# NARRATION AS A TOOL FOR ANALYZING BELIEFS ON CALCULUS – A CASE STUDY

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Abstract: In the following research report the question is pursued concerning which specific beliefs are present in the field of calculus in the university course for prospective mathematics teachers. Narration is methodically employed as a research tool, allowing answers to a number of research questions to be expected. On the one hand they entail content-specific beliefs on calculus which are researched in view of their development in the sense of when they were developed. Due to the open method of data collection we receive, on the other hand, information on which dimensions calculus is perceived by prospective teachers. Finally, we obtain information to what extent permanent temporal lines of consistent views can be determined. This allows conclusions to be drawn on the efficiency of university courses for prospective mathematics teachers.

## 1. The context of the case study

In the German educational system calculus plays a central role in the secondary grades (age groups 16-19) (see details of the German system in [Robitaille 97]). This circumstance dates back to the initiative of the famous mathematician Felix Klein at the beginning of this century. His concepts are still markedly present in the German mathematics curriculum of today. This results in the fact that calculus contents are always compulsory parts in the *Abitur* examination (the final school-leaving examination). The demands in the mathematics *Leistungskurse* in Germany (equivalent to A-Level courses in Great Britain) are greater compared to equivalent examinations in the United States (NCTM 1991). Calculus also already plays a dominant role in the first semester of university prospective teacher courses for the *Sekundarstufe II*. These students visit the same lectures and seminars as those studying for a purely academic degree.

However, it can not be ignored that calculus lessons in schools are increasingly slipping into an orientation crisis. The unfulfilled, exaggerated expectations towards New Maths can be held partially responsible: it views calculus primarily as a propaedeutic field of science. After all, the present discussion is also being pushed forward by the availability of CAS-Tools in pocket calculators featuring graphics display facilities (DERIVE). These calculators downgrade the usual traditional curve discussions and calculus

extrema tasks (suitable for schema-oriented lessons) to literally trivial example applications. This view amongst teachers, however, has at present not yet affected the traditional-oriented calculus canon at university. In contrast to the United States, a calculus debate has not yet got underway in Germany.

Nonetheless, new demands are being discussed in occasionally offered lectures on mathematics education for prospective teachers. However, such seminars do not predominantly serve the acquisition of methodical aspects of calculus but intend to exemplary introduce general and specific didactic aspects related to mathematics lessons. The subject material employed in this paper originated from such a seminar.

Reflection on mathematics lessons can itself be carried out on different levels and be subject to widely differing objectives. In this paper the question is discussed as to what extent self-reflection in the form of narration elements can offer highly differing information on system-internal states which might reveal concealed parameter. The innate ideas of the learner are also articulated here. The subject material employed for this question were topics of papers by 7$^{th}$-semester students who were requested to write essays on this in a seminar on mathematics education focussing in particular calculus.

In the literature, reflection is emphasized as having a central role as a structuring facility. Both for oneself and for the external observer, reflections reveal beliefs and also conceptual systems expressing teachers' thoughts and actions. Reflection has, therefore, gained significant acceptance as a basis of prospective teacher education. Chapman (1998) (cf. the literature quoted there) emphasizes that, in general, preservice teachers tend to rely on their personal experience as learners in constructing meaning for classroom events. Self-reflection is also employed as a means to accompany and support processes of change. Self-reflection, therefore, is also a basis for a prospective teacher´s professional progress. Different possibilities arise for the initiation of self-reflection. The use of narration in our study is based on the view that narration is a way in which one makes sense of the world (Bruner, 1986, see Chapman, 1998) by incorporating in particular one's own experiences.

## 2. The research questions

According to Schonfeld (98), the interdependency of teacher's goals, beliefs and knowledge is decisive for the teaching process. From this we derive the following questions for our case study:

2.1 Which beliefs on mathematics, in particular on calculus, are characteristic for prospective teachers? When were they developed?

2.2 In which dimensions are calculus lessons perceived by prospective teachers: (A) the self-experienced lessons (in school, at universities)? (B) lessons considered worth having according to one's own experiences at university and on the basis of knowledge gained in seminars on mathematics education? (C) lessons given in the future as a qualified teacher?

2.3 To what extent can permanent temporal lines of consistent views be determined? To what extent are self-experienced teaching situations criteria for self-planned lessons?

## 3. Method and procedure

Within the framework of a seminar on calculus the students ($n = 10$) were asked to write essays (1 – 2 pages) on the topics: (A) Calculus and I: how I (have) experienced calculus at school and university. (B) How I would have liked to have learned calculus. (C) How I would like to teach calculus.

These essay topics were subsequently handed out at intervals of three weeks. The students knew nothing of the actual case study approach of the author. Altogether six students handed in contributions on all three topics. Their essays are used here for this case study.

In the first viewing of the essays the given topics can be categorized as being similar. The research approach realized here is classified as triangulation (Cohen; Manion 1994). This procedure can be justified through the following basic assumptions: Learning and teaching are dual processes that can be individually considered as linked together. Possibly experienced deficits are categorized - when viewed positively - as points of emphasis of one's own responsibly conducted lessons. Positive experiences lead to reinforcement of one's own actions towards others. In this respect a temporal invariant consistency in the evaluation of one's own teaching and learning processes is presumed, whereby one must bear in mind that repeated mentioning of its aspects can lead to its confirmation. Due to the limitation of a maximum of 2 pages (per essay topic) an exhausting presentation of the three topics by the students cannot be expected here. No content-related expectations were placed on the students so that freely written essays were ensured. The location and topic change, induced by the respective question formulation, enables new reflection impulses and recapitulates new aspects of a topic from the viewpoint of the other students. As the three topics are intended to illuminate different time concepts (A – past, B – present, C – future) it is to be expected that through the essays it will be possible to determine time-invariant lines.

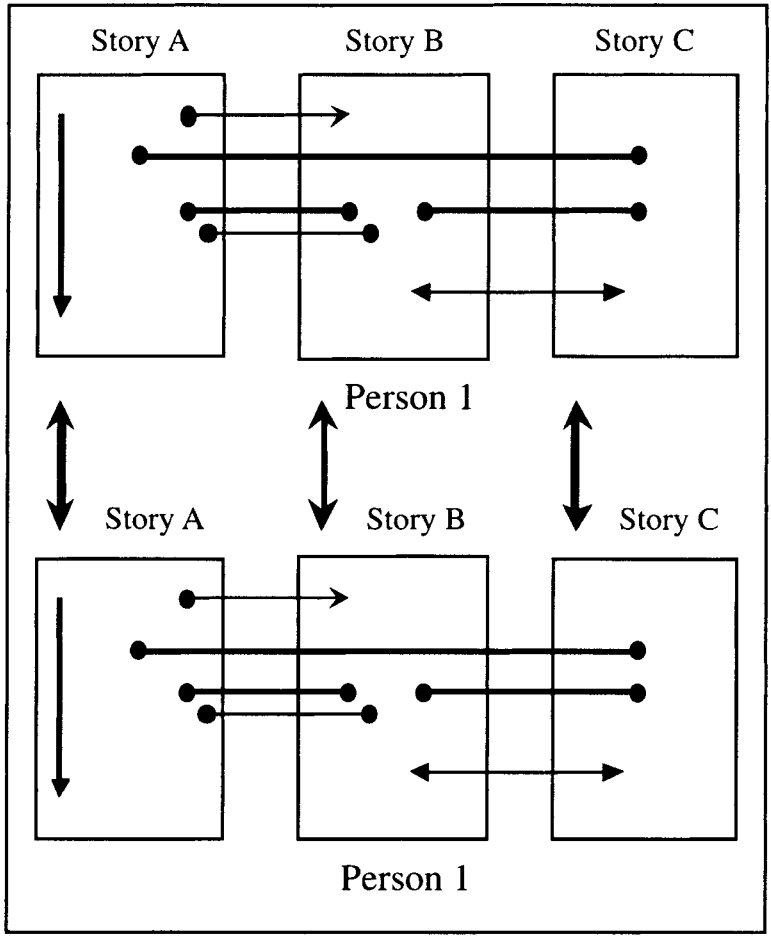


*Figure 1.* Possible clusters of evaluation

## 4. Results and evaluation

With respect to research question 2.2 the lines given by the three topics ((A), (B), (C)) offer text contributions which can be assigned to the following fields: (1) objectives of mathematics lessons and calculus, (2) views on mathematics, in particular calculus, (3) calculus and formal elements of mathematics, (4) calculus and learning / teaching mathematics, (5) calculus and demonstration, (6) calculus and the emotional dimensions of mathematics, (7) personal specifica. Not all essays contain explicit contributions to each of these fields. All contributions to these fields cover aspects on cognition, beliefs and goals (see Schoenfeld [98]). For reasons

of concise presentation only a few general results can be presented here, reported rather as a tendency.

Concerning the beliefs (research question 2.1) implicit in the essays of the students, these beliefs are in particular found in the text when the essay writers explicitly remark their contents as self-estimations being subjected to alterations in the course of time (from the perspective of the sectional levels (A), (B) and (C). They concern, e.g. calculus as a field of mathematics in view of its place within mathematics: Calculus in the upper secondary grades is perceived by pupils as an independent and new mathematics field, whereas on the lower secondary grades (age groups 10-16) mathematics is viewed rather as a whole. Possible „colourations" of this mathematics field are also to be classed as beliefs. University mathematics reinforces the impression of calculus being an independent field of mathematics, in particular as the student becomes familiar with further fields of mathematics. While the school view of calculus is schema-oriented, university students possess primarily a formal orientation. It seems that stimulation towards a stronger problem-oriented viewpoint is hardly given. In this respect their views towards calculus and mathematics in general present on the whole a one-sided picture. Also the role that may be placed in calculus by aspects of logics or application can be understood as subjective beliefs. Formulations such as „calculus training", „breaking the code of inequations", „differentiation as handicraft", or „integration as an art" reveal something about calculus learning picture of the individual essay writer.

Beliefs are expressed more clearly when evaluations in the form of described emotions are explicitly mentioned. Their views on ideal calculus lessons seem to be nurtured from their own positive experiences in lessons or to be complemented through self-experienced deficits in school and at university. Within this framework, calculus lessons are experienced at university as being not very constructive. Admittedly, calculus seminars serve a universal function in academic mathematics, whereby aspects of educating prospective teachers how to teach mathematics only plays a subordinate role. On the other hand, the few chances for illuminating the relevance of academic mathematics for school teaching are apparently hardly made use of.

In all the six essays emotional value judgments are given towards calculus, whereby in particular negative statements stand out towards calculus experienced at university. The scope lies between, 'interesting but difficult', 'found it very hard to', 'more brutal than any other mathematics seminar'.

One must hereby note that in particular those students who successfully completed calculus under considerable effort have the tendency to

exaggerate the formal aspect (for instance the logical structures of calculus terms) as the most important characteristic of calculus. Is such a formalistic view of calculus for prospective teachers, however, possibly more than slightly responsible for learning difficulties of their future pupils?

## Conclusions

It hardly needs mentioning that these results cannot be considered representative either for calculus lessons in the upper secondary grades in Germany, or in respect to statements of prospective teachers in general.

Nonetheless, the evaluations offer the author a number of valuable insights, in particular it provides a deeper and better understanding of the psychological aspects of teaching and learning topics around calculus.

Calculus is understood as a central part of mathematics; insofar beliefs may be conveyed onto mathematics *pars pro toto*. Thus it would be complementary, however, in the interest of the students, to go beyond the individually specialized fields and instead enable (again) an integral all-encompassing view of mathematics to be experienced. Here deficits were clearly articulated. Whereas the literature offers an abundance of papers on teachers' and pupils' beliefs, descriptions of specifically related beliefs are, however not abundant at all. In comparison, papers on calculus (in the upper secondary grades) are hard to find (see e.g. Fox 1998).

More concern must be given to the circumstance that calculus in the evaluation of future mathematics teachers is emotionally highly loaded - however not primarily in a positive manner. Finally, action is required when one feels compelled to note that university seminars on calculus do not seem to offer a convincing contribution towards the professionalization of future teachers. It appears that the well-known quotation of F. Klein (1908) on the so-called *'two-fold discontinuity'* still has its relevance after almost one hundred years:

> 'The young student sees himself at the beginning of his university course confronted with problems which in no point remind him of things he was concerned with at school; of course this is why he forgets all these things rapidly and thoroughly. However, when he enters a teaching position after completion of study, he is expected to teach traditional elementary mathematics in the traditional school manner; as he can hardly relate this to his university mathematics, he will in most cases embrace traditional teaching within short time, and the university course will remain to him only a more or less pleasant memory that has no influence on his lessons.'

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