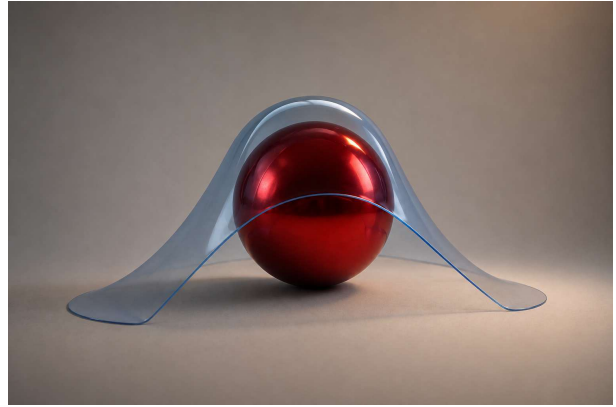


Analysis von Variationsungleichungen / Variational inequalities

Winter term 2026/27

Lecture Monday/Wednesday 10:00–12:00,
Exercise Monday 12:00–14:00



Dozent:

Prof. Dr. Georg Sebastian Weiss

Variational inequalities provide the mathematical framework for optimization problems with constraints and arise naturally in many areas of mathematics, physics, engineering, and economics. Typical applications include elastic contact problems, obstacle problems, free boundary problems, plasticity, fluid flow in porous media, and mathematical finance.

Learning Outcomes

After completing the course, students will be able to

- formulate constrained minimization problems in Hilbert spaces;
- derive variational inequalities from energy principles;
- understand the role of convexity and projections;
- prove existence and uniqueness results for elliptic variational inequalities;
- analyze the classical obstacle problem;
- understand the basics of free boundary problems;
- apply elementary numerical methods to variational inequalities;
- recognize important applications in mechanics, finance, and engineering.

Course Outline

1. Motivation (Constrained energy minimization, Elastic membranes, Obstacle problems, ...)
2. Minimisers, First and Second Variation
3. Convex Analysis (Convex sets, Convex functionals, Projection theorem, Subdifferentials)
4. Weak Formulation of Elliptic Problems (Energy minimization, Sobolev spaces, Weak derivatives, Weak solutions, The Lax–Milgram theorem)
5. Variational Inequalities
6. Existence and Uniqueness (Direct Method, Method of Monotone Operators)
7. The Obstacle Problem

- 8. Capacity
- 9. Free Boundary Problems

Sprache / Language:

German or English (depending on the participants)

Voraussetzungen / Prerequisites:

Although knowledge of functional analysis and partial differential equations helps, it will for most parts not be necessary as I will start with one-dimensional variational calculus and introduce necessary concepts such as Sobolev functions in this course.

Analysis 1-3.