice by themselves during class".

5. Prospects

At least the following questions came up from our analyses:

- To what extent are teacher beliefs related to "success" of their students? What kind of success do we want?
- What are possible obstacles which prevent many teachers to implement methods in their classroom, which they esteem very often high? What are barriers between wish and reality?

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[1] Furthermore, this teaching-method has been already applied for several years in a gifted project in Hamburg, cf. e. g., WAGNER/ZIMMERMANN 1986, ZIMMERMANN 1986.
[2] Cf. BEATON et al. 1996, p. 138 - 143.

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