Exercises p-adic Galois representations **summer term 2015**
Prof. Dr. J. Kohlhaase **problem sheet 4**
Due date: May 12, 2015

Problem 1 (8 points): Let F be a complete discretely valued field with residue class field k.

(i) Show that for any finite separable field extension ℓ of k there is a finite unramified field extension E of F with residue class field ℓ.

(ii) Assume that E_1 and E_2 are two finite unramified field extensions of F with $k_{E_1} = k_{E_2}$. Show that $E_1 = E_2$.

(iii) Let E be a finite unramified field extension of F, and let k_E denote the residue class field of E. Show that $E|F$ is Galois if and only if $k_E|k$ is Galois.

Hint: As for (i), write $\ell = k[\overline{x}]$ and let $f \in \mathfrak{o}[t]$ be a monic lift of the minimal polynomial of $\overline{x}$ over k. Use Gauss' lemma to show that f is irreducible. Now let $E = F[x]$ where x is a suitable root of f. As for (ii), write $k_{E_1} = k[\overline{x}]$ with $\overline{x} = x + \mathfrak{m}_{E_1}$ as in the proof of Proposition 1.27. Use Hensel's lemma to show that the minimal polynomial of x over F has a root in E_2.

Problem 3 (4 points): Let F be a complete discretely valued field, and let F^{nr} be the maximal unramified extension of F. Let k denote the residue class field of F, and let k^{sep} denote a separable closure of k. Show that F^{nr} is complete if and only if $k^{sep}|k$ is finite.

Hint: First have a look at the special case $F = k((t))$ and then try to find a strategy which works in the general setting.