VIEWS OF GERMAN MATHEMATICS TEACHERS ON MATHEMATICS

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In a questionnaire with 77 items, more than 300 mathematics teachers of secondary schools in Germany were asked for details on their views on mathematics. Thus, we were questioning their mathematical beliefs by which we understand the teachers´ attitudes towards mathematics. We were particularly interested in whether their views could be recognized as a structure. The aspects of "formalism," "scheme," "process" and "application," which are known from former research, were central dimensions of attitudes in the teachers´ answers. These four global dimensions formed a global part structure which we derived as a graph through the significant partial correlations. Thus, the purpose of the talk is two-fold: first of all, to present the results of such an investigation for which only scattered results are available in Germany; and secondly, to provide a more precise insight into the structure of beliefs and to reveal their interrelationships using the multivariate methods.

The Relevance of the Investigation

The central role of beliefs for the successful teaching and learning of mathematics has been pointed out again and again by numerous educational researchers. Pehkonen / Törner (1996) mentioned four aspects, in particular, which justified a close investigation of beliefs and belief systems: (i) mathematical beliefs as a regulating system, (ii) mathematical beliefs as an indicator, (iii) mathematical beliefs as an inertia force and (iv) mathematical beliefs as a prognostic tool. With this in mind, the teachers´ beliefs play a key role in the teaching and learning process so that priority is given to this in research.

Theoretical Framework

In the German language there does not exist an adequate translation for the widely-used word *beliefs*, because each translation is inhibited with limited associations. On the other hand, many publications avoid laying a clear, understandable foundation. We will identify *beliefs* in the following with *attitudes*, whereby we understand attitudes as being conceptional constructions which we assume *a priori* to contain a certain consistency among cognitive, emotional and behavioral components.

Mathematics as a world of experience and action can be assumed to be an extremely complex field. This also applies to the corresponding attitudes. On the *cognitive level* we can assume that the subjective knowledge of mathematics and teaching mathematics contains ideas in several different categories: (a) beliefs about mathematics, (b) beliefs about learning mathematics, (c) beliefs about teaching mathematics and (d) beliefs about ourselves as practitioners of mathematics. At the same time, category (a), "beliefs about mathematics," comprises a wide spectrum of beliefs which, at least, contains the following components: (a1) beliefs about the nature of mathematics as such, (a2) the subject of mathematics (as taught in school or at the university), (a3) beliefs on the nature of mathematical tasks and problems, (a4) beliefs on the origin of mathematical knowledge and (a5) beliefs on the relationship between mathematics and empiricism (particularly on the applicability and utility of mathematics) and so on. It is obvious that there are easily understood *affections* as well as *behavioral dispositions* and intentions associated with each component (a) to (d) and its subcategories.

It is therefore evident that there cannot exist an absolute smallest attitude unit which cannot be broken down and analyzed further, just as an atom may not broken down into its respective sub-particles. In this context there will always exist a smaller particle. In most of the cases it was impossible to differentiate between beliefs and "belief systems." With this in mind, we define a belief system as a hypothetical attitude construction which, concerning particular attitudes towards mathematics, is yet to be proven and is, therefore, of no empirical, but rather of heuristic value. In the German language the expression *belief system* is often replaced by a term occasionally used in analogy by Schoenfeld and referred to as a "mathematical world view." With this in mind, the expression, "mathematical world view" will be understood as a synonym for a "belief system" with respect to mathematics.

Thus, information gained from two levels is significant to a definite expression of a 'mathematical world view' or belief system respectively: on the one hand, expressions of single beliefs; on the other hand, the relationships between different beliefs within the 'world view.' The relationships between single beliefs form a structure which is probably more important to the representation of a 'mathematical world view' and its relevance to action than to all the beliefs it contains.

The following question then arises: *Which structural parameters do mathematical world views possess?* It may be that these structures offer better explanations and predictions of certain ways of acting rather than single beliefs. Furthermore, changing a belief system requires a detailed knowledge of the interfering parameters as well as the number and strength of the connections which are intricately woven into a net.

Research literature on this subject offers some approaches to a possible structuring of belief systems. For example, Rokeach (1960) organized beliefs along a dimension of *centrality* to the individual. The beliefs most centralized were those on which there was a complete consensus; beliefs about which there were some disagreement would be less central. In contrast to this idea, Green (1971) discusses three dimensions of belief systems: *quasi-logicalness, psy-*

chological centrality, and *cluster structure*, which will be considered here more closely (see also Pehkonen 1994). We prefer a *multivariate method* in order to visualize belief systems, which will be described later. This approach is a central theme of the in-depth examination of more than 1,600 students in Grigutsch's dissertation (1996).

The Investigation

Approximately 400 teachers participating in the German Annual Mathematics Education Conference in Duisburg were asked to fill out a questionnaire containing 77 items. A total of 310 questionnaires were filled out and returned. By conducting this investigation, Grigutsch, Raatz & Törner (1995) tried to explore teachers' beliefs concerning mathematics. This spot check cannot, however, be classified as fundamentally representative of mathematics teachers at the secondary level, because participation in this continued teacher training conference was voluntary. It can be assumed that the returned questionnaires were filled out more or less by innovative teachers.

Methodology

The 'attitude' concept is, among other things, signified by a consistency in reactions. Thus, the existence of an attitude may only be inferred and empirically recorded if a class of similar stimuli reacts to similar situations.

In relation to our survey, this means (even if the situation of the survey is no real situation of action) at least the following: it is not enough to try to infer an attitude from a certain reaction to an single item during one observation. A group of statements of similar content must be answered in a similar way. Only in this case can the existence of a characteristic (i.e. the object of an attitude) be assumed in mathematics, producing a necessary but not sufficient prerequisite.

We decided upon a method of designing a questionnaire which reflected more of an antagonistical idea of mathematics, on the one hand, as a static product and mathematics as a dynamic process on the other. Our presumptions were based on investigating mathematical world views of mathematics students (Törner, Grigutsch 1994). In the teachers' questionnaires we grouped the questions under three headings: (i) my experiences with the normal, classroom, teaching situation in the school; (ii) mathematics as a field, according to my perspective; (iii) the origin of mathematics; and (iv) mathematics and reality.

First of all, we did a factor analysis to form groups of statements which were part of the questionnaire. The items were scaled as follows: 5 = totally agree, 4 = agree for the most part, 3 = undecided, 2 = partly agree, 1 = do not agree. Furthermore, we used listwise deletion provided a person did not have a positive value in one of the items. Thus, 207 persons were left for the statistical procedures through the factor analysis.

Each factor analysis consisted of 75 items. Items 1 to 77 were included except for items 22 and 23. As for items 22 and 23, the large number of refusals to answer them resulted in too many observations (subjects) being excluded from the factor analysis. Furthermore, it was questionable in principle whether items to which many subjects refused to respond could be taken into consideration during the evaluation of the questionnaire.

The data was analyzed through factor analysis. First of all, an analysis of the principal components was calculated in order to determine the eigenvalues and to carry out the Scree-test. There were 25 eigenvalues which exceeded 1. When taking data from the scree-plot, an analysis based on *four* factors seemed to be recommended. In performing principal component analysis, the four factors were assumed, and then the varimax-rotation as a transformation method was applied (using StatView Software 4.5). As to the orthogonal solution, each factor was determined by items whose loadings exceeded .39.

We were able to prove that the components represented different aspects or views on mathematics, namely the *tool* or *schematic orientation aspect* (S) resp., the aspect of *formalism* (F), the aspect of *process* (P) and the aspect of *application* (A). According to statistical analysis, these aspects were independent dimensions of attitudes toward school mathematics.

Each of these four aspects is operationalized through 8 to 13 items; the methodological and statistical approach demanded this.

Results

In the mathematical world views of teachers, these four global dimensions formed a global partial structure. Using the matrix of partial correlations (n = 253) in Table 1 we obtained the following diagram (Figure 2).

The partial correlations resulted in a partial structure of the 'mathematical world view' or the belief system, respectively, which corresponded to our theoretic assumption of antagonistic ideals. The formalism and scheme (= tool aspect) scale represents both aspects of the static view of mathematics as system and intercorrelate highly. Both parts of the static paradigm correlate with the process scale in a significantly negative way. This confirmed our original hypothesis

Table 1 *The Intercorrelation Matrix of the Four Factors: Tool aspect (S), Formalism (F), Process (P), and Application (A)*

	Formalism (F)	Application (A)	Process (P)
Application (A)	,042		
Process (P)	–,127*	,127*	
Tool aspect (S)	,364***	,087	–,146*

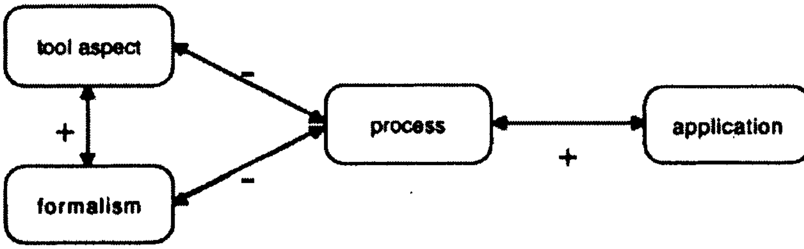

Figure 2. Structure of teachers' belief system defined by the significant partial correlations (Grigutsch, Raatz & Törner 1995)

that both views are in direct opposition of one another (at least if a paradigmatic analysis is carried out). The application aspect of mathematics correlated only significantly with the process aspect of mathematics. This corresponded to our pre-theoretic assumptions in that scheme and formalism express a static property, which does not include, however, that solving problems of reality is not a primary aim. From a formalist point of view, mathematics largely refers to itself, a precise conceptualization, a purely formal-logical verification of statements and to its logical-systematic structure. From a schematic point of view, mathematics is a collection of calculation techniques and algorithms which (the non-connection with the application scale has to be interpreted this way) are considered suitable for mathematics-related routine rather than for concrete applications and solutions to problems of reality. On the other hand, the process aspect aims at developing knowledge through a problem-related cognitive process, emphasizing the importance of seeing connections of ideas and of intuition. This dynamic concept of mathematics is more likely to be suitable for application, and this is expressed by the teachers' attitudes. On account of the sample-related findings, the 'mathematical world view' is not uniformly, but differently marked. As to all four belief objects, the frequency distribution covers certain values. For this reason, there are individually different ways in which teachers look at mathematics, ranging from rejection to agreement. Furthermore, those differences are supported or stabilized by the structure formed by these attitude objects. While they are negatively connected to the process aspect, the attitudes toward the scheme aspect and the formalism aspect show mutual support. Looking at mathematics as being dominated by schemes corresponds with this attitude, expressing the idea that formalism is of great importance whereas a process; a like view of mathematics is less significant.

Each attitude of a certain dimension, therefore, supports other attitudes belonging to other dimensions. As for these four dimensions, the "mathematical world view," in the very least, is highly consistent and stable. But, then, this only scarcely implies that any changes of the 'mathematical world view' concerning these dimensions come about. Thus, every attempt must manipulate effects on all four dimensions simultaneously. For this very reason, a change of beliefs will be carried out most effectively if it imparts experiences to all four dimensions which may cause a change.

Further Observations — Comparison of the Means

We shall briefly mention some more evaluations on the basis of our data. Scale values referring to each of the four dimensions were set for each person involved. These dimensions were operationally defined by 8 to 13 items, respectively. The score concerning the statements of each dimension was added up for each teacher involved. A transformation and stretching of the scale resulted in each teacher having a scale value in each dimension ranging from 0 to 50; 0 to 10 represent utter rejection; 40 to 50 in full agreement.

There was no unique view of mathematics in the sample. Teachers had different individual attitudes towards each of the four dimensions, ranging from rejection to approval. The attitudes were 'normally distributed'. Because the distributions in each scale were "normal," there was no preference of a specific value except for the mean which the distribution optimally characterizes. Because the variances are principally the same, we cannot neglect them when comparing their characteristics or underlying features. The overall attitudes of the teachers were (with respect to formalism, schema, process and application) represented by their mean and marked as single points on the scale. The average attitude of the teacher in each dimension was optimally identified by its mean. Due to the fact that the means were different, their were also varying attitudes, which contrasted to the four attitude objects. In the eyes of the mathematics teacher, these four factors were not of equal importance. As a result, in an accented mathematical view there exists much more in that some elements were found to display more emphasis while others, less. According to the average view of the teacher on mathematics, the aspect of scheme was estimated rather low and, in a way, somewhat refused, with formalism ranking in the upper middle sector of a scale. In contrast, the aspects of application and process were valued relatively high. The views relating to the application and process aspects were considered as being quite meaningful and were not distinguishable by their average estimations.

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