SELF-ESTIMATING TEACHERS' VIEWS ON MATHEMATICS TEACHING - MODIFYING DIONNE'S APPROACH

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The following report has originated in a joint, extensive investigation with E. Pehkonen (Helsinki) on investigating teachers' conceptions on teaching mathematics. However, in the following presentation we restrict to focus on one aspect of the research, namely different *self-estimations* of the teachers on their views on mathematics teaching and their representations. It was Dionne (84) who proposed some numerical self-estimation of personal views on mathematics by distributing 30 points to three components. This rough adhoc-approach, however, vividly visualizes differences in the personal constructs on mathematics. Here, we present some methodological modifications leading to a more detailed description of underlying teachers' views. Our test subjects amounted to a total of 6 experienced German teachers. The research was conducted during the spring and summer of 1994.

The literature contains controversial and abundant contributions concerning teachers' beliefs as well as the process by which teachers change in regard to their professionalism. Marginal attention is dedicated, however, to the corresponding methodological questions concerning the research of these beliefs (Grigutsch 1994).

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It was Dionne (1984) who proposed some numerical self-estimation on personal views on mathematics by distributing 30 points to three components. In this research note, we present some methodological modifications leading to a more detailed description of underlying teachers' views. Originally, our test subjects amounted to a total of 13 experienced German teachers, five of whom teach in the *Gymnasium*, two in the *Realschule*, one in the *Hauptschule*, and five in the *Gesamtschule*. Here, we will limit ourselves to the researching of six people.

1. Theoretical Frame - What are Beliefs?

The concept of belief has many definitions in the literature (e.g. Thompson 1992, Pehkonen 1994, 1995). Here, we understand *beliefs* as one's stable subjective knowledge (which also includes his feelings) of a certain object or concern to which undisputable ground may not always be found in objective considerations.

As stated elsewhere, an individual's beliefs form a structure: We will call this structure of beliefs, or belief system, his/her *view of mathematics*.

This wide spectrum of beliefs (and conceptions) contains, among others, four main components: (1) beliefs about mathematics, (2) beliefs about oneself as a user of mathematics, (3) beliefs about teaching mathematics, and (4) beliefs about learning mathematics. These main groups of beliefs can be, in turn, split into smaller pieces. However, generally it is not easy to differentiate between the beliefs on mathematics and teaching resp. learning of mathematics. This is particularly true if there is no 'world outside of school mathematics' for the test subject. Along these lines, the term 'mathematical world view' which originated from Schoenfeld (1985), was recently used by Törner & Grigutsch (1994), and elaborated in a newer paper by Grigutsch; Raatz & Törner (1998).

1.1. Different Views of Mathematics

For his research, Dionne (1984) used the following three perspectives of mathematics: (A) Mathematics is seen as a set of skills (*traditional* perspective); (B) Mathematics is seen as logic and rigor (*formalist* perspective); (C) Mathematics is seen as a constructive process (*constructivist* perspective). In his book, Ernest (1991b) describes analogously three views of mathematics: *instrumentist, platonist* and *problem solving*. These correspond more or less to the three perspectives Dionne (1984) mentioned above. Furthermore, the same three-component model was rediscovered ten years later by Törner & Grigutsch (1994) calling (A) the *toolbox aspect*, (B) the *system aspect* and (C) the *process aspect*. Recent research shows that there are at least four basic components in an individual's view of mathematics: the fourth component seems to be application (Grigutsch, Raatz & Törner 1998).

1.2. The German Educational System

For sake of shortness of this report it is not possible to provide the reader with the framework of the underlying German education system. The reader is refered to Robitaille (1997).

2. The Basic Ideas of this Research

The teachers we interviewed in the study came from each type of school present in the German educational system. This was cause to take the simpliest possible approach as a plane of projection of their views. As a result, we asked the participating teachers in one of the inquiries to share with us their self-assessment with regard to both the actual and the intended teaching methods of mathematics and to employ the catagories of Dionne. It is obvious that the tendency between their vision and the reality of their accomplishments contain pertinent information. With this statement the raised beliefs of teaching mathematics can be understood in regard to the type of data as three dimensional vectors through which the entries as weights stand for the different dimensions in which teaching

mathematics is presented. Thus, the question arises how to visualize these vectors.

In this paper we shall introduce a new pictorial method of self-estimation to serve as a supplement to the numerical Dionne-data. Although these two methods, presenting data in a tabular and representing them graphically, are mathematically equivalent, there are not redundant in a *methodological* sense. With respect to the considerations above, our main research question reads: *How well do information from these two different methodological sources investigating teachers' views of mathematics fit together?* where we will mainly restrict only to the informations derived from the self-assessments. It will be reported in (Pehkonen & Törner 98) how valid information will be received with the method of Dionne compared also with the rich data sources through the questionnaire and interview method.

3. The Realization of the Research

Since in our earlier attempts with the same methods there was some confusion with respect to the view on mathematics under consideration, we let the teacher explicitly *distinguish* between his/her *real* and *ideal* view of mathematics. So, not only the self-estimation with respect to the categories mentioned above were of interest for us, but also we paid much attention the pattern of transition in the self-estimations with respect to their real and ideal view respectively.

We asked the teachers to share 30 points between their three real and ideal perspectives of mathematics written it in a tabular form. In addition, we decided to use another type of inquiry, namely a pictorial representation of the self-estimation. Dionne's categories are assumed to be represented by the vertices of an equilateral triangle. Any normalized estimation towards the three categories can be visualized by some point in the interior of triangle with the distances to the vertices as a measure of closeness. Thus, the teachers were asked to mark their positions of the scores within an equilateral triangle.

4. The Results

4.1. The Numerical Self-Estimation

In Table 4.1 we give the scores which teachers attribute to three components of the view of mathematics. At first glance the following is made obvious by the table: (4.1.1) None of the teachers chose an extreme position, neither in their real view nor in their ideal view. (4.1.2) Each teacher with the exception of K wanted to change, in the direction of process; e.g., H1 wanted to emphasize the process aspect more. (4.1.3) Each teacher regarded the process aspect as the most important factor. It was H1 who gave the process aspect the highest loading. (4.1.4) It is notable that the estimation of K and L, the teachers (having the same formal qualification!) not

having met each other before. The same applies to their formal qualification. (4.1.5) Teachers D, J and H2 share the highest loadings with respect to the toolbox aspect. It should be noted that the interviews supported also these entries in Table 4.1. (4.1.6) Again, the mentioned three persons are exactly those teachers who are not quite satisfied with their own teaching in mathematics and would like to change their situation, however, through quite different ways.

Table 4.1
Scoring to the Self-Estimation

Teacher	Tool	System	Process	Tool	System	Process
	real			ideal		
D	**15**	5	10	5	5	**20**
H1	9	1	**20**	4	1	**25**
H2	**14**	8	8	6	**12**	**12**
J	**15**	3	12	10	5	**15**
K	8	10	**12**	10	8	**12**
L	9	9	**12**	6	9	**15**

Tool = math as a tool box, System = system aspect of mathematics, Process = process aspect of mathematics. The leading positions are marked in boldface.

On the basis of the interview (Pehkonen & Törner 98) we may classify D and H1 as the most innovative among these six persons. The tendencies in their ideal view of mathematics are the same, however there are small differences concerning the role of systems and structures in mathematics (D, System = 5; H1, System = 1). However, on the basis of the figures for the real classroom lesson the assessment of D is considerably more rational than H1. Perhaps this discrepency is explained by the fact that D, in contrast to H1 has passed through a full academic university course of study, so D's mathematical horizon can be regarded as broader.

It seems evident to us, that primarly the **weights** set by the teachers, not the exact numerical scoring, **indicate their understanding of mathematics teaching** which leads to a linear ordering of the components. Thus, we derive table 4.2.

(4.1.7) Apparently, with the exception of K, all teachers held the formalism aspect (system) in last place in their real teaching; however, H2 and L would like to change the order. This can be understood by the interviews: K was extremely in favor of the formalism aspect in the past. (4.1.8) It is notable that (only) K and L give the formalism aspect a second ranking in their real teaching. Probably, this fact can be explained through their

Table 4.2
The Ranking of the Components Derived from Table 4.5

Teacher	real	ideal
D	T > P > S	P > T = S
H1	P > T > S	P > T > S
H2	T > P = S	P = S > T
J	T > P > S	P > T > S
K	P > S > T	P > T > S
L	P > S = T	P > S > T

T = Tool, S = System and P = Process aspect

teaching career at *Gymnasium* affording a high mathematical qualification. (4.1.9) On the other hand, with respect to ideal teaching, the process aspect is ranked first among all of the teachers and, in case of H2, on the same level with the system aspect. H2 seems to be obligated to mathematics which he has been taught to be of structural importance at the university and, therefore, feels guilty since his actual situation makes it impossible to present this subject in an adequate manner according to his view.

4.2 Triangular approach

In Figure 4.3 we illustrate the marks within the equilateral triangle from the teachers' responses.

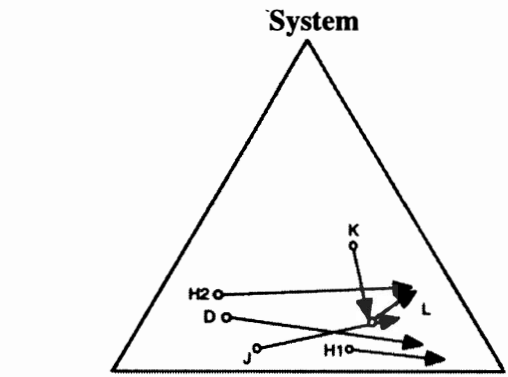

Figure 4.3. The self-estimation-data in graphical form as given by the teachers. Arrows (real to ideal) indicating tendencies are drawn by the authors

First, the *outer distribution* underlines previous observations (see 4.1.1, 4.1.2 and 4.1.3). Next: (4.2.1) It becomes obvious that there is more or less a common view on ideal mathematics teaching. The positions are closer than they are in the actual teaching of mathematics teachers of different school forms. (4.2.2) Teachers D, J and H2 are found along with their implemented lessons in the toolbox corner (4.1.5). These estimations can be supported by corresponding quotes (Pehkonen & Törner 98).

Secondly, the predominant *tendency* of the change of the teachers underlines the importance of the process aspect. In particular: (4.2.3) The two *Gymnasium* teachers K and L show some slight differences (in contrast to 4.1.4).

One should note the fact that L does not estimate the necessity of change in his own classes to be very high. As it became clear in the interview, possibly the daily disturbances taking place his classes lead only to marginal frustration, because for him (as is with K) mathematics exists outside of the classroom and is, as a result, a pure and philosophical discipline worthy of respect.

Thirdly, if one takes the *length* of the arrows as an indicator for the magnitude of change these metric informations are *not made transparent* by Table 4.1. Note: (4.2.4) Again, teachers D, J and H2 claim the largest change with respect to their teaching (see the length of the arrows). (4.2.5) Probably, the arguments of 4.2.2 may also explain why K and L are claiming only limited changes.

4.3. Comparison of Both Self-Estimations

The question arises which data one should hold to be of primary importance: the graphical or numerical data? Both positions could be supported by arguments; both information sources have their own message. It is evident that the numerical and graphical data do not correspond exactly. This fact should therefore not be overemphasized. It seems to us that the test subjects unconsciously intend to express different information through these mathematical equivalent representations. Thus, they serve as two independent data sources covering different aspects!

For example, H2 estimated his math view by Toolbox = 14, System = 8, Process = 8, thus the aspects of System and Process are playing an equivalent but low estimated role. And, in the ideal teaching his scores are Toolbox = 6, System = 12, Process = 12. This feature is not reflected in the graphical representation where the aspect of System seems to remain unchanged. However, the length of vector indicates his feeling that his real teaching differs greatly from ideal teaching.

Apparently there is some inconsistency in the numerical and graphical data of K. His estimations of real and ideal teaching show some interchanging of the roles of Toolbox and System which should be represented by a reflection of positions within the equilateral triangle. On the other hand,

the arrow in the graphical mode calls for some change in particularly towards more Process aspect and less System.

Whereas it is easier to realize the tendencies and the direction of the changes, the table may also show some clues as to how the changes should take place. Note that all three, D (System = 5), H1 (System = 1), and L (System = 9) would not be likely to change the absolute value of the factor System; they only prefer an exchange between the Tool aspect in favor of the Process aspect. It must remain an open question whether or not it is an intentional exchange, or perhaps if it is merely just a strategy to treat the data which is to follow the Dionnian catagorization and to distribute 30 points twice among three entries.

5. Conclusions

The derivable conclusions are two-fold in nature. They reflect our considerations on the different methodological sources as well as there are informations on German teachers' view of mathematics. Here, we are restricted to a discussion of methodological aspects.

First, without having done interviews (see Pehkonen & Törner 98) it would have been difficult to give an extensive interpretation of the numerical or pictorial data. Thus, the Dionne-data is of limited importance if there are no additional information sources.

Furthermore, it is indisputable that Dionne´s three-pole must be seen only as a primitive model. Nevertheless, in spite of its simplicity it still possesses a high degree of clarification, especially when concerning a first approach to the problem. The detailed comparision of the self-estimations of the Dionnian components makes clear that numerical data is in need of commentary.

It is decisive that one highlights in such examinations, whether the data under question concerns the real teaching of mathematics or if they are valued as the ideal teaching of mathematics.

Furthermore, it seems that the use of pictorial self-estimations completes in some sense the 'rough' and primitive, yet illustrative, method of Dionne. The additional expenditure for the test subjects is of marginal magnitude. Although, it is a matter of processing redundant data, the noticeable inconsistancy should not be overly valued, because both sources of data make allowances for varying emphasis.

Metrical aspects especially play a strengthened role in the process through pictorial illustrations. The examinee can highlight his/her basic discrepencies, and, finally, a direction of change will become evident in relation to the three components, which are represented by the three corners.

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