Uninterpretable symbols, algebra, and instrumentalism in Boole

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September 1, 2021
Text-Driven Approaches to the Philosophy of Mathematics

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This is joint work with Dirk Schlimm from my time as post-doc with him at McGill University.

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See: Waszek, D., Schlimm, D. 'Calculus as method or calculus as rules? Boole and Frege on the aims of a logical calculus.' Synthese (2021).
DOI: 10.1007/s11229-021-03318-x

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which means that z consists of those elements that are both x and y, plus 'an indefinite remainder (some, none, or all)' of things that are neither x nor y.

A typical assessment of this procedure today:

> *Despite the impressive power of Boole's method, it falls well short of modern standards of rigor. The solution of equations generally involves eliminating factors, and so dividing class terms—even though Boole admits that no logical interpretation can be given to division. [...] Boole, clearly influenced by Peacock, argued that there was no necessity in giving an interpretation to logical division, since the validity of any "symbolic process of reasoning" depends only upon the interpretability of the final conclusion. Jevons thought this an incredible position for a* logician *and discarded division in order to make all results in his system interpretable. Venn retained logical division but interpreted it as "logical abstraction"—as had Schröder, whose 1877 book introduced Boolean logic into Germany.*

(Heis, 'Attempts to Rethink Logic', 2012, 108–109)

Our questions

Boole's peculiar procedure raises several question, first about Boole's methods, then about his justification for his methods.

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2. Keep the method in place but provide a general argument that it yields correct results, irrespective of the symbols' meaning (if any).

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For this, we first need to understand where, exactly, Boole's interpretability problems resided.

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- $x - y$, defined only if y is included in x, is their difference; $[x - y = x \cap \bar{y}]$
- 1 is the universe and 0 the empty class, so that $1 - x$ is the complement of x.

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- The dodgy case: $\frac{1}{0}$ is any class whose intersection with the empty class is the universe... which is impossible. So whenever this pops up in their calculations, Venn (and Boole) conclude that the term multiplying it should be zero.

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indeed yields that z is the intersection xy plus an arbitrary portion of $(1-x)(1-y)$, with the added (correct) relation $x(1-y) = 0$, meaning that there is nothing that is both x and not y (x is included in y).

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Now, Boole clearly understood this. It is explained in many places and in detail in his manuscripts (ed. Grattan-Guinness & Bornet), e.g., (p. 88)

> *[. . .] let the four elementary logical operations of Addition Subtraction Composition and Abstraction be expressed by the same signs as the respective arithmetical operations of Addition Subtraction Multiplication and Division [. . .]*

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> *[I]t cannot be inferred from the equation $zx = zy$ that the equation $x = y$ is also true. In other words, the axiom of algebraists, that both sides of an equation may be divided by the same quantity, has no formal equivalent here. I say no* formal equivalent, *because, in accordance with the general spirit of these inquiries, it is not even sought to determine whether the mental operation which is represented by removing a logical symbol, z, from a combination zx, is in itself analogous with the operation of division in Arithmetic. That mental operation is indeed identical with what is commonly termed Abstraction, and it will hereafter appear that its laws are dependent upon the laws already deduced in this chapter.*

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Perhaps he found it somewhat murky. More importantly, he clearly felt there was no need: he did believe that interpretation of the intermediate stages of a computation was not required, and crucially, other problems of interpretability, more serious than division, also arose in his system.

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The actual problem is that his addition, subtraction, and division only make sense under certain conditions (e.g., $x + y$ is only defined if x and y are disjoint; $x - y$ if y is included in x) while his computations sometimes require using these signs when these conditions are not met.

As he puts it:

> *We have [. . .] investigated, among other things, the laws of that logical process of addition which is symbolized by the sign $+$. Now those laws have been determined from the study of instances, in all of which it has been a necessary condition, that the classes or things added together in thought should be mutually exclusive. The expression $x + y$ seems indeed uninterpretable, unless it be assumed that the things represented by x and the things represented by y are entirely separate [. . .].*

The question then arises, whether it is necessary to restrict the application of these symbolical laws and processes by the same conditions of interpretability under which the knowledge of them was obtained. [...] If such restriction is necessary, it is manifest that no such thing as a general method in Logic is possible. On the other hand, if such restriction is unnecessary, in what light are we to contemplate processes which appear to be uninterpretable in that sphere of thought which they are designed to aid?

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So that's Boole's problem: the laws of his signs (first and foremost $+$ and $-$) are based on the cases in which they are well-defined, but he needs to use them beyond such cases.

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- We observe empirically that it works in particular instances, which reveals to us a general law of the mind.

Whatever our a priori *anticipations might be, it is an unquestionable fact that the validity of a conclusion arrived at by any symbolical process of reasoning, does not depend upon our ability to interpret the formal results which have presented themselves in the different stages of the investigation.*

...

[T]he principle in question may be considered as resting upon a general law of the mind, the knowledge of which is not given to us a priori, *i.e. antecedently to experience, but is derived, like the knowledge of the other laws of the mind, from the clear manifestation of the general principle in the particular instance.*

...

[T]hat principle [. . .] seems to deserve a place among those axiomatic truths which constitute, in some sense, the foundation of the possibility of general knowledge, and which may properly be regarded as expressions of the mind's own laws and constitution.

Summary: Boole's outlook

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The symbols $+, \times, -, \div$ correspond to logical operations (on classes or propositions). The laws governing these symbols reflect the laws governing the corresponding operations. As is constantly done in algebra, it is legitimate to follow these laws even in cases in which the interpretation in terms of classes, say, breaks down, as long as a meaningful ('interpretable') result is obtained in the end.

Is this instrumentalism (or formalism)?

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Mic Detlefsen ('Formalism', 2005, 263): Formalism centers

> *on an intrumentalist conception of language and its use in reasoning. [...] The instrumentalist conception of language allows for purely* symbolic *uses of signs in our reasoning—uses that do not depend in any essential way on the semantic content of the signs involved or on their even having such content. [...] This central doctrine is itself composed of two key elements. The first is what I will call the* creative *or* creativist element*—the idea that the mathematician is free to introduce or "create" methods out of considerations of convenience or efficiency as distinct from evaluation of content. The second is the distinctively* symbolic *element—namely, that nonsemantical uses of signs may, at least on occasion, constitute such conveniences.*

I do not see the creativist element in Boole—his justification seems to me to be of an altogether different sort.

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- Let's clarify our concepts and the meaning of our symbols so that our computations are perfectly clear and meaningful again.
- Let's give up on this dream of perfect meaningfulness: instead, let's codify our computational machinery precisely; whether the cogs mean something is unimportant as long as we get correct results. (These, for him, are the formalists and instrumentalists of various kinds.)

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- Let's give up on this dream of perfect meaningfulness: instead, let's codify our computational machinery precisely; whether the cogs mean something is unimportant as long as we get correct results. (These, for him, are the formalists and instrumentalists of various kinds.)
- Let's try to explain the problems away by philosophizing about, e.g., the constitution of the human mind.

This, of course, is a caricature. But using this as orientation, Boole's justification seems to be closer to the third category.

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