Parameter identification problems with non-Gaussian noise

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Consider the inverse problem $S(u) = y^\delta$ for a (possibly nonlinear) operator S between two Banach spaces X and Y and noisy data y^δ. One possible approach for computing an (approximate) solution to the inverse problem is minimizing the Tikhonov functional

$$\mathcal{F}(S(u), y^\delta) + \alpha\mathcal{R}(u)$$

for an appropriate discrepancy term $\mathcal{F}$ and regularization term $\mathcal{R}$. Just as the regularization term incorporates a priori information on the solution, the discrepancy term should be chosen based on a priori information on the noise. Here, the standard L^2 data fitting term is statistically motivated by the assumption of Gaussian noise. For non-Gaussian noise, however, other data fitting terms turn out to be more appropriate. For impulsive noise (appearing in digital image acquisition, e.g., as salt-and-pepper noise) L^1 fitting is more robust. Similarly, uniform noise (e.g., arising from quantization errors) has a statistical connection to L^∞ fitting. Both formulations lead to non-differentiable problems which are challenging to solve numerically.

This talk presents an approach that combines an iterative smoothing procedure with a semismooth Newton method. Specifically, the L^1 norm is replaced with a Huber norm

$$\|u\|_\beta := \int |u(x)|_\beta \, dx, \quad |t|_\beta = \begin{cases} t - \frac{\beta}{2} & t > \beta \\ -t - \frac{\beta}{2} & t < -\beta \\ \frac{1}{2\beta}t^2 & |t| \leq \beta \end{cases}$$

which has a semismooth Fréchet derivative. The L^∞ fitting problem has the equivalent formulation

$$\min_{u,c} c + \alpha\mathcal{R}(u) \quad \text{subject to} \quad \|S(u) - y^\delta\|_{L^\infty} \leq c,$$

for which the Moreau–Yosida smoothing

$$\min_{u,c} c + \alpha\mathcal{R}(u) + \frac{\gamma}{2}\left[\|\max(0, S(u) - y^\delta - c)\|_{L^2}^2 + \|\min(0, S(u) - y^\delta + c)\|_{L^2}^2\right]$$

is introduced. Again, this functional has a semismooth Fréchet derivative. In both cases, under a standard second order condition, the semismooth Newton method converges locally superlinearly for fixed smoothing parameter β or γ, and the family of minimizers of the smoothed problems converge (subsequentially) to a minimizer of the original Tikhonov functionals as $\beta \to 0$ or $\gamma \to \infty$. The semismooth Newton method is thus combined with a continuation strategy with respect to the smoothing parameter, which in practice has a globalizing effect.

The efficiency of this approach is illustrated for the inverse potential problem of recovering u from noisy measurements of $y = S(u)$ solving $-\Delta y + uy = f$.

References

[1] C. Clason, B. Jin *A semi-smooth Newton method for nonlinear parameter identification problems with impulsive noise*, SIAM Journal on Imaging Sciences **5** (2012), 505–536.
[2] C. Clason, L^∞ *fitting for inverse problems with uniform noise*, Inverse Problems **28** (2012), 104007.