UNIVERSITÄT DUISBURG ESSEN

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A proposal for the establishment of a DFG-Priority Program in

Homotopy theory and algebraic geometry

Prof. Dr. Marc Levine (Coordinator)

University Duisburg-Essen – Faculty of Mathematics

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Coordinator

Prof. Dr. Marc Levine
University of Duisburg-Essen
Faculty of Mathematics
Universitätsstrasse 2
45117 Essen

Phone: (0201) 183-3114
Fax: (0201) 183-2426
E-Mail: marc.levine@uni-due.de

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Prof. Dr. Marc Levine

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1 General Information

1.1 Initiators

Name	Field of research within the Priority Program	Institution
Prof. Dr. Hélène Esnault	algebraic geometry	Freie Universität Berlin
Prof. Dr. Marc Levine	algebraic geometry, motivic homotopy theory	Universität Duisburg-Essen
Prof. Dr. Birgit Richter	derived algebraic geometry, homotopy theory	Universität Hamburg
Prof. Dr. Stefan Schwede	derived algebraic geometry, homotopy theory	Universität Bonn

1.2 Theme

The cross-fertilization of homotopy theory and algebraic geometry, especially through motivic homotopy theory, derived algebraic geometry and differential homotopy theory.

1.3 Fields involved

Mathematics: homotopy theory, algebraic geometry, motivic homotopy theory, derived algebraic geometry, differential homotopy theory.

1.4 Keywords

Algebraic geometry, homotopy theory, motivic homotopy theory, derived algebraic geometry, rigid analytic spaces, K-theory, algebraic cycles, algebraic cobordism, string cobordism, Gromov-Witten theory.

1.5 Expected duration

6 years, divided into two 3 year periods.

1.6 Commencement of support

1. half-year 2015

2 Introduction

2.1 Overview

The Priority Program in homotopy theory and algebraic geometry will build upon recent developments in two central pillars of modern mathematics, *algebraic geometry* and *homotopy theory*, to bring the synergistic interactions between these two disciplines to a new level, to draw in mathematicians from both disciplines to profit from and contribute to these interactions, and to exploit these interactions to further both fields. The ideas, methods and techniques that spring from these interactions lead to new and innovative mathematics, going beyond the well-established research in each of the individual fields and breaking down the classical boundaries between homotopy theory and algebraic geometry.

As its name suggests, algebraic geometry has its roots in two classical areas of mathematics, algebra and geometry. Algebraic geometry is concerned with the study of the solution sets of algebraic equations, that is to say, polynomial equations, using not only techniques of algebra, but relying as well on geometric considerations. This two-sided approach to the study of equations can be traced back to Descartes, and is familiar to us all when we graph the solutions to an equation in variables x and y to yield a curve (for example, a circle or parabola) in the plane. Of course, modern algebraic geometry has come a long way since Descartes and now applies methods from all branches of mathematics, from algebra, analysis and topology, and has wide-ranging applications to other branches of mathematics as well as to physics, engineering and even logic.

Homotopy theory is a considerably newer area of mathematics, being an important branch of algebraic topology, the modern development of what is popularly known as “rubber-sheet geometry”, that is, the study of the properties of curves, surfaces and objects of higher dimension which are preserved under operations such as bending and stretching; in homotopy theory one allows additional modifications by “continuous deformation”. Since its creation in the late 19th and early 20th centuries by Poincaré and others, algebraic topology has become an essential component of modern mathematics. Homotopy theory has numerous applications both in and out of mathematics, including aspects of physics such as string theory.

Algebraic geometry and homotopy theory are both recognized as being in the forefront of modern mathematics. These two fields together account for more than a third of all Fields Medals (the mathematician's equivalent of the Nobel Prize) awarded since 1945, and are well represented at research centers throughout the world.

Algebraic geometry and topology have both profited from a long history of fruitful interaction. This Priority Program is based on several recent expressions of this interaction:

• *Motivic homotopy theory*, which for the first time brought the full weight of homotopy theory into the realm of algebraic geometry,
• *Derived algebraic geometry* and the introduction of modern algebraic geometry into homotopy theory
• *Differential homotopy theory and Arakelov theory*, which uses analytic methods to relate and conjoin arithmetic and algebraic invariants

Besides these main fields, this program will rely a great deal on support from classical homotopy theory, as well as having connections with numerous other fields, such as algebraic stacks and moduli problems, representation theory, formal groups, computer algebra, quadratic forms, Gromov-Witten theory and tropical geometry.

2.2 Pathways between algebraic geometry and homotopy theory

2.2.1 Motivic homotopy theory

One basic setting for homotopy theory is the *stable homotopy category*; this is the mathematical universe in which the modern homotopy-theorist works. The initial breakthrough leading to the creation of the subject of motivic homotopy theory was the construction by Morel-Voevodsky [79] of new versions of this category, which brought the "classical" versions from homotopy theory together with inputs from algebraic geometry. Voevodsky's construction of the motivic stable homotopy category enabled one for the first time to work with the basic material of algebraic geometry, solutions of polynomial equations, with the flexibility and power previously only available in homotopy theory.

The motivic theory has been a notable success. The Fields Medalist Voevodsky used these homotopical methods in his proof of the celebrated Milnor conjecture [108], and motivic homotopy theory played an even more central role in his contribution to the proof of the Bloch-Kato conjecture [106], the successful culmination of thirty years of intensive research. Besides these quite spectacular applications, the fact that one could now use the ideas and methods of homotopy theory to solve problems in algebraic geometry has drawn in mathematicians from both fields and has led to a wealth of new constructions and applications, such as classification results for algebraic vector bundles (for example, Asok-Fasel [4, 5]), Vishik's work [104] on the Kaplansky conjecture on quadratic forms (1953), Tzeng's solution [103] of the Göttsche conjecture in Gromov-Witten theory and parts of the so-called MNOP conjectures in Donaldson-Thomas theory (Levine and Pandharipande [71]).

Motivic homotopy theory has been useful for the classical homotopy theorists as well. The recent work of Hill, Hopkins and Ravenel [44] on the Kervaire invariant one conjecture, settling one of the main open problems in stable homotopy theory, used in an essential way the "slice filtration" in equivariant stable homotopy theory, which in turn was inspired by Voevodsky's slice filtration in motivic stable homotopy theory (see section PA1 below).

2.2.2 Derived algebraic geometry

From the point of view of modern algebra, the fundamental object in local algebraic geometry is the *commutative ring*; these are glued together to form the basic objects of global algebraic geometry. Passing from algebra to stable homotopy theory, one replaces a commutative ring with a *commutative ring spectrum*. Driven forward by contributions of the Fields Medalist Kontsevich and many others, the field of derived algebraic geometry is devoted to broadening this transformation from commutative ring to commutative ring spectrum by tranferring the constructions of local and global algebraic geometry to the setting of stable homotopy theory.

Besides giving a better understanding of aspects of algebraic geometry, derived algebraic geometry has been essential to some of the most important recent developments in homotopy theory. A large portion of research in stable homotopy theory in the last two decades has been devoted to *elliptic cohomology theories*, which relate stable homotopy groups, manifolds and modular forms, and have close ties to string theory in mathematical physics. Results from derived algebraic geometry have quite recently enabled researchers in elliptic cohomology to achieve two of their main goals: the construction of a certain universal spectrum called "topological modular forms" (TMF) and the construction of the String orientation of a closely related construction, tmf (the connected cover of TMF). This String orientation refines the Witten genus [116] from cobordism to elliptic cohomology, by which Witten describes the index of the operator known as the supercharge of the supersymmetric nonlinear sigma model. The original construction of TMF, due to Goerss-Hopkins-Miller [39, 46], has recently been revised by Lurie from a point of view that is very appealing to algebraic geometers, as it represents TMF as coming from a derived version of an object that is of central importance in algebraic geometry, the moduli stack of elliptic curves. Current research suggests that there are similar results and constructions related to higher dimensional versions of elliptic curves, involving derived versions of some other familiar and important objects from algebraic geometry, such as abelian varieties, p-divisible groups and automorphic forms.

2.2.3 Differential homotopy theory and Arakelov theory

Differential homotopy theory is based on refining the restriction to manifolds of classical homotopy invariants of spaces by incorporating additional structures, such as differential forms or connections. In one direction, this approach appears in algebraic geometry through Arakelov theory, while index theory forms another important direction. Recently this theory has taken on a "motivic" character, in that objects are constructed as presheaves on various categories of smooth manifolds, just as the motivic theory is based on presheaves on the category of smooth schemes. From both points of view, the exchange of ideas and methods between motivic homotopy theory and differential homotopy is both natural and desirable.

Ordinary differential cohomology has been introduced by Cheeger and Simons [27] as a refinement of ordinary cohomology with integral coefficients, and serves as a target for refined characteristic classes and characteristic forms. This approach is based on an explicit description of the relevant groups by cycles and relations. Using a more homotopy theoretic approach, Hopkins and Singer [49] show how one can refine a generalized cohomology theory, such as K-theory, to a differential one. Connections to physics are discussed in [35] and a uniqueness theorem for differential cohomology theories is achieved in [23]. In [21], a general setup using infinity categories is developed to define differential extensions as a sheaf of spectra on the category of smooth manifolds and is applied to the construction of differential algebraic K-theory of number rings. Generalizations to regular schemes over the integers are given in [24]. Work of Holmstrom-Scholbach [43] uses aspects of motivic homotopy theory to construct Arakelov motivic cohomology and Arakelov K-theory; a similar approach is used by Hopkins-Quick [48] in their construction of "Deligne"-cobordism.

2.3 Building upon success

Recent developments (see §4 for details) have made possible an exchange of ideas and methods between these areas at a level that is unique in the history of the subject. Effective application of these developments requires mathematicians working in algebraic geometry, motivic homotopy theory, homotopy theory, differential homotopy theory and derived algebraic geometry to come together, to learn from one another and to work with each other on common projects. Many individual researchers and research groups throughout the world have contributed to these recent advances and are now starting to use these new results to make the leap to the next level in their respective fields. However, there is at present *no single group of researchers* that brings together experts in all the fields that are taking part in these exciting developments. The *long-term goal* of this Priority Program is the creation of just such a collaborative research network. This network, comprised of top researchers in all five fields: algebraic geometry, homotopy theory, motivic homotopy theory differential homotopy theory and derived algebraic geometry, will be *unique* among mathematical research groups world-wide.

This Priority Program in homotopy theory and algebraic geometry sets up a framework, via targeted program areas, promotion of young researchers through summer schools and workshops, and conferences on the international level, that will promote collaboration, encourage the participation of new groups of young mathematicians, create new mathematics of interest to the wider mathematical community, and in doing so, build the links needed to form an effective mathematical network.

We have identified *key program areas* which build upon recent progress in algebraic geometry and homotopy theory and which hold promise of significant progress in the near and middle term. In our detailed scientific program (see §5), we outline specific *goals for short- and middle-term progress* in these areas. This Priority Program will focus efforts, coordinate individual projects and facilitate larger collaborations through the use of *planning workshops* held at the start of each of the three-year funding period. This coordination and exchange of ideas will continue through the funding period, aided by frequent workshops, summer schools and conferences, allowing us to link together the efforts of individuals and smaller groups in a way that up to now has been either non-existent or at best accidental. At the same time we will be making our activities known to the international mathematics community through our individual contacts, exchange programs supported by the Priority Program and yearly conferences held by this Priority Program. This will encourage mathematicians from outside of Germany, especially early career researchers, to come and take part in the research activities of the Priority Program.

Ultimately, this Priority Program will establish in Germany a vibrant and long-lasting network, forming a community of researchers within Germany that will be unique in the mathematical world, and resulting in the creation of new and exciting mathematics that will continue to draw in young researchers for a long time to come. We believe that the creation of such a collaborative network is the most effective way for mathematics in Germany to rise to the forefront of this exciting field of mathematics and this Priority Program is an excellent vehicle for accomplishing this task.

3 Summary of the Research Topics

This Priority Program comprises areas of algebraic geometry and homotopy theory which import ideas, methods or results from one field to advance the other, or which develop methods of interaction between these fields. The interactive links between algebraic geometry and homotopy theory have been built up in the areas of motivic homotopy theory, differential homotopy theory and derived algebraic geometry. Progress in motivic homotopy theory and derived algebraic geometry over the past ten years have made possible a number of new directions in these fields, as well as applications to algebraic geometry and homotopy theory.

The goal of this Priority Program is to further research in algebraic geometry and homotopy theory by using this interplay between these fields. As a further goal, this Priority Program seeks to promote the development of classical homotopy theory, motivic homotopy theory, differential homotopy theory and derived algebraic geometry, including the cross-fertilization of these areas.

To focus the activities of this Priority Program in particularly promising directions, and to describe more precisely the recent work upon which this Priority Program will build, we have delineated five separate program areas (**PA**). These main program areas are

PA 1 Chromatic and motivic aspects of stable homotopy theory
PA 2 Equivariant homotopy theory
PA 3 Classification problems
PA 4 Cobordism
PA 5 Unstable homotopy theory

In the description of the scientific program, §5, we briefly illustrate a total of 24 separate example project directions. The following chart describes the expected contributions of the main research groups to the various projects described here:

Table of program areas

	PA 1.					**PA 2.**					**PA 3.**			**PA 4.**					**PA 5.**					
	1	2	3	4	5	1	2	3	4	5	1	2	3	1	2	3	4	5	1	2	3	4	5	6
Motivic homotopy theory	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Derived algebraic geometry	x	x		x	x	x	x				x	x	x	x										x
Differential homotopy thy.	x			x		x	x	x	x		x						x					x		
Algebraic geometry	x	x	x				x	x	x	x		x		x	x	x	x	x	x	x	x	x	x	x
Classical homotopy theory	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Other fields[1]	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x

4 Reasons for funding this Priority Program

The main points arguing for the establishment of the Priority Program in homotopy theory and algebraic geometry are briefly stated as follows:

• Both algebraic geometry and homotopy theory are central areas of modern mathematics. Both areas are historically and currently well-represented in Germany. There is at present no German network devoted to aspects of homotopy theory.

• The area of concentration of this Priority Program, namely, the links between algebraic geometry and homotopy theory represented by motivic homotopy theory, differential homotopy theory and derived algebraic geometry, has seen striking developments in recent years, with researchers in Germany making significant contributions. These results form the basis for the next surge in activity in these areas. Research in these areas will

[1] such as: number theory, formal groups, representation theory, computer algebra, tropical geometry

continue to lead to new and innovative mathematics, going beyond the well-established research in each of the individual fields and breaking down the classical boundaries between homotopy theory and algebraic geometry.

• International interest in these recent developments is evidenced by recent conferences and workshops, as well as several group programs and individual funded projects in the U.S. and Europe. There are several large networks in the US and Europe devoted to these topics.

• The planned Priority Program will effectively channel efforts to utilize these recent developments by bringing researchers from the different areas together via joint projects as well as through workshops, conferences summer schools and exchange programs. The particular problem areas delineated in this program have been carefully chosen to maximize this cooperation and at the same time take advantage of the existing interests and skills of researchers in Germany.

• The program will heighten the international stature of this area of research in Germany and will draw in researchers from outside Germany.

• The development program, workshops, summer schools, conferences, will further a long lasting establishment of this area in the German research landscape, as well as drawing in potential new researchers to this field at the doctoral and post-doctoral levels.

In more detail:

• **Recent breakthroughs form the basis for new development**. A surprising number of recent breakthroughs and successful completions of foundational programs in the boundary areas between algebraic geometry and homotopy theory set the stage for a new surge of activity. These recent developments include:

1. The construction of a geometric theory of algebraic cobordism [70], and its application to problems in Donaldson-Thomas theory [71] and Gromov-Witten theory [63], for example, the proof of Göttsche's conjecture by Tzeng [103].
2. Vishik's symmetric operations in algebraic cobordism and their application to Kaplansky's 1953 conjecture on the u-invariant of quadratic forms [104]

3. The computation of the essential dimension of p-groups by Karpenko-Merkurjev [58]
4. Ayoub's construction of Grothendieck's six operations in the motivic stable homotopy category [10] and the motivic homotopy category for rigid analytic spaces [11].
5. Voevodsky's introduction of the slice tower in motivic homotopy theory [109, 110] and the verification of nearly all of Voevodsky's conjectures concerning the slice tower [47, 52, 65, 67, 86, 87, 98, 99, 107, 111].
6. The solution of the Kervaire invariant one conjecture by Hill-Hopkins-Ravenel [44], relying on equivariant homotopy theory and ideas taken from Voevodsky's slice tower construction.
7. Work of Lawson-Naumann on Brown-Peterson theory [62].
8. Morel's computation of motivic stable π_0 of the sphere spectrum [78], followed by first computations of motivic unstable π_1 (Asok-Morel [7]) and higher motivic homotopy groups, with applications to vector bundles (Asok-Fasel [4, 5])

• **The area is actively pursued on an international level**. The interaction of homotopy theory and algebraic geometry is one of the key areas of research at many of the top mathematics departments throughout the world, seen for example, in the work of Harvard professors Michael Hopkins (ICM[2] speaker 1994, ICM plenary speaker 2002, National Academy of Sciences Award in Mathematics, 2012) and Jacob Lurie (ICM speaker 2010), who are recognized leaders in this area.

These topics form the subject of several high-level workshops and special programs, in the recent past as well as planned for the near future. Programs discussing these and related works include the special program in "$\mathbb{A}^1$-Homotopy Theory and its Recent Developments" at the Institute for Advance Study (Princeton, 2009-10), the week-long program in algebraic K-theory and equivariant homotopy theory at the Banff Institute (Feb. 2012), and the upcoming semester program in Algebraic Topology at the MSRI[3], Berkeley (2014).

The NSF has recently funded Focused Research Groups in *Homotopical methods in Algebra* (Friedlander, Haesemeyer, Walker, Weibel) and *Homotopy Theory: Applications and New Dimensions* (Hopkins, Lurie, Miller, Barwick, Behrens), both of which deal with topics closely paralleling those selected for this Priority Program, as well as numerous individual grants in this same area. There are also networks devoted to such research areas in France and Norway (see §8 for details and possible collaborations).

[2]International Congress of Mathematics
[3]Mathematical Sciences Research Center

• **The Priority Program creates value beyond individual projects**. Although homotopy theory and algebraic geometry share to a large extent a common background, the problems, constructions and methods in one area are often not familiar to those in another. Central goals of this Priority Program are to broaden this common background, to accelerate the already existing cross-fertilization and collaboration between these areas, and to promote the exchange of ideas, methods and problems between algebraic geometers and homotopy theorists, on the level of both junior and senior researchers.

The areas covered by this Priority Program are actively pursued in Germany, however, researchers are scattered in small groups throughout the country, with no concentration at any one university. This Priority Program will link these geographically scattered groups, intensifying their activities in spite of their small size.

This Priority Program is designed to create new collaborations and break down existing barriers between fields. The program areas identified in our scientific program form an emerging field that draws from five different established fields: algebraic geometry, classical homotopy theory, motivic homotopy theory, differential homotopy theory and derived algebraic geometry. Due to the complexity of the methods in these established fields, cooperation among experts in each of these fields is necessary for further progress. The coordinated planning of projects, combined with the workshop, summer school and conference program and the exchange of students and post-docs between the various research locations will create a community of researchers from what is now largely a collection of isolated groups. This will foster the exchange of ideas between these groups and the different fields they represent far more than would occur if researchers would continue to work in isolation.

This Priority Program will thereby create a collaborative network of researchers in algebraic geometry, classical homotopy theory, motivic homotopy theory, differential homotopy theory and derived algebraic geometry, which will be distinguished from other international research groups working in these areas in that it will actively promote collaborations between researchers in these fields. This network will be the unique collection of researchers that can draw upon expertise in all these research fields, *creating value far beyond a collection of isolated individual projects.*

• **Long-term development, sustainability and international visibility** The Initiators of this Priority Program have had a wide experience in organizing conferences at the international level, both inside and outside Germany, they have received numerous grants for research visits at top-level research institutions throughout the world, and are active as editors of highly respected mathematical journals.

Due to its unique nature, this Priority Program will significantly heighten the visibility of this internationally active area of research in Germany. Combined with selection of research areas and the program of conferences, workshops and summer schools, the Priority Program will be able to attract excellent young mathematicians from within and from outside Germany and build up the existing groups in Germany. The prospects for securing this area as a lasting component of the German mathematical landscape are excellent.

The program can draw from researchers located at more than 20 universities throughout Germany for active participation in the Priority Program. In addtion to senior researchers, a sizable number a junior researchers have also expressed interest in this program, giving a career-stage distribution suitable for both both stability and long-term sustainability of the Priority Program.

• **Suitability of the program**. The Initiators form a body of experts for a wide range of research areas covered by this Priority Program.

Professor Esnault: Hélène Esnault is a leading expert in areas of algebraic geometry such as: rational points on algebraic varieties, algebraic cycles, algebraic vector bundles, flat connections and motives. She has a wide knowledge of many other fields in algebraic and arithmetic geometry. Her expertise will be invaluable in helping guide the aspects of this Priority Program that deal with connection with and applications to algebraic geometry.

Professor Levine: Marc Levine has worked in various aspects of algebraic geometry, dealing with algebraic cycles and higher algebraic K-theory, since the beginning of his career. In the last twenty years, he has concentrated his research in the study of motivic cohomology and motivic categories and more recently in the area of motivic stable homotopy theory. Together with Fabien Morel, he is the originator of algebraic cobordism.

Professor Richter: Birgit Richter works on stable homotopy theory with a focus on structured ring spectra and their algebraic properties. She studies these via algebraic K-theory and related invariants such as Picard and Brauer groups. Her focus in the Priority Program will be on derived algebraic geometry, in particular on the transfer of methods from algebraic geometry to stable homotopy theory and the investigation of properties of algebraic K-theory such as Galois descent.

Professor Schwede: Stefan Schwede works in algebraic topology, specifically in stable homotopy theory and equivariant homotopy theory. Schwede studies foundational questions about structured spectra and structured ring spectra; he is interested in triangulated categories that arise from topology, and how they compare to triangulated categories that arise in algebra and algebraic geometry. He brings expertise in homotopy theory and derived algebraic geometry to this Priority Program.

This Priority Program has been planned with the active participation of a large majority of the prospective participants and as such accurately reflects the research interests of its potential project applicants. The type of activities planned for this Priority Program, and the number of internationally respected participants, located at more than 15 universities through Germany, all fit perfectly into the concept of the Priority Program, as set out by the DFG.

5 Scientific Program

PA1 Chromatic and motivic aspects of stable homotopy theory

Chromatic homotopy theory refers to various aspects of the Adams-Novikov spectral sequence, and its interplay with the theory of formal group laws. This is a central part of modern stable homotopy theory (see e.g. [90]). Recently, aspects of chromatic homotopy theory have been introduced into the motivic setting, see [34, 54, 64, 80, 85]. Additional information comes from the point of view of differential homotopy theory through recent work of Bunke-Naumann [22]. Recent developments in motivic homotopy theory point to a new point of view on chromatic homotopy theory, incorporating one of the central themes in algebraic geometry: Grothendieck's coniveau filtration.

PA1. Previous work

One of the most powerful computational tools to compute the stable homotopy of spheres is the Adams-Novikov spectral sequence [89]. Its E_2-term is the quasi-coherent cohomology of the stack of formal groups with coefficients all twists of the canonical line bundle. Once localized at a prime, the structure of the abelian category of quasi-coherent sheaves is governed by the height filtration of the stack, and this has been discovered in the guise of structural results for BP_*BP-comodules in the 1970s by Landweber, Miller, Ravenel, Morava and others [77]. Ravenel went on to formulate precise conjectures which roughly speaking say that this structure should be visible on the level of (finite, p-local) spectra. All except one of these conjectures were settled by Hopkins and others in the mid 1980s and constitute what now is commonly referred to as the chromatic approach to stable homotopy [90]. This circle of ideas received new attention in the early 1990s with Goerss's and Hopkins's multiplicative ring spectrum project, which tries to explain the above results by directly lifting the algebra of the stack of formal groups into the homotopy category (of E_∞-ring spectra) [38]. One of the first successes of this approach, due to Hopkins-Miller-Goerss and Devinatz is that the local rings of the stack can be lifted. This leads to an E_∞-action of the Morava stabilizer group on Lubin-Tate theory [33].

Replacing spaces and spectra with presheaves on the category $\mathbf{Sm}/S$ of smooth schemes over a chosen base-scheme S, performing the appropriate localization, and replacing S^1 with $\mathbb{P}^1$, one arises at the *motivic* version of classical homotopy theory. Voevodsky defined the *slice tower* in motivic stable homotopy theory as an analog to the classical Postnikov tower, with $\mathbb{P}^1$-connectivity replacing the usual notion. He presented a series of conjectures [109] on the layers of the slice tower for a number of objects of interest, such as the algebraic K-theory spectrum $\mathcal{K}$, the algebraic cobordism spectrum MGL and the sphere spectrum, as well as some conjectures regarding the convergence of the slice tower for finite spectra and a general structure result for the layers. Through work of Voevodsky [111], Levine [67], Hopkins-Morel and Hoyois [47, 52], Naumann, Panin, Röndigs, Østvær [86, 98], most of these conjectures have been proven in varying levels of generality. This foundational work sets the stage for the next phase of development. At present, the full range of the classical chromatic theory is not available in the motivic setting.

Hu-Kriz-Ormsby [54, 55] and Dugger-Isaksen [34] have constructed and studied motivic liftings of the Adams, Adams-Novikov and May spectral sequences. Besides proving convergence properties, they have made extensive explicit computations, relying in part on the extra grading to give limitations on the differentials, which makes computations significantly easier. Playing the motivic and classical spectral sequence off each other, Dugger-Isaksen have extended the range of the explicit computations for the differentials in the classical sequences.

PA1. Scientific goals

1. A motivic version of the chromatic package. This is a long-term goal, to recast the entire chromatic theory in the motivic setting. There are obstructions to a straightforward translation of the classical theory into a motivic version, and it is not clear what reasonable expectations one should have. However, using the classical theory as a guide one can certainly uncover interesting properties of the motivic stable homotopy category. The motivic version of parts of the Goerss-Hopkins multiplicative ring spectrum project are discussed more fully in §PA3.

One expects that the classical theory is most closely reflected in the motivic setting for the base scheme an algebraically closed field, for instance, results of Levine [64] show that the motivic theory of torsion objects is very closely related to that of their Betti realizations in the algebraically closed case in characteristic zero.The next step is to fields of finite cohomological dimension, with the most difficult and most interesting case being that of fields with a real embedding, leading to non-torsion elements in the Witt group. One can hope to study the case of fields of finite cohomological dimension by comparison with the theory over the algebraic closure, which brings one to the equivariant theory discussed in §PA2. For fields with a real embedding, the homotopy theory acquires a completely new character, which one already sees in the π_0 of the sphere spectrum, through Morel's identification of this with the Grothendieck-Witt group of the field [78].

In the case of fields of finite cohomological dimension, one knows that the distinction between motivic stable homotopy theory and motivic "homology theory" come down to essentially torsion phenomena; this may be viewed as an analog of Serre's theorem that the higher stable homotopy groups of spheres are all torsion. As this is no longer the case if the field admits a real embedding, one can ask if this torsion principle still holds for the *higher* homotopy sheaves of the sphere spectrum. This is probably the first question that needs to be answered in the study of the motivic stable homotopy category over such fields.

2. The algebraic geometry of the Adams-Novikov spectral sequence. As mentioned above, it is now known that the Adams-Novikov spectral sequence agrees with the slice spectral sequence for the weight zero part of the sphere spectrum. The next step is to investigate in more detail the underlying algebraic geometry of the Adams-Novikov spectral sequence and the chromatic tower as reflected by this connection.

One should also use the classical structures to help understand the motivic side. It has up to now been very difficult to compute the layers in the slice tower for the suspension spectrum of a given variety X, even for smooth projective X. Using the various base-change functors, the essential case is the sphere spectrum over an arbitrary field, where Voevodsky's (by now verified) conjecture [109] on the slices of the sphere spectrum give the answer. The motivic versions of the May spectral sequence and the motivic chromatic filtration give a structure to the slices for the sphere spectrum that is not at all apparent from the original construction. Using Ayoub's four-functor formalism [10], one can hope to extend this "Adams-Novikov package" on the Tate-Postnikov layers from fields to a general variety.

3. Arithmetic aspects of classical homotopy theory. Since classical homotopy theory sits as a full subcategory of the motivic stable homotopy category over $\bar{\mathbb{Q}}$, one should be able to utilize the arithmetic structure inherent in $\mathcal{SH}(\bar{\mathbb{Q}})$ to attack problems in classical homotopy theory. For instance, the natural action of $\mathbb{Z}_p^\times$ on $BP(p)$-theory should arise from the cyclotomic character of $\mathrm{Gal}(\mathbb{Q})$; it would be of interest to see how the p-local actions fit together to give an action of the full Galois group on the integral spectral sequence. One would like to apply some of the more arithmetic aspects of formal groups (e.g. p-divisible groups) to chromatic homotopy theory in a direct way via motivic homotopy theory.

4. Analytic invariants. Differential homotopy theory has been already used by Bunke-Naumann [22] to construct invariants detecting parts of the chromatic tower. One would like to reinterpret and extend these invariants as realization functors in Deligne-type cohomology theories. Bunke-Tamme [24] have constructed cycle class maps into differential algebraic K-theory for quite general schemes over number rings; following the recent work of Spitzweck [97], there should be an extension of this construction to motivic cohomology for regular schemes over number rings. There is at present very little use of classical index theory in the study of motivic invariants; this is an area ripe for development. For instance, one should extend the role played by analytic torsion in the Arakelov Riemann-Roch theorem to a wider range of motivic theories, such as algebraic cobordism.

5. Computations. The works of Dugger-Isaksen [34] and Isaksen-Shkembi [57] point out the computational advantage flowing from the motivic aspects of classical homotopy theory. New aspects of the chromatic theory now seems approachable by this method, leading to numerous projects in homotopy theory. There would also be algebraic, algorithmic and computer aided aspects to these types of computations, touching on fields such formal groups, representation theory and computer algebra.

PA2 Equivariant homotopy theory

Equivariant stable homotopy theory studies spaces with group actions, up to suspension by the spheres of orthogonal representations. The most successful and widely used frameworks apply to actions of finite groups and compact Lie groups. The corresponding motivic foundations, based on actions of finite group schemes or algebraic groups on motivic spaces, are currently under investigation.

PA2. Previous work

One recent push for equivariant stable homotopy theory came when in 2009 Hill, Hopkins and Ravenel announced a proof that there do not exist manifolds with Kervaire invariant 1 in dimensions $2^k - 2$ for any $k \geq 8$. Their proof makes serious use of equivariant stable homotopy theory, in particular a C_2-equivariant refinement of the complex bordism spectrum, the multiplicative norm construction and a slice filtration inspired by Voevodsky's slice filtration in motivic stable homotopy theory.

Another current strand of research is a systematic study of *global* equivariant stable homotopy theory. Here 'global' refers to simultaneous and compatible actions of a whole class of groups, for example all finite groups or all compact Lie groups, on a spectrum. A rigorous framework is given by the category of orthogonal spectra, endowed with a new 'global' model structure that takes the Mackey functor valued homotopy groups for all groups in the desired class into account. This gives a natural refinement of traditional stable homotopy theory; one aim here is the definition and study of 'hyper-commutative' ring spectra, where 'hyper-commutative' refers to the expectation that the global model structure should lift to the category of commutative orthogonal ring spectra.

The topological complex bordism spectrum has two universal properties: on the one hand, it is the universal multiplicative complex oriented cohomology theory; on the other hand, its coefficient ring carries the universal 1-dimensional commutative formal group law. The equivariant and motivic extensions and refinements of these properties are still under investigation. In equivariant homotopy theory, one has to distinguish between geometric equivariant bordism (made from bordism classes of stably almost complex G-manifolds) and homotopical bordism (represented by an equivariant Thom spectrum over Grassmannians in a complete G-universe). This difference also occurs in motivic homotopy theory, in the form of homotopical versus algebraic bordism. The homotopical equivariant bordism is the universal equivariant cohomology theory that is complex oriented, but there is not yet a really satisfactory theory of equivariant formal group laws.

One approach to an equivariant motivic homotopy theory has been discussed in [26]. The slice filtration as exploited in [53] is one connection between equivariant and motivic homotopy theory. Another connection comes from the observation that the topological realization function from the motivic homotopy category of a field has a natural lift to an equivariant homotopy category with action of the Galois group. This has been exploited in the study of hermitian K-theory and Real algebraic K-theory (see for instance [18, 53]).

Real homotopy theory concentrates on the $\mathbb{Z}/2$-action coming from complex conjugation,and is thus related to aspects of algebraic geometry over the real numbers.

PA2. Scientific goals

1. Foundations for equivariant homotopy theory. Well-known structures in classical and motivic homotopy theory, such as algebraic K-theory, should have natural extensions to the setting of global stable homotopy theory. For such constructions, one requires a new framework for global stable homotopy theory: the category of orthogonal spectra, endowed with a new 'global' model structure that takes the Mackey functor valued homotopy groups for all groups in the desired class into account. An obvious questions to settle is the precise relationship between the frameworks for global homotopy theory by Greenlees-May and Bohmann and the approach via orthogonal spectra.

One should see that the well-known smash product for orthogonal spectra can also be derived globally. Moreover, the global model structure should lift to the category of commutative orthogonal ring spectra; the fibrant objects there may be thought of as 'hyper commutative ring spectra', as they refine commutative equivariant ring spectra and possess a host of structures, such as a global Green functor with power operations and norm maps on the homotopy groups. Global algebraic K-theory ought to be a prominent example of such a hyper-commutative ring spectrum, introducing derived algebraic geometry into the picture.

There should be a corresponding differential/analytic version of this global stable homotopy theory as well.

2. Equivariant motivic cohomology theories. Concerning equivariant motivic homotopy theory, there should be a reasonable motivic stable model structure on linear spectra which is similar to the one for topological orthogonal spectra. The equivariant version of this model structure should then be a particularly useful strictly monoidal model for doing equivariant stable motivic homotopy theory. It would be interesting to see how existing explicit constructions of equivariant (co-)homology theories such as equivariant algebraic K-theory and K-homology relate to the corresponding equivariant (co-)homology theories represented in the equivariant stable motivic homotopy category. Finally, one should establish the correct Bredon-style equivariant motivic cohomology theory, including a spectral sequence relating it to equivariant algebraic K-theory, which then should be compared to the one of Levine and Serpé [72], or a G-equivariant version of the Grayson spectral sequence [41].

3. Real homotopy theory. The special case of Real homotopy theory and its motivic counterpart have been used by Kriz-Hu-Ormsby [53] to study hermitian K-theory. One should extend their method by constructing an equivariant version of the slice tower, which would give a better understanding of the relation of hermitian K-theory and Chow-Witt theory.

The extension of differential invariants of K-theory to the setting of hermitian K-theory, as well as a general extension to a theory of real differential homotopy theory would be desirable.

4. Real and tropical enumerative geometry. Computations (Hoyois-Fasel-Levine) of Euler characteristics in the motivic stable homotopy category of homogeneous spaces for reductive groups give an example of an interesting connection between motivic homotopy theory over the reals, real enumerative geometry and tropical geometry. In fact, one can make the computation of this Euler characteristic, which turns out to be the rank of the Chow groups of even codimension minus the rank of the Chow groups of odd codimension, by a homotopy-theoretic approach, by a tropical approach, and by computing an intersection product in the Chow-Witt groups. This confluence points the way to a three-pronged approach to problems in real enumerative geometry, in which the techniques available to motivic stable homotopy theory complement those arising from the more well-understood methods from real intersection theory and tropical geometry.

Similarly, using analytic invariants arising from equivariant realization functors would introduce methods from equivariant differential homotopy theory into the picture.

One could also shift the emphasis from the reals to other interesting fields, such as p-adic fields, using the motivic approach to replace classical real enumerative geometry with information arising from quadratic forms, and then specializing back to the Grothendieck-Witt rings of p-adic fields.

5. Hermitian K-theory and Chow-Witt groups. The use of the slice filtration in the equivariant case by Hu-Kriz-Ormsby [53] has given one of the first applications of this technique to the study of Hermitian K-theory. The Chow-Witt groups of a scheme, defined as the cohomology of the Chow-Witt sheaves, are thought to be a "Witt"-pendant to the classical Chow groups. As such, one would like to establish a close relation with hermitian K-theory, the Witt-pendant to algebraic K-theory and the Chow-Witt theory. Although the slice tower for algebraic K-theory does give a close relation with motivic cohomology, the situation for hermitian K-theory is much more complicated and unclear. Investigating an equivariant version, relying on the connection between hermitian K-theory and real algebraic K-theory, would be a promising approach.

In another direction, Chow-Witt groups could be used to give a "motivic" viewpoint on real enumerative geometry; this is related to the project PA2.4. The idea here would be reinterpret results in real enumerative geometry as computations in Chow-Witt groups for varieties over the rationals and then specializing to identities in the Witt groups of $\mathbb{Q}_p$. This would yield some interesting p-adic interpretations of results from real enumerative geometry. As a test case, one could try to give such a treatment of the recent results of Okonek-Teleman [83, 84].

PA3 Classification problems

Via surgery the classification of manifolds can be achieved if one understands bordism relations in various forms. These bordisms are again to be understood via homotopy theory with the help of various spectra like KO (real K-theory), or *TMF* (topological modular forms). The latter can only be constructed by using derived algebraic geometry, that is, algebraic geometry over enriched ring spectra. Other ways of understanding the subtleties of such theories is via the K-theory of ring spectra; this too requires the use of derived algebraic geometry.

This gives two main points of contact between algebraic geometry and topology: the use of derived algebraic geometry to accomplish the goals of topology, and the extension of the methods of topology to the motivic level, in order to better understand the algebraic geometry of smooth varieties.

PA3. Previous work

A *String structure* on a smooth manifold M is a lift of the classification map of its stable tangent bundle to the 7-connected cover of BO. String structures on M are related to Spin structures on the free loop space LM. The bordism ring of String manifolds coincides up to dimension 7 with the homotopy groups of the spheres and also plays a role in surgery theory, that is, in the classification of manifolds. However, a complete calculation has not been established yet.

Spin-bordism has been calculated. By a theorem of Anderson, Brown and Peterson the Thom spectrum *MSpin* splits at the prime 2 into a sum of connective covers of KO and an Eilenberg-Mac Lane part. In particular, two Spin manifolds are bordant if and only if all KO and Stiefel Whitney classes coincide. A corresponding result for the Thom spectrum *MString* is not yet known, but such a splitting would be crucial for the understanding of *String*-manifolds. Although KO is not strong enough to detect all String bordism classes, the theory of topological modular forms, *TMF*, together with its derivates at various levels, seems to have this property. The analogy between *MSpin* and KO on the one hand and *MString* and *TMF* on the other hand has been extensively studied during the last decade by Ando, Hopkins, Laures, Strickland and others [1, 2, 59, 60]. There has been substantial progress, but a bordism classification and the relation of *TMF* to index theory and particle physics is still not completely worked out.

The theory *TMF* can also lead to a better understanding of the $K(2)$ local homotopy category. There is a resolution which generalizes the fibre sequence of the self map $\psi^g - 1$ of KO from the $K(1)$ to the $K(2)$ local sphere. The resolution involves a spectrum Q which can be viewed as a higher analogue of the J-spectrum and a duality map which has not been fully understood. Also its relation to the f-invariant still has to be worked out.

For chromatic levels $n \geq 2$ Behrens and Lawson [16] investigate higher dimensional abelian varieties and automorphic forms in homotopy theory which give rise to spectra of topological automorphic forms (TAF). These can lead to even higher invariants, resolutions of $K(n)$-local spheres and possibly to the understanding of bordism spectra which are yet closer to the sphere spectrum. Other approaches (see [101]) use other moduli stacks, such as that of polarized K3 surfaces, and other constructions of formal group laws, to construct spectra applicable to higher chromatic levels.

Another area of research that implements ideas and concepts from algebraic geometry into homotopy theory is algebraic K-theory of ring spectra. Waldhausen extended the study of algebraic K-theory to ring spectra and showed that the algebraic K-theory of the sphere spectrum and more general spherical group rings captures information about geometric topology [113].

In classical stable homotopy, the construction of a spectrum TMF (topological modular forms) was finally achieved in the last couple of years by Goerss-Hopkins-Miller [15], [39] and Lurie [73]. This spectrum represents a deep relationship between modular forms and more generally the stack of elliptic curves on the one hand side and $L_2\mathbf{S}$, that is the second chromatic layer in stable homotopy theory on the other side. The construction of TMF relies (notably in Lurie's approach) on a number of lifts of constructions in algebraic geometry, such as the moduli stack of elliptic curves, the cotangent complex and deformation theory, from commutative rings to commutative (that is E_∞-) ring spectra. It is natural to ask to what extent these constructions may be generalized to the motivic setting, *cf.* [92].

The paper [51] provides the necessary model structures on motivic ring spectra and shows that these satisfy a motivic version of the axioms of the obstruction theory of [39]. In the considerably easier case of chromatic height one a motivic generalization of the desired result has been recently established as a combination of [37], [51] and [100]. The existence of motivic elliptic cohomology theories via a motivic Landweber exactness theorem has been proved in [80], , but rigidifying this to a diagram in commutative motivic ring spectra remains a challenge.

Symmetric monoidal categories of spectra are available since the 90's, and several concepts from algebraic geometry and number theory have successfully been transferred to the setting of structured ring spectra. This allows one to relate arithmetic properties of ring spectra to their algebraic K-theory. First examples have been studied by Ausoni and Rognes, who investigated the algebraic K-theory of connective complex K-theory, ku, and related ring spectra in detail [8, 9]. This helped to show that algebraic K-theory of ku is a form of elliptic cohomology and classifies categorified vector bundles on spaces [12, 13]. The guiding questions here were whether applying algebraic K-theory actually raises the chromatic type as predicted by Rognes' chromatic red shift conjecture and whether there is a suitable notion of Galois descent for the algebraic K-theory of ring spectra, *cf.* [25].

The most successful strategies for computing K-theory use trace maps mapping to topological Hochschild and topological cyclic homology. Until recently, these trace methods covered only the case of connective ring spectra, while Galois theory mainly involves non-connective spectra. To remedy this, Rognes and Sagave studied how to adapt the notion of logarithmic structures to the setup of ring spectra [93].

PA3. Scientific goals

1. Study MString and characteristic classes for String bundles. At the prime 2 one conjectures that

MString additively splits in a copy of *tmf*, some suspended copies of *tmf* with level structures and an Eilenberg-Mac Lane part. For such a splitting one needs a good understanding of the *tmf* characteristic classes for *String* bundles. A good start for the study of these is the Kitchloo-Laures-Wilson sequence of Hopf algebras

$$K(2)_*K(\mathbb{F}_2,2) \rightarrowtail K(2)_*K(\mathbb{Z},3) \longrightarrow K(2)_*BO\langle 8\rangle \twoheadrightarrow K(2)_*BSpin.$$

This leads to a computation of $TMF(3)^*BO\langle 8\rangle$ and $TMF_1(3)^*BO\langle 8\rangle$; the first goal is to extend these results to *TMF* itself or to its connective version *tmf*. One could try to approach this problem through methods of equivariant homotopy or using other approaches to constructing spectra from moduli stacks.

While these are computations in classical homotopy theory, they may benefit from being looked at motivically, as in the case of the Adams-Novikov spectral sequence discussed in §PA1. The goal here is to construct motivic liftings of $MU\langle n\rangle$ or even $MO\langle n\rangle$ to learn about the classical Adams spectral sequence computing $\pi_*MO\langle 8\rangle$, refining the existing motivic lifting MGL of MU. For a start, this requires finding correct connective covers of $\Omega^\infty(MGL) = \mathbb{Z}\times BGL$ and studying the resulting motivic Thom spectra.

***2. Towards a motivic lift of* TMF**. A key observation due to Goerss and Hopkins is that the classical Landweber exactness theorem can be phrased as lifting the structure sheaf on the stack of formal groups from a sheaf in rings to a sheaf in commutative ring spectra *up to homotopy*. The motivic extension of Landweber exactness is available through [80]. All relevant applications require rigidifying this construction *coherently up to all higher homotopies*. The most straightforward approach to this is through obstruction theory and works for complex K-theory, Lubin-Tate theories and *TMF*. A motivic variant of this is in its infancy but has already been applied successfully to the easiest test case of the algebraic K-theory spectrum KGL. It should not be hard to extend this to obtain the desired sheaf of motivic commutative ring-spectra on the moduli stack of formal groups of height 1. Lurie's approach is to observe the classical moduli problems remain meaningful over commutative ring spectra. The details of this are daunting and a serious prediction of whether this approach will be suitable in the motivic context will have to await the detailed account of Lurie's work.

In somewhat more detail, one should investigate the problems of the following list related to the Goerss-Hopkins-Miller approach:

i) establish a general obstruction theory for motivic ring spectra (à la Robinson [91] and Goerss-Hopkins [39]).
ii) apply this to construct E_∞-structures on motivic Lubin-Tate spectra and the actions of Morava stabilizer groups on them.
iii) construct homotopy fixed-point spectra à la Devinatz-Hopkins [33]. That is, construct an action of $\mathbb{Z}_p^*$ on $\mathrm{KGL}^{\wedge p}$ (p-completed motivic K-theory), and analyze the relationship between its homotopy fixed points and the $K(1)$-local motivic sphere.
iv) study multiplicative properties of localized quotients of MGL, (non-highly structured and highly-structured), that is, motivic Morava-K-theories (this would solve a problem of Voevodsky).
v) In recent work [3] it was shown that Morava K-theories admit a unique structure of S-algebra; a surprise since it has been known for a while there are uncountably many such structures of MU-algebra. One would like a motivic generalization of this result which should also yield multiplicative properties of Voevodsky's conjectured spectral sequence computing algebraic Morava K-theories from motivic cohomology.

Lurie's approach for defining TMF uses a derived algebraic geometry construction of the sheaf of E_∞-ring spectra on the moduli stack of elliptic curves. Although the details of this approach have not yet appeared, it would still be interesting to see if portions of Lurie's constructions go through in the context of motivic derived algebraic geometry. Here one considers (cellular) motivic E_∞-ring spectra for building blocks of motivic derived algebraic schemes and more generally (Deligne-Mumford) stacks. There are already candidates for the basic concepts of the theory, e.g. of motivic derived algebraic groups. The main steps on the motivic side will be the construction of the stack of pre-oriented motivic derived elliptic curves and the control over the passage from this stack to the stack of oriented motivic derived elliptic curves.

Many of the problems encountered in the construction of motivic versions of TMF concern motivic generalizations of chromatic homotopy theory in general, which is of independent interest (as noted in §PA1.5). There has been some work in that direction (see for example the section on previous work above and [19], [50]) but most of the interesting problems remain open so far.

3. Aspects of derived algebraic geometry in connection with algebraic K-theory. The homotopy cofiber sequences for logarithmic topological Hochschild homology developed in work in progress by Rognes, Sagave, and Schlichtkrull give a promising tool for understanding algebraic K-theory for such examples as non-connective ring spectra such as the periodic complex K-theory spectrum KU, the spectrum of real topological K-theory, ko, its hypothetical residue field, and its periodic version KO. In addition, work by Baker and Richter [14] shows that C_2-Galois extensions of $KO[1/2]$ are purely algebraic, thus an understanding of Galois descent in these cases should be within reach.

Further examples include the algebraic K-theory of Galois extensions of finite fields: If $\ell \to k$ is a finite G-Galois extension, then $K(\ell) \to K(k)$ is not Galois in general, but one should investigate whether the $K(1)$-localized extension $L_{K(1)}K(\ell) \to L_{K(1)}K(k)$ is G-Galois. These will serve as testing devices for the Galois condition and lead to an approximation of the algebraic K-theory.

PA4 Cobordism

Bordism theories are a central aspect of homotopy theory and its application to the study of differentiable manifolds. The analogous algebraic/motivic theory has up to now been developed mainly in the setting corresponding to complex cobordism.

The topological theory of (co)bordism is at present much more extensive and flexible than its motivic counterpart and thus there is much that needs to be done on the motivic side. The bordism theory of manifolds with corners developed in the 1960's and 70's has had a wide range of applications, including giving a geometric interpretation of the Adams-Novikov spectral sequence (for details see [61]). The theory of framed bordism gives a direct connection of the homotopy groups of spheres with the theory of differentiable manifolds, and the theory of h-cobordism and s-cobordism is similarly central to the problem of homotopy-theoretic classification of differentiable manifolds. At present, none of these theories have a good motivic counterpart; in light of the manifold applications of bordism theories in topology, the development of algebraic versions is an area ripe for development.

PA4. Previous work

There is a well-developed motivic theory of complex cobordism, starting from [94]. From the homotopy-theoretic side, Voevodsky [109, 112] has defined a direct motivic analog MGL of the Thom spectrum MU as an object in the motivic stable homotopy category over a general base-scheme S. The structural properties of MGL have been studied (see [80, 82, 81, 85]): MGL enjoys the analogous universal property for oriented cohomology theories as does MU, as well as a Landweber exactness theorem and a Conner-Floyd theorem for algebraic K-theory.

On the geometric side, one has the Levine-Morel theory of algebraic cobordism, $X \mapsto \Omega^*(X)$, defined as an oriented cohomology theory (in the sense of [70]) on smooth varieties over a field of characteristic zero, and, in analogy with MU^*, Ω^* is the universal such theory. In addition, Ω^* is equal to the "geometric part" $\mathrm{MGL}^{2*,*}$ of MGL-theory [66]; a concrete description of the rest of MGL-theory is not known.

The motivic Steenrod operations and Landweber-Novikov operations [20, 70, 85, 107] have been successfully applied to problems in quadratic forms and the study of homogeneous spaces over non-algebraically closed fields. Vishik [105] has defined a number of operations that do not as yet fit into the general theory, but have had significant applications to quadratic forms. Recent work of Levine and Dai-Levine [29, 69] have laid open the way for a study of operations on other theories, such as connective algebraic K-theory.

The algebraic cobordism of a point is a model for a number of problems in Gromov-Witten theory and Donaldson-Thomas theory. In these theories one constructs generating functions that encode the enumerative information to be found in virtual fundamental classes associated to certain Hilbert schemes or moduli problems. Via the work of Levine-Pandharipande [71], a connection has been established between the generating functions for some classes of problems in Donaldson-Thomas theory (see [75, 76]) and the Levine-Morel theory of algebraic cobordism. This has been extended by work of Lee-Pandharipande [63] and applied to give a positive answer to a conjecture of Göttsche by Tzeng [103].

There has been a series of works on the homotopy theory of moduli spaces of algebraic curves and more generally *cobordism categories*. Work of Galatius-Madsen-Tillmann-Weiss [36, 74, 102] has given a systematic description of the infinite loop spaces that arise from cobordism categories, generalizing the Madsen-Tillmann-Weiss weak equivalence $\mathbb{Z} \times B\Gamma_\infty^+ \sim (\Omega^\infty \mathbb{CP}^\infty)_{-1}$, which in turn is the essential identity used to prove Mumford's conjecture on the stable homology of the moduli spaces of curves.

PA4. Scientific goals

1. Develop new theories of algebraic cobordism. One needs to develop both the motivic homotopy theory and the algebro-geometric theory for "manifolds with corners", as well as bordism theories for other structures, such as framed bordism. We expect that this would have applications to both a better understanding of the motivic sphere spectrum as well as to Gromov-Witten theory. On the geometric side, one should try to develop theories along the lines of the Levine-Pandharipande double-point cobordism description of algebraic cobordism. One should also look for the motivic homotopy theory side of the extensions of double-point cobordism found by Lee-Pandharipande [63], and use decomposition phenomena arising in Gromov-Witten theory to guide the construction of new motivic theories.

2. Landweber exact theories and their truncations. Recent work of Dai-Levine [29] computes the geometric part of connective algebraic K-theory in terms of algebraic cobordism, in spite of the fact that connective algebraic K-theory is not a Landweber exact theory, and give a description of the connective algebraic K_0 in terms of the Grothendieck groups of coherent sheaves with support in bounded codimension. There should be a similar theory for all "slice-connective" versions of Landweber exact theories, and with it a corresponding collection of interesting degree formulas. There should be numerous applications of this extension of algebraic cobordism to problems such as incompressibility, canonical dimension and essential dimension of algebraic groups.

3. Motivic cobordism categories. The gluing construction used in the definition of cobordism categories does not have an evident algebraic counterpart. However, the motivic tubular neighborhood constructions of Ayoub [10] and Levine [68] could possibly be used to give some motivic analog of stable homology of the moduli spaces of curves. It would be interesting to construct in this way a general motivic version of cobordism categories and see if one can use these to give a motivic version of the space $(B\Gamma_\infty)^+$ as well as the Madsen-Tillmann-Weiss weak equivalence $\mathbb{Z} \times B\Gamma_\infty^+ \sim (\Omega^\infty \mathbb{CP}^\infty)_{-1}$.

The approach via rigid analytic motivic homotopy theory (see §PA5 below) could be fruitful, as one can hope that the necessary gluing construction can be done at the unstable level and possibly for higher dimensional varieties as well, where the purely algebraic approach seems to present numerous technical difficulties.

4. Deligne cobordism and Arakelov motivic cohomology. The recent works of Holmstrom-Scholbach [43] and Hopkins-Quick [48] define extensions of Deligne cohomology to the setting of motivic cohomology and algebraic cobordism; Holmstrom-Scholbach [43] define an Arakelov motivic cohomology as well. One should refine the Hopkins-Quick theory to a good integral theory, and extend the construction to a Arakelov-like arithmetic theory. This should help in applications, where one would like to detect interesting elements in algebraic cobordism. In its current state, the Arakelov motivic theory does not detect classes that are topologically non-trivial; an extension of this theory and its cobordism counterpart to cover the topologically non-trivial classes would be very desirable, especially as the major uses of classical Arakelov theory involve exactly these classes.

5. Operations. Construction and study of motivic operations on connective algebraic K-theory is a fruitful direction, possibly following the approach used in [69] for the construction of Steenrod operations via algebraic cobordism. This should allow one to recover some at present mysterious operations constructed by Vishik in a more natural way, allowing for a better understanding of their properties, and leading to new applications to the study of quadratic forms and homogeneous varieties.

PA5 Unstable Homotopy Theory

Many of the methods of differential topology are up to now inapplicable in $\mathbb{A}^1$-homotopy theory. The recent construction by Ayoub of a motivic homotopy theory for rigid analytic spaces opens up the possibility of a more systematic application of differential topology to the algebraic/analytic setting.

Classical applications of unstable homotopy theory to geometric problems rely on a combination of obstruction theory and the computations of the homotopy groups of symmetry groups. The versions of obstruction theory appropriate to the algebraic setting are available, but the corresponding computations of the $\mathbb{A}^1$-homotopy groups of algebraic groups are considerably more difficult and are only at the beginning stages. Extending these computations is one of the central problems in the area, both for giving a fundamental understanding of the "structure constants" in the motivic homotopy categories, as well as having ocnrete applications to problems in the study of vector bundles and algebraic cycles.

PA5. Previous work

Ayoub [11] has laid some of the foundations for a homotopy theory for rigid analytic varieties over a complete non-archimedean field. In analogy with the construction of motivic homotopy theory, Ayoub [11] has defined a rigid analytic motivic homotopy theory. On the one hand, this has connections to motivic homotopy theory: there exists a reduction functor from rigid analytic varieties to schemes over the residue field. On the other hand, it is possible to use analytic methods to study rigid analytic geometry. Up to now, the main application of the rigid analytic theory is to give information on the motivic nearby cycles functor, this being the main result of [11].

There are many ways that a knowledge of the $\mathbb{A}^1$-homotopy groups of a particular variety may be applied, including determination of obstructions to rational points, or as receptors for invariants of vector bundles (see [4, 5]). Such computations are correspondingly difficult. The works of Morel [78], Asok-Morel [7] and Asok-Fasel [4] have laid some of the groundwork for the computations of the $\mathbb{A}^1$-homotopy groups of special varieties, such as algebraic groups and their homogeneous spaces.

Starting with work of Chen, Morgan, Hain and others, the methods of rational homotopy theory have been applied to define mixed Hodge structures on fundamental groups. Through results of Deligne-Goncharov [32], these constructions have been lifted to categories of motives, if one assumes the underlying varieties are of mixed Tate type.

PA5. Scientific goals

1. Develop the unstable motivic homotopy theory of non-archimedean analytic spaces. Ayoub constructs motivic theories for rigid analytic spaces in the setting of the motivic S^1-stable category $\mathcal{SH}_{S^1}(k)$ and the motivic stable homotopy category of T-spectra, and well as triangulated categories of motives; the unstable version still needs to be developed. Many of the results of Ayoub require the base (non-archimedean) field to be of equal characteristic zero; it would be very useful if this hypothesis could be removed.

For further applications, it would be useful if the motivic tubular neighborhood and punctured tubular neighborhood constructions given in [68] can be performed in the unstable rigid analytic setting. It is known that the unstable motivic version of the punctured tubular neighborhood does not exist. However, a rigid analytic version may be possible; this would enable rigid analytic versions of many of basic constructions of differential topology that at present are not available in unstable motivic homotopy theory.

There are at present a number of competing theories for "analytic spaces" in algebraic geometry. The various associated motivic theories should be constructed and compared. Berkovich spaces [17], Huber's adic-spaces [56] or Scholze's perfectoid spaces [95], are natural candidates for these constructions.

2. Develop non-archimedean analytic versions of aspects of classical unstable homotopy theory. Many aspects of classical unstable homotopy theory are quite difficult to extend to the motivic setting. However, in certain cases, an extension to the non-archimedean analytic setting appears to be more promising. For instance:

• One case of the *h-principle* (in the formulation of Gromov [42]) states that for X a complex manifold with a spray and U a Stein manifold, the spaces of continuous maps $U \to X$ and of holomorphic maps $U \to X$ are weakly equivalent. A special case is Oka's principle which states that on Stein manifolds continuous and holomorphic classification of vector bundles agree.

An analogue of this case of the h-principle can be formulated in motivic homotopy theory, where the relevant condition is called the affine Brown-Gersten property, *cf.* [78] or [114]. This property allows one to identify the motivic and continuous mapping spaces.

Gromov's h-principle applies to a wide variety of complex manifolds, such as complex Lie groups and their homogeneous spaces, or spherical varieties, and is stable under excision of codimension ≥ 2 subsets, blowups, and other operations. In motivic homotopy theory, only rationally trivial homogeneous spaces under (split) groups are known to have the affine Brown-Gersten property. Even in this restricted situation, this property is very useful. It enables one to identify motivic homotopy groups with unstable K-groups and therefore relates stabilization properties in Karoubi-Villamayor K-theory to the connectivity of homogeneous spaces [114, 115].

Via non-archimedean analytic geometry, one can study all these things at the same time and compare the analytic vs. algebraic angles of h-principles. In particular, one can study versions of Gromov's h-principle in rigid analytic geometry. Of course, due to the existence of the reduction functor, one cannot expect a much better h-principle in a given non-archimedean analytic setting than one would have in the usual motivic setting for the reduction to the residue field. This still leaves a lot of interesting examples.

• In the case of completely degenerate reduction, where one is able to handle the motivic theory of the reduction directly, one should be able to introduce a wide range of other methods from differential topology, for example: Morse theory, leading to a determination of the non-archimedean analytic homotopy type of an analytic space, or surgery theory. Is there an analytic motivic version of the Hirsch-Smale theorem [45, 96] on immersions, or its dual counterpart, the Phillips submersion theorem [88]? This latter would be especially relevant for the development of a motivic version of cobordism categories as discussed in §PA4.

3. Compute the $\mathbb{A}^1$-homotopy groups of special varieties. Morel's connectedness theorem tells us the $\mathbb{A}^1$-connectivity of many varieties, for example $\mathbb{A}^n \setminus \{0\}$ is $n-1$-connected. One knows $\boldsymbol{\pi}_n^{\mathbb{A}^1}(\mathbb{A}^n \setminus \{0\})$ for $n = 1, 2$; recent work of Asok-Fasel [4] computes the case $n = 3$ (up to extensions) and goes on to make the following general conjecture:

Conjecture. *For $n \geq 4$, we have an exact sequence of Nisnevich sheaves*

$$0 \longrightarrow \mathbf{K}^M_{n+1}/24 \longrightarrow \boldsymbol{\pi}^{\mathbb{A}^1}_n(\mathbb{A}^n \setminus 0) \longrightarrow \mathbf{GW}^n_{n+1} \longrightarrow 0.$$

This computation would verify the following conjecture of Asok-Fasel:

Conjecture. *Let X be a smooth affine d-fold over an algebraically closed field k and let E be a vector bundle of rank $d-1$ over X. Then $E \simeq E' \oplus \mathcal{O}_X$ if and only if $c_{d-1}(E) = 0$ in $CH^{d-1}(X)$.*

One can go in a different direction, asking for a computation of $\mathbb{A}^1$-homotopy groups of other special varieties, such as algebraic groups. On the topological side, this is the problem of explicit computations of homotopy groups of compact Lie groups and their homogeneous spaces, which is not difficult to reduce essentially to the computation of homotopy groups of spheres, this being of course hopeless. However, some fruitful steps can nevertheless be taken. The first non-trivial information is the v_1-periodic homotopy groups, the first layer in the chromatic filtration, the part of homotopy of Lie groups that can be detected by K-theory. The portion of the homotopy of Lie groups detectable by K-theory has been computed over the past 20 years by Bendersky, Davis, Mahowald, Mimura, etc., [30, 31].

An analogous (partial) computation of the $\mathbb{A}^1$-homotopy groups of algebraic groups and their homogeneous spaces would be more challenging, but should be an approachable problem, and would have numerous applications, for instance, to the classification of rationally trivial G-torsors and stability questions for algebraic vector bundles. One can also approach these computations via the slice spectral sequence and there may be a close relation between this approach and the one via the chromatic filtration (see §PA1).

4. Develop a motivic version of rational homotopy theory and apply homotopy-theoretic methods to the Hodge theory of homotopy groups. There is already a nascent motivic rational homotopy theory derived for example from the Deligne-Goncharov [32] construction of the motivic π_1. However, this theory is applicable only in rather limited settings; a more extensive and purely motivic theory should be developed. Another aspect that is lacking is a homotopy-theoretic foundation for these constructions. It would be interesting to construct a suitable motivic rational homotopy theory, possibly one more closely related to the $\mathbb{A}^1$ derived category. From another direction, one should apply more sophisticated methods from the homotopy theory of cdgas and related model category structures to the task of giving Hodge structures to all aspects of the homotopy groups of algebraic varieties.

5. Obstruction theory and higher rational connectedness. The geometric notions of rational connectedness and rational simple connectedness have been successfully applied to questions of the existence of rational sections to maps of schemes to curves and surfaces over algebraically closed fields. The relation of rational connectedness to $\mathbb{A}^1$-homotopy has been clarified by Asok-Haesemeyer [6], who show that rational connectedness of a smooth projective variety X over a given field F is equivalent to the triviality of $\pi_0^{\mathbb{A}^1}(X)(F)$. The theorem of Graber-Harris-Starr [40] on the existence of a rational sections for rationally connected fibrations over a smooth curve over $\mathbb{C}$ may be thereby viewed as a Galois descent property of the Nisnevich sheaf $\pi_0^{\mathbb{A}^1}(X)$. There is an obstruction theory in place for the existence of a section in the $\mathbb{A}^1$-homotopy category [28], but it is not at all clear what relation the $\mathbb{A}^1$-homotopy sheaves have to do with the geometric notions of rational simple-connectedness.

6. Operads and recognition principles. Operads play an important role in stability questions and in recognition of loop space structures in classical homotopy theory. One central open problem in the motivic theory is the construction of models for $\mathbb{P}^1$-loop spaces and a usable recognition principle; at present it is not at all clear if this can be accomplished through the use of operads as in the classical case, but models based on mixing sheaf theory with the classical constructions could be tried. Related topics would include the rational homotopy properties of operads, a study of formality of operads and modules over operads and spaces of (derived) maps between operads.

6 Gender equality measures

6.1 Integration of and funding opportunities for participating female researchers

We plan to organize a yearly meeting "Young Women in Homotopy Theory and Algebraic Geometry". These meetings will be a continuation of the meetings "Young Women in Topology" that have been organized successfully by the Graduiertenkolleg 1150 "Homotopy and Cohomology" in 2010, 2011, 2012 and 2013.

For each meeting, taking place over the course of 2-3 days, we would specifically invite young women – on the master, PhD and postdoc level – working in topology or algebraic geometry from Germany and neighboring countries. One senior female mathematician is invited to give a series of 2-3 lectures about a recent development in the field; the lecture series is complemented by shorter talks of the junior female participants. There will also be ample time for discussion and networking, We envision a group of about 20 female participants; prospective speakers of the lecture series include for instance: Ulrike Tillmann (Oxford), R. Sujatha (Tata Institute), Claire Voisin (CNRS, Jussieu), as well as participants in the Priority Program such as Héléne Esnault, Annette Huber and Birgit Richter. The talks in the "Young Women in Homotopy Theory and Algebraic Geometry" are public, i.e., men are also encouraged to attend but funding and the possibility to give talks are reserved to women.

We note that two of the four Initiators are women; we hope this will have a "role-model" effect for younger female researchers. We also plan on reserving a portion of the guest funds for visiting female researchers.

6.2 Family-friendly options

Besides the options offered to employees at the respective institutions, we will work with the hosting universities to offer childcare as needed during our workshops, summer schools and conferences. A budget item requests funds for this purpose.

7 Coordinator project: Integration and exchange

The problem areas delineated in this Priority Program's Scientific Program require expertise in several different fields, whose practitioners are widely distributed throughout Germany. To link these separate locations, to help bring in outside talent and to further the integration of early career researchers into the Priority Program, we will use three different approaches: We will organize a series of conferences, summer-schools and workshops, we will use regional seminars to establish links between relatively nearby locations in this Priority Program and we will offer exchange programs for post-docs and Ph.D. students. A website for the Priority Program will help with the organization and publicizing of these activities, and will be used for disseminating information about projects within the Priority Program. The *Steering Committee* will organize these activities and will consist of: Prof. Dr. Hélène Esnault (FU Berlin), Prof. Dr. Jens Hornbostel (Univ. Wuppertal), Prof. Dr. Marc Levine (Univ. Duisburg-Essen), Prof. Dr. Birgit Richter (Univ. Hamburg) and Prof. Dr. Stefan Schwede (Univ. Bonn).

7.1 Integration of early career researchers into the program

Ph.D. students and post-docs are integrated into this Priority Program through their active participation in regional seminars, workshops and summers schools, as well as through their participation in exchange programs, as detailed in §7.2 below. In addition, we will promote the integration of Ph.D. students and post-docs by

- setting funds aside for the attendance of young researchers at meetings of the Priority Program,
- including recent PhDs among speakers, achieving a good mix of established and young speakers,
- using travel funds as well as the exchange program to allow young researchers short-term visits.

7.2 Conferences, summer-schools, workshops

Our program of meetings forms the connective tissue that will link the research groups taking part in the Priority Program. We plan five types of meetings:

• *Planning workshops*: At the beginning of each three-year funding period, we will hold a one-day planning workshop for the purpose of fine-tuning and coordinating potential projects. The workshop would be devoted to presentation of project ideas, discussion of possible modifications and creation of potential team projects, forming a basis for the furhter development of finished proposals.

• *Yearly conference*: Each year (following the initial year) we will hold a week-long conference presenting the latest results in an area or areas represented in the Priority Program, with speakers and invited participants on the international level.

• *Workshops*: We will hold two workshops in selected areas of research within the Priority Program; depending on the topic selected, the workshop would run from a few days to a week. The core purpose of the workshop will be to promote the interaction and exchange of expertise between the different research directions represented in the Priority Program through lectures and discussions.

• *Compact workshops*: These 2-3 day workshops would be planned and organized by the post-docs and Ph.D. students and will allow a part of the Priority Program to inform those working in other areas about foundation aspects of their field in a quick and efficient manner. Those directly connected with the chosen topic would assist those from outside the area, outside experts could also be invited to help organize the workshops.

• *Summer schools*: The week-long summer schools will be held once a year, and are directed at the younger members of the research community. Foundational topics and methods, for example, *An introduction to motivic homotopy theory*, will be presented in a short-course format, with lectures by a mix of experts and younger mathematicians, along with problem sessions and discussion groups to encourage active participation.

7.3 Regional seminars

In addition to these large-scale meetings, the Priority Program will help organize smaller regional seminars. These seminars will take advantage of regional concentrations among the Priority Program participants to set up smaller scale cooperative efforts. For example, the organization of the current Bochum-Bonn-Düsseldorf-Wuppertal topology seminar, run as part of the GRK 1150 "Homotopy and Cohomology", will be taken over by this Priority Program after the conclusion of the GRK. We plan to organize other regional seminars on this model to help link together regional clusters of participants in this Priority Program.

7.4 Exchange programs

We plan on the exchange of personnel between the various research groups affiliated with this Priority Program, through short-term and long-term exchanges.

On the short-term side, the Coordinator and Steering Committee will encourage the exchange of researchers involved in the Priority Program through their participation in the regularly scheduled seminars and colloquia taking place under the various research groups participating in the Priority Program. This will give the participants opportunities in an informal setting to disseminate their work on topics relevant to the Priority Program.

Longer term exchanges, by either Ph.D. students or post-docs, will be financed in part through the Priority Program. Upon the return of the visiting researchers, lectures and discussion groups will be used to spread the knowledge acquired at the host group. A second advisor from the host group will be assigned to any visiting Ph.D. student and the planning of the visit would actively involve the student and both advisors.

In addition to such internal exchanges, this Priority Program will encourage the involvement of other research groups, both inside and outside of Germany, through exchange programs on various levels. For example:

Other DFG-funded groups: Targeted short-term exchange programs involving post-docs funded by the Priority Program and other DFG projects, such as the SFB Transregio 45, form an effective method for promoting collaboration and the transfer of information among the young researchers involved in these projects.

Cooperation with other research networks: We are discussing a similar exchange with the French network GATHO and the Norwegian "Topology in Norway" project, as well as joint conferences, workshops and summer schools.

7.5 Webserver

The Coordinator will arrange for the creation of a website for this Priority Program, serving as information nexus for the various activities of the Priority Program, including a calendar of conferences, workshops and summer schools, lists of participants, projects funded, and the like, as well as serving as an advertising platform to the larger mathematical community.

8 International Cooperation

The initiators and potential participants for this Priority Program have extensive ties to researchers on the international level, too numerous to list here. For a selected list of collaborators with potential participants of this Priority Program, and other researchers with related interests, please see the Appendix §4.

Homotopy theory and allied areas are internationally recognized as central areas in mathematics and there are a number of extensive networks devoted to these topics. There is at present no research network in this area in Germany.

In the U.S., the NSF is currently funding two Focused Research Groups in areas close to those in our program: *Homotopy Theory: Applications and New Dimensions* (Hopkins, Lurie, Miller, Barwick, Behrens) and *Homotopical methods in Algebra* (Friedlander, Haesemeyer, Walker, Weibel).

In France, there is the long-running program funded by the CNRS: GDR 2875 "Topologie Algébrique et Applications", with a wide range of areas of research, and approximately 100 members at 14 different universities. The project GATHO (Groupes algébriques et Théories Homologiques) with topics quite close to ours is funded by the ANR. Represented in this project are the Université Paris 6, Université d'Artois and Université Paris 13.

Norway has the "Topology in Norway" project sponsored by the Research Council of Norway involving the Norwegian topologists in Bergen, Oslo, and Trondheim, including M. Brun, B. Dundas, P.A. Østvær, J. Rognes, C. Schlichtkrull, and N. Baas.

There are numerous connections on the level of individual collaborations and joint organization of past conferences between members of these groups and the initiators and potential participants of this Priority Program. We are currently discussing possibilities for cooperation and collaboration with both the "Topology in Norway" group and the GATHO group; these include joint conferences or workshops as well as exchange programs.

9 Connections to and separation from other DFG funded activities

This SPP will *not* allow any double funding of projects supported in other DFG programs. We intend to support projects that further the following goals of this SPP:

• to encourage interaction among researchers in the different research fields: algebraic geometry, classical homotopy theory, motivic homotopy theory and differential homotopy theory, represented in this SPP
• to encourage researchers to engage in projects that develop connections between other fields (such as number theory, representation theory, tropical geometry) and the main research fields represented in this SPP. This includes joint projects between researchers within the "mainstream" fields in this SPP and those outside.

Overall, even though some existing or proposed DFG programs do involve individual projects which would fit into this SPP, these projects occur in essentially isolated form; no DFG program is devoted to the constellation of research directions which form the heart of this SPP. Even though a number of researchers intending to submit projects to this SPP are already participants in existing or proposed DFG programs, projects funded by this SPP will not overlap with the funding within these DFG programs. Additionally, there is great potential for constructive collaboration among researchers throughout Germany through this SPP, that would not be possible within the currently existing or proposed DFG programs.

The SFB/Transregio 45 "Periods, moduli spaces and arithmetic of algebraic varieties"[4] contains a few projects that would overlap with this SPP. However, the overall goals and direction in this SFB are quite different from those of this SPP. Additionally, the current funding period of the SFB would be ending; the individual projects of this type that are currently being funded in the SFB could easily move over into the SPP, as the second funding period for the SFB will be ending some few months after projects could be funded through this SPP.

The specific projects involved are: Prof. Levine's projects M02-3 (Algebraic cobordism: extensions and applications), M02-4 (Motivic homotopy theory) and project M4-2 (Motivic obstructions to rational points). Each of these projects may simply be completed in the current funding period of the SFB/Transregio 45, or may form the basis for possible project applications to continue the project in this Priority Program; care will be taken in any case to avoid double funding. We thus expect that this limited overlap between these two programs would not carry over to a possible third funding period for this SFB and would therefore be short-lived. There would be no overlap with Prof. Levine M02-2 (motivic structure on the nilpotent orbit), Profs. Levine/Müller-Stach M02-5 (Regulators for Tate motives), Dr. Semenov M02-6(Algebraic groups and Chow motives).

The Research Unit at the Universität Heidelberg has contact with this SPP through a few projects dealing with Suslin homology and motivic cohomology, but as the emphasis in the Research Unit is not on homotopy theory, the approach being more arithmetic, we see no overlap with this program.

There is one subproject in the SFB 878 (Münster) entitled "Equivariant homotopy and homology" which has connections with PA2, but the SFB subproject is largely concerned with aspects of $\mathbb{C}^*$-algebras, the Farrell-Jones and Baum-Connes conjectures, and does not significantly overlap with the areas in this SPP.

An SFB at the Universität Regensburg is currently under consideration and involves some projects in homotopy theory, motivic cohomology and differential homotopy theory, which would fit into this SPP. In this case, we would take care to see that no double funding of any projects would occur. Although these projects in this SFB do overlap with this SPP, the SFB in Regensburg is entirely restricted to researchers resident in Regensburg,

[4] Currently in its 2nd funding period: July 2011-June 2015

and thus does not have the geographically widespread character of this SPP; for instance, the Regensburg SFB would not be funding a joint project carried out by a researcher at Regensburg together with one at another location.

An SFB/Transregio involving the FU Berlin, Frankfurt University and the University of Warsaw is in the initial proposal stage, but it appears that the emphasis of this SFB would preclude any significant overlap with the type of projects expected in this SPP.

Overlap with other DFG funded programs would be very small or non-existent. There are connections to the Research Training Group 1150GRK "Homotopy and Cohomology" Bonn; Bochum; Düsseldorf (funding period 2005-2014), 1692 GRK "Curvature, Cycles, and Cohomology" Regensburg (funding started 2010); and 1821GRK "Cohomological Methods in Geometry" Freiburg (funding started 2012), however, 1150GRK will be ending shortly and the other two programs deal with topics that are not directly involved in this Priority Program.

Similarly, the Priority Program 1388 SPP "Representation Theory" (funding started 2009), the recently established GK 1916 "Combinatorial structures in geometry" and the SFB 701 "Spectral Structures and Topological Methods in Mathematics" (funding stated 2005, running through 2017) have some small connections with this SPP but no project overlap. The SFB 676 "Particles, strings and early universe" (currently in its 2nd funding period 2010-2014) and the Research Training Group GRK 1670 "Mathematics inspired by string theory and quantum field theory" (funding started 2011) each have a small connection to this SPP via mathematical aspects of string theory, again, not involving specific program topics.

We expect there will be numerous opportunities for constructive collaborations with these other programs, especially those which we have identified as related to the Priority Program.

10 Requested funding

Budget

Post-docs	10 × 190.000€	=	1.900.000€
Ph.D. students	14 × 135.000€	=	1.890.000€
Travel			216.000€
Coordinator project			554.000€
Total (3 years)			4.560.000€
Total/year			1.520.000€

Coordinator project

Visitors			332.000€
Conferences	2 × 16.000€	=	32.000€
Planning workshop			3.000€
Workshops	2 × 10.000€	=	20.000€
Summer schools	3 × 16.000€	=	48.000€
Compact workshops	4 × 4.000€	=	16.000€
"Young women in..."	3 × 8.000€	=	24.000€
Exchange program	12 × 4.000€	=	48.000€
SHK/Website			21.000€
Childcare			10.000€
Total(Coordinator)			554.000€

The budget items are estimates for a 3-year funding period, based on an estimate of the equivalent of approximately 20 single funded projects. The item "SHK/Website" is for setting up and maintaining the Program website. "Childcare" refers to the measures discussed in §6.2. The estimates for the item "Travel" follow the standard lump-sum (for single projects) of 9.000 Euro per person per 3-year period as set out by the DFG-Fachkollegium in Mathematics.

11 Literature cited

A "•" marks the publications having an author who is a prospective participant in this Priority Program; the relevant author is in bold. Literature that has not appeared in print is marked as "Preprint". Each preprint is available for down-load, either from the Math ArXiv `http://arxiv.org/archive/math` using the arXiv identifier listed in the article reference or at the URL listed in the article reference. For the reader's convenience, we have also set up a web-site for the down-loading of all preprints at `http://www.esaga.uni-due.de/marc.levine/SPP/`

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[64] •**M. Levine**, *A comparison of motivic and classical homotopy theories.* Journal of Topology (2013) `DOI:10.1112/jtopol/jtt031`

[65]•**M. Levine**, *Convergence of Voevodsky's slice tower* Documenta Math. **18** (2013) 907–941.

[66]•**M. Levine**, *Comparison of cobordism theories.* J. Algebra 322 (2009), no. 9, 3291–3317.

[67]•**M. Levine**, *The homotopy coniveau tower.* J. Topol. **1** (2008), no. 1, 217–267.

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[69]•**M. Levine**, *Steenrod operations, degree formulas and algebraic cobordism.* Pure and Applied Mathematics Quarterly Journal, volume in honor of the 60th birthday of Robert MacPherson **3**, No. 1 (2007) 283-306.

[70]•**M. Levine**, F. Morel, *Algebraic cobordism.* Springer Monographs in Mathematics. Springer, Berlin, 2007.

[71]•**M. Levine**, R. Pandharipande, *Algebraic cobordism revisited.* Invent. Math. **176** (2009), no. 1, 63–130.

[72]•**M. Levine**, C. Serpé *On a spectral sequence for equivariant K-theory.* K-Theory **38** (2008), no. 2, 177–222.

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[80]•**N. Naumann**, **M. Spitzweck**, P. A. Østvær, *Motivic Landweber exactness*. Doc. Math. **14** (2009), 551–593.
[81]•**N. Naumann**, **M. Spitzweck**, *Brown representability in A^1-homotopy theory*. J. K-Theory **7** (2011), no. 3, 527–539.
[82]•**N. Naumann**, **M. Spitzweck**, P.A. Østvær, *Chern classes, K-theory and Landweber exactness over nonregular base schemes*. Motives and algebraic cycles, 307–317, Fields Inst. Commun., **56** Amer. Math. Soc., Providence, RI, 2009.
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[84] C. Okonek, A. Teleman, *A wall crossing formula for degrees of real central projections*. Preprint 2012, rev. 2013 `arXiv:1206.4271`
[85]•I. Panin, K. Pimenov, **O. Röndigs**, *A universality theorem for Voevodsky's algebraic cobordism spectrum*. Homology, Homotopy Appl. **10** (2008), no. 2, 211–226.
[86]•I. Panin, K. Pimenov, **O. Röndigs**, *On the relation of Voevodsky's algebraic cobordism to Quillen's K-theory*. Invent. Math. **175** (2009), no. 2, 435451.
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[90] D.C. Ravenel, **Nilpotence and periodicity in stable homotopy theory**. Annals of Mathematics Studies **128**, Princeton University Press, Princeton, NJ 1992.
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[97]•**M. Spitzweck**, *A commutative $\mathbb{P}^1$-spectrum representing motivic cohomology over Dedekind domains*. Preprint 2012. `arXiv:1207.4078`
[98]•**M. Spitzweck**, *Slices of motivic Landweber spectra*. J. K-Theory **9** (2012), no. 1, 103–117.
[99]•**M. Spitzweck**, *Relations between slices and quotients of the algebraic cobordism spectrum*. Homology, Homotopy Appl. 12 (2010), no. 2, 335–351.
[100]•**M. Spitzweck** and P. A. Østvær, *The Bott inverted infinite projective space is homotopy algebraic K-theory*. Bull. Lond. Math. Soc. **41** (2009), no. 2, 281–292.
[101] M. Szymik, *Crystals and derived local moduli for ordinary K3 surfaces*. Adv. Math. **228** (2011), no. 1, 1–21.
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[107] V. Voevodsky, *Reduced power operations in motivic cohomology*. Publ. Math. Inst. Hautes Études Sci. No. 98 (2003), 1–57.
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[114]•**M. Wendt**, *$\mathbb{A}^1$-homotopy of Chevalley groups*. J. K-Theory **5** (2010), 245–287.
[115]•**M. Wendt**, *Rationally trivial torsors in $\mathbb{A}^1$-homotopy theory*. J. K-Theory **7** (2011), no. 3, 541–572.
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Appendices

1. Topic-related publications
2. Curriculum vitae
3. Prospective participants
4. International collaborators
5. Literature not in print

1 List of topic-related publications

Here are 18 topic-related publications (co)authored by the Initiators; the relevant author is underlined.

(a) Works which have appeared or are accepted for publication

[a1] H. Esnault, *Algebraic differential characters of flat connections with nilpotent residues*, in Algebraic Topology, Abel Prisen **4** (2007), Springer Verlag, 83–94.

[a2] H. Esnault and O. Wittenberg, *On abelian birational sections*. Journal of the American Mathematical Society 23, (2010), 713-724.

[a3] H. Esnault and E. Viehweg, *On a rationality question in the Grothendieck ring of varieties*. Acta Mathematica Vietnamica 35, (2010), 31-41.

[a4] H. Esnault and J. Nicaise, *Finite group actions, rational fixed points and weak Néron models*, Pure and Applied Quarterly Q. 7:4, in the Special issue in memory of Eckart Viehweg (2011), 1209–1240.

[a5] A. Baker, B. Richter, *Gamma-cohomology of rings of numerical polynomials and E_∞-structures on K-theory*, Commentarii Mathematici Helvetici 80 (4), (2005), 691–723.

[a6] A. Baker, B. Richter, *Realizability of algebraic Galois extensions by strictly commutative ring spectra*, Trans. Amer. Math. Soc. 359 (2), (2007), 827–857.

[a7] N. Baas, B. I. Dundas, B. Richter, J. Rognes, *Stable bundles over rig categories*, Journal of Topology (2011), 623–640.

[a8] A. Baker, B. Richter, M. Szymik, *Brauer groups for commutative S-algebras*, Journal of Pure and Applied Algebra 216 (11), (2012), 2316–2376.

[a9] N. Baas, B. I. Dundas, B. Richter, J. Rognes, *Ring completion of rig categories*, Journal für die reine und angewandte Mathematik **674** (2013), 43–80.

[a10] S. Schwede, *The n-order of algebraic triangulated categories*, Journal of Topology (2013) `DOI: 10.1112/jtopol/jtt014`

[a11] S. Schwede, *The p-order of topological triangulated categories*, Journal of Topology (2013) `DOI: 10.1112/jtopol/jtt018`

[a12] S. Schwede, *On the homotopy groups of symmetric spectra*. Geometry and Topology **12** (2008), 1313–1344.

[a13] S. Schwede, *The stable homotopy category is rigid*. Annals of Math. **166** (2007), 837–863.

[a14] F. Muro, N. Strickland and S. Schwede, *Triangulated categories without models*. Invent. Math. **170** (2007), 231–241.

[a15] M. Levine and F. Morel, **Algebraic Cobordism**. Monographs in Mathematics, 246 pp., Springer, Berlin 2007.

[a16] M. Levine *The homotopy coniveau tower*. J Topology **1** (2008) 217–267.

[a17] M. Levine and R. Pandharipande, *Algebraic cobordism revisited*. Invent. Math. **176** (2009), no. 1, 63–130.

[a18] M. Levine, *A comparison of motivic and classical homotopy theories*. Journal of Topology (2013) `DOI:10.1112/jtopol/jtt031`

2 Curriculum Vitae of the Initiators

September 2013

Curriculum Vitae

Prof. Dr. Hélène Esnault

Fachbereich Mathematik und Informatik
Freie Universität Berlin
Institut für Mathematik
Arnimallee 3, 14195 Berlin

Born:
17.07.1953 in Paris, French and German citizenship

Academic training and academic degrees:

1975	Diplôme d'E.A., Paris VII
1976	Agrégation, Paris
1976	Thèse de 3-ième cycle, ParisVII
1984	Doctorat d'Etat, Paris VII
1985	Habilitation, Universität Bonn

Academic positions:

1977 – 1983	Assistent, Paris VII
1983 – 1985	Gastwissenschaftlerin, MPI Bonn
1985 – 1989	Heisenberg Stipendiat
1989 – 1990	Maître de Conférences, Paris VII
1990 – 2012	Professor (C4) at the Universität Essen
2012 –	Einstein Professur at the Freie Universität Berlin

Awards and honors:

2001	Prix Paul-Doisteau Blutet of the Academy of the Sciences in Paris
2002	Invited Speaker, Section Algebra, International Congress of Mathematics, Beijing
2003	Leibniz Preis der deutschen Forschungsgemeinschaft (with Eckart Viehweg)
2005 –	Academy of Nordrhein-Westfalen (Full Member)
2009 –	European Research Council Advanced Grant
2009	Honoray Doctorate of the Vietnam Academy of Sciences and Technology in Hanoi
2010 –	Academy Leopoldina (German National Academy) (Full Member)
2010 –	Academy of Berlin-Brandenburg (Full Member)
2011	Chaire d'Excellence de la Fondation Mathématique de Paris
2012	Einstein Professur
2012	Invited Speaker, Section Algebra, European Congress of Mathematics, Krakow
2013	Honorary Doctorate of the University of Rennes, France

Academic service:

Several years	Faculty Council
Until 1989	Scientlfic Committee MPI Bonn
1997 – 2003	Scientific Committee Humboldt Foundation
2001	Jury of Cantor Medaille
2002	IMU delegate ICM
2002 – 2003	National Council CNRS
2004 – 2007	DFG-Senat-Council for Graduate schools
2005	Visiting Committee of the Department of Mathematics of the Harvard University
2005 –	Institute for Sciences and Ethics

Academic service (continued):

	NSF grants panel
2006 –	Scientific Council of Oberwolfach
2007 –	Scientific Committee of the DFG-Leibniz-Preis
2007 –	Advisory Board of the Institute of Mathematics, Academia Sinica in Tapei
2008 –	Advisory Board of the Dahlem Conference, Berlin
2009 –	Advisory Board of the Tsinghua Research Institute
1995 –	co-editor, Duke Mathematical Journal
1998 – 2010	co-editor, Mathematische Annelen
2000 –	co-editor, Acta Mathematica Vietnamica
2002 –	co-editor, Cubo Journal
2007 –	co-editor, Mathematical Research Letters
2007 –	founding editor, Algebra and Number Theory
2011 –	co-editor, Astérisque
2005	co-organizer (with A. Beilinson, S. Bloch, N. Katz, R. MacPherson) on the occasion of P. Deligne's 61-st birthday
2002	distinguished lecture, Kuweit Lectures
2005	distinguished lecture, Manin's retirement conference
2006	distinguished lecture, MPI's 25-th anniversary conference
2007	distinguished lecture, Abel Lectures
2012	distinguished lecture, Arithmetic-Motives-and-Moduli Spaces
2012	distinguished lecture, VIASM Yearly Meeting

Five most important publications: (a) works published or accepted for publication.

1. (with E. Viehweg) *Logarithmic De Rham complexes and vanishing theorems*, Invent. Math. **86** (1986) no. 1, 161 – 194.
2. (with V. Srinivas, E. Viehweg) *The universal regular quotient of the Chow group of 0-cycles on a singular projective variety*. Invent. Math. **135** (1999), no. 3, 595 – 664.
3. *Varieties over a finite field with trivial Chow group of 0-cycles have a rational point.* Invent. Math. **151** (2003), no. 1, 187191.
4. *Deligne's integrality theorem in unequal characteristic and rational points over finite fields.* With an appendix by P. Deligne and Esnault. Ann. of Math. (2) **164** (2006), no. 2, 715 – 730.
5. (with V. Mehta) *Simply connected projective manifolds in characteristic $p > 0$ have no nontrivial stratified bundles.* Invent. Math. **181** (2010), no. 3, 449465.

Funding during the last five years:

Research project	Funding period	Funding souce
Einstein Professorship	2012	Einstein Foundation Berlin
SFB Transregio/45	2007-2013	DFG
ERC Advanced Grant "Rational points"	2008 – 2013	European Research Council Grant 226257
Leibniz Preis	2003 – 2010	DFG

Ph.D. theses directed during the last five years:

Name	Topic	Ph.D. awarded
Dzmitry Doryn	Cohomology of graph hypersurfaces associated to certain Feynman graphs	2008
Emel Bilgin	Classes of some hypersurfaces in the Grothendieck ring of varieties	2012
Annabelle Hartmann	On rational points of varieties over complete local fields algebraically closed residue field	2012
Lars Kindler	Regular singular stratified bundles in positive chracteristic	2012
Lei Zhang	Homotopy sequence for fundamental groups	2012
Abolfazl Tarizadeh	Galois Theory of Coverings and Algebraic Properties of Rings of Continuous Functions	2013

September 2013

Curriculum Vitae

Prof. Dr. Marc Levine

Fakultät Mathematik
Universität Duisburg-Essen
Universitätstrasse 2, 45117 Essen
Telefon: +49 201 183 3114
Fax: +49 201 183 2624
Email: marc.levine@uni-due.de

Born:
29.07.1952 in Detroit, U.S.A, U.S. citizen

Academic training and academic degrees:

1974	B.S. in mathematics, Massachusetts Institute of Technology
1976	M.S. in mathematics, Brandeis University
1979	Ph.D. in mathematics, Brandeis University Ph.D. advisor, Professor Teruhisa Matsusaka Thesis title: “On the deformations of certain classes of uni-ruled varieties”.

Academic positions:

1974 – 1979	Teaching Fellow, Brandeis University
1979 – 1984	Assistant Professor, University of Pennsylvania
1984 – 1986	Assistant Professor, Northeastern University
1986 – 1988	Associate Professor, Northeastern University
1988 – 2009	Full Professor, Northeastern University
2009–	W3-Professor (Humboldt Professorship) in Mathematics at the Universität Duisburg-Essen

Awards and honors:

2013–	Full Member, German National Academy of Sciences Leopoldina
2009 – 2014	Humboldt Professorship, Universität Duisburg-Essen
2006	Humboldt Senior Research Fellowship
2002	Invited address at the International Congress of Mathematics
2001	Wolfgang Paul Award, Humboldt Foundation

Academic service:

2004, 2006, 2007, 2010, 2013	NSF grants panel member
2009 – 2012	co-editor, Journal für die Reine und Angewandte Mathematik
2008 –	co-editor, Journal Inst. Math. Jussieu
2007 –	co-editor, Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg
2013	co-organizer, conference in “Arithmetic Algebraic Geometry”, June 10-14, 2013, FU Berlin.
2006, 2009 2013	co-organizer, Oberwolfach K-theory conference
2010	co-organizer, Oberwolfach conference on motives
2010	co-organizer, “Geometric aspects of motivic homotopy theory”, Hausdorff Center for Mathematics, Bonn
2010	co-organizer, “Algebraic and arithmetic geometry”, Universität Duisburg-Essen, Essen

Five most important publications: (a) works published or accepted for publication.

1. (with T. Geisser) *The K-theory of fields in characteristic p*, Invent. Math. **139** (2000), no. 3, 459–493.
2. *Techniques of localization in the theory of algebraic cycles*, J. Alg. Geom. **10** (2001) 299-363.
3. (with F. Morel) **Algebraic Cobordism**. Monographs in Mathematics, 246 pp., Springer, Berlin 2007.
4. *The homotopy coniveau tower.* J Topology **1** (2008) 217–267.
5. (with R. Pandharipande) *Algebraic cobordism revisited* Invent. Math. **176** (2009), no. 1, 63–130.

Funding during the last five years:

Research project	Funding period	Funding souce
SFB Transregio/45	1. 2010-2011 (member SFB) 2. 2011-2014 (several projects)	DFG
Humboldt Professorship	2009-2014	Humboldt Foundation
Motivic homotopy theory (co-PI J. Weyman)	2008-2011	NSF: grant DMS-0801220

Ph.D. theses directed during the last five years:

Name	Topic	Ph.D. awarded
Thomas Hudson	A Thom-Porteus formula in algebraic cobordism	2012
Anandam Banerjee	Tensor structure on smooth motives	2010
Pablo Pelaez	Multiplicative properties of the slice filtration	2008

September 2013

Curriculum Vitae

Prof. Dr. Birgit Richter

Fachbereich Mathematik der Universität Hamburg
Bundesstrasse 55, 20146 Hamburg.
Telefon: +49 40 42 838-5173
Fax: +49 40 42 838-5190
email: birgit.richter@uni-hamburg.de

Born:
14.07.1971 in Dortmund

Academic training and academic degrees:

1991 – 1997	Studium der Mathematik an der Rheinischen Friedrich-Wilhelms-Universität Bonn. Diplomarbeit "Dialgebren, Doppelalgebren und ihre Homologie" unter der Betreuung von Prof. Dr. Carl-Friedrich Bödigheimer.
1997 – 2000	Promotionsstudium (wissenschaftliche Mitarbeiterstelle) der Mathematik an der Rheinischen Friedrich-Wilhelms-Universität Bonn. Dissertation "Taylorapproximationen und kubische Konstruktionen von Gamma-Moduln" betreut durch Prof. Dr. Carl-Friedrich Bödigheimer.

Academic positions after Promotion:

2000 – 2005	Assistentin (C1) in Bonn.
2005 – 2010	Professorin (W2) am Fachbereich Mathematik der Universität Hamburg.
2009	Ruf auf eine W3-Professur in Mathematik an der Bergischen Universität Wuppertal (abgelehnt).
2010 –	Professorin (W3) am Fachbereich Mathematik der Universität Hamburg.

Awards, honors, plenary lectures:

March 2002	Northwestern University, Northwestern conference (Emphasis Year on Algebraic Topology), Plenarvortrag: *Topological André-Quillen cohomology – an overview*

Academic service:

Seit 05/2006 Schriftleitung der "Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg", zusammen mit Vicente Cortés.

Mitglied in 21 Promotionskommissionen in Hamburg, einer in Straßburg (Bruno Vallette), einer in Lille (Eric Hoffbeck) und drei in Bonn (Steffen Sagave, Julia Singer, Boryana Dimitrova). Mitglied in 4 Habilitationskommissionen.
Mitglied der Auswahljury des Programmes *Topologi* des Norwegischen Forschungsrats (Bergen, Oslo, Trondheim): Promotions- und Postdoc-Stellen.

Gutachterin für *Advances in Mathematics*, *Geometry and Topology*, *Documenta Mathematica*, *International Mathematics Research Notices*, *Inventiones*, *Journal of Pure and Applied Algebra*, *Journal of Topology*, *K-theory*, *Mathematische Annalen*, *Mathematische Zeitschrift*, *Memoirs of the AMS*, *Topology*, *Theory and Applications of Categories*, Springer Buchpublikationen.
25 Reviews für die *Mathematical Reviews*.
Gutachterin für den Schweizerischen Nationalfond, für das *Engineering and Physical Sciences Research Council* (EPSRC), UK, für das MIT (tenure Gutachten), und für die DFG.

Buchherausgabe: *Structured Ring Spectra*, eds.: A. Baker, B. Richter, London Mathematical Society Lecture Note Series 315, Cambridge University Press, 2004 und *New Topological Contexts for Galois Theory and Algebraic Geometry*, eds.: A. Baker, B. Richter, G & T monographs no 16, (2009).

Academic service(continued):
Conference organized:

08. – 14.10.2000	Oberwolfach, , Arbeitsgemeinschaft: "Operaden und ihre Anwendungen", (with C.-F. Bödigheimer (Bonn), J.-L. Loday (Straßburg))
21. – 25.01.2002	Glasgow, Schottland, "Workshop on Structured Ring Spectra", (with A. Baker (Glasgow))
20. – 24.09.2004	Bonn, "Workshop on Structured Ring Spectra", (with A. Baker (Glasgow), S. Schwede (Bonn))
05. – 07.06.2007	Hamburg, "Workshop on Elliptic Cohomology", (with K. Fredenhagen, U. Schreiber, C. Schweigert (Hamburg))
09. – 14.03.2008	Banff, Kanada, Workshop "New Topological Contexts for Galois Theory and Algebraic Geometry", (with A. Baker (Glasgow))
01. – 05.08.2011	Hamburg, workshop Structured Ring Spectra - TNG, (with A. Baker (Glasgow))

Five most important publications: (a) works published or accepted for publication.

1. (with T. Pirashvili) *Robinson-Whitehouse complex and stable homotopy*, Topology **39** (2000), no. 3, 525–530.
2. (with A. Baker) *Gamma-cohomology of rings of numerical polynomials and E_∞ structures on K-theory*, Commentarii Mathematici Helvetici **80**, (2005), 691–723.
3. with (A. Baker) *Realizability of algebraic Galois extensions by strictly commutative ring spectra*, Trans. Amer. Math. Soc. **359** (2007), no. 2, 827–857.
4. (with N. Baas, B. I. Dundas, J. Rognes) *Stable bundles over rig categories*, Journal of Topology (2011), 623–640.
5. (with N. Baas, B. I. Dundas, J. Rognes) *Ring completion of rig categories*, Journal für die reine und angewandte Mathematik **674** (2013), 43–80.

Funding during the last five years:

Research project	Funding period	Funding souce
Mathematics inspired by string theory and QFT	2011 –	DFG-GRK 1670
Zusatzstrukturen auf E_n-Kohomologie	2011-2014	DFG-Sachbeihilfe Einzelantrag
"Lehramtsausbildung im Fach Mathematik nachhaltig verbessern", zusammen mit Gabriele Kaiser und Jens Struckmeier	2012–2016	Universitätskollegs Brücken in die Universität -Wege in die Wissenschaft des Qualitätspakts Lehre (BMBF)

Ph.D. theses directed during the last five years:

Name	Topic	Ph.D. awarded
Fridolin Roth	Galois and Hopf-Galois Theory for Associative S-Algebras	Hamburg 08/2009
Hermann Soré	The Dold-Kan correspondence and coalgebra structures	Hamburg 03/2010
Hannah König	A Segal model for a multiplicative group completion	Hamburg 02/2011

September 2013

Curriculum Vitae

Prof. Dr. Stefan Schwede

Mathematisches Institut
Universität Bonn
Endenicher Allee 60, 53115 Bonn
Tel. (0228) 73-3158 (-2941), Fax. (0228) 73-6198
Email: schwede@math.uni-bonn.de
Homepage: www.math.uni-bonn.de/~schwede

Born:
June 23, 1969 in Bielefeld

Academic training and academic degrees:

1994	Diplom (Bielefeld)
1996	Dr. math. (Bielefeld)
2001	Habilitation (Bielefeld)

Academic positions:

1997 – 1998	Post-doc (MIT, Cambridge, USA)
2002 – 2003	Head of Junior Reseach Group, SFB 478 (Münster)
2003 –	Professor (C4) Universität Bonn

Awards, honors and plenary lectures:

18. – 22. 06.2007	Joint International Meeting UMI – DMV, Perugia, Plenary lecture
29.02 – 02.03.2008	European Mathematical Society – Joint Mathematical Weekend, Kopenhagen, Plenary lecture

Academic service:

2003–	co-editor, Documenta Mathematica
2006 – 2012	co-editor, Mathematische Zeitschrift
2007	co-organizer (with Nils A. Baas, Bjørn Ian Dundas, Bjørn Jahren, John Rognes, Eric Friedlander, Graeme Segal) of The Abel Symposium 2007 "Algebraic Topology", August 5th - 10th 2007, Oslo, Norway
2007	co-organizer (with J. Greenlees, P. Goerss) of the Oberwolfach Workshop 'Homotopy theory', 16.-22. September 2007
2011	co-organizer (with J. Greenlees, P. Goerss) of the Oberwolfach Workshop 'Homotopy theory', 18.-24. September 2011
2015	co-organizer (with S. Galatius, H. Miller, P Teichner) Trimester program "Homotopy theory, manifolds, and field theories" Hausdorff Research Institute for Mathematics (Bonn) May - August 2015

Five most important publications: (a) works published or accepted for publication.

1. *On the homotopy groups of symmetric spectra.*
   Geometry and Topology **12** (2008), 1313–1344
2. *The stable homotopy category is rigid.*
   Annals of Math. **166** (2007), 837–863
3. (with F. Muro, N. Strickland) *Triangulated categories without models.*
   Invent. Math. **170** (2007), 231–241
4. (with B. Shipley) *Stable model categories are categories of modules.*
   Topology **42** (2003), 103–153
5. (with B. Shipley) *Algebras and modules in monoidal model categories.*
   Proc. London Math. Soc. **80** (2000), 491-511

Funding during the last five years:

Research project	Funding period	Funding souce
Graduiertenkolleg 1150 "Homotopy and Cohomology"	1. Funding Period: 10.2005 – 03.2010 2. Funding Period: 04.2010 – 09.2014	DFG
Exzellenzcluster "Mathematics: Foundations, Models, Applications"	1. Funding Period: 11.2006 – 10.2012 2. Funding Period: 11.2012 – 10.2017	DFG

Ph.D. theses directed during the last five years:

Name	Topic	Ph.D. awarded
Julia Singer	Äquivariante λ-Ringe und kommutative Multiplikationen auf Moore-Spektren	2008
Martin Langer	On the notion of order in the stable module categories	2009
Arne Weiner	Homotopy theory of S-bimodules, naive ring spectra and stable model categories	2009
Moritz Groth	On the theory of derivators	2011
Boryana Dimitrova	Obstruction theory for operadic algebras	2012
Lennart Meier	United elliptic homology	2012
Katja Hutschenreuter	On rigidity of the ring spectra $P_m\mathbb{S}_{(p)}$ and ko	2012
Irakli Patchkoria	Rigidity in equivariant stable homotopy theory	2013

3 Prospective participants

Here is a list of prospective participants in the Priority Program, all of whom have been contacted and have expressed a desire to participate in the Priority Program. Those marked with a ∗ have expressed an interest in submitting a project proposal to this Priority Program:

	Name	Faculty/Institute	University
*	Prof. Dr. Ulrich Bunke	Fakultät für Mathematik	Universität Regensburg
*	Dr. Joana Cirici	Fachbereich Mathematik und Informatik	Freie Universität Berlin
	Dr. Benjamin Collas	Mathematisches Institut	Universität Münster
*	Prof. Dr. Hélène Esnault	Fachbereich Mathematik und Informatik	Freie Universität Berlin
*	Dr. Jean Fasel	Fakultät Mathematik	Universität Duisburg-Essen
*	Dr. Gereon Quick	Mathematisches Institut	Universität Münster
*	Dr. Olivier Haution	Mathematisches Institut	LM Universität München
*	Dr. Philip Herrmann	Fachbereich Mathematik	Universität Hamburg
*	Prof. Dr. Detlev Hoffmann	Fakultät für Mathematik	TU Dortmund
*	Prof. Dr. Jens Hornbostel	Fachgruppe Mathematik und Informatik	Bergische Universität Wuppertal
*	Prof. Dr. Annette Huber	Mathematisches Institut	Universität Freiburg
	Prof. Dr. Moritz Kerz	Fakultät für Mathematik	Universität Regensburg
*	Prof. Dr. Kai Köhler	Mathematisches Institut	Universität Düsseldorf
*	Prof. Dr. Jürg Kramer	Institüt für Mathematik	Humboldt Universität Berlin
*	Prof. Dr. Gerd Laures	Fakultät für Mathematik	Universität Bochum
*	Prof. Dr. Marc Levine	Fakultät Mathematik	Universität Duisburg-Essen
	Prof. Dr. Christian Liedtke	Fakultät für Mathematik	Technische Universität München
*	Prof. Dr. Hannah Markwig	Fachrichtung Mathematik	Universität des Saarlandes
*	Prof. Dr. Stefan Müller-Stach	Fachbereich Physik, Mathematik und Informatik	Universität Mainz
*	Prof. Dr. Niko Naumann	Fakultät für Mathematik	Universität Regensburg
*	Prof. Dr. Birgit Richter	Fachbereich Mathematik	Universität Hamburg
*	Prof. Dr. Oliver Röndigs	Fachbereich Mathematik/ Informatik	Universität Osnabrück
*	Prof. Dr. Andreas Rosenschon	Fakultät für Mathematik, Informatik und Statistik	LM Universität München
*	Dr. Steffen Sagave	Mathematisches Institut	Universität Bonn
*	Prof. Dr. Thomas Schick	Fachbereich Mathematik	Universität Göttingen
*	Prof. Dr. Alexander Schmidt	Mathematisches Institut	Universität Heidelberg
*	Prof. Dr. Alexander Schmitt	Fachbereich Mathematik und Informatik	Freie Universität Berlin
*	Dr. Jakob Scholbach	Mathematisches Institut	Universität Münster
	Prof. Dr. Peter Scholze	Mathematisches Institut	Universität Bonn
*	Prof. Dr. Stefan Schwede	Mathematisches Institut	Universität Bonn
*	Jun.-Prof. Dr. Nikita Semenov	Fachbereich Physik, Mathematik und Informatik	Universität Mainz
*	Jun.-Prof. Dr. Markus Spitzweck	Fachbereich Mathematik/ Informatik	Universität Osnabrück
*	Dr. Florian Strunk	Fakultät für Mathematik	Universität Regensburg
*	Dr. Georg Tamme, AR auf Zeit	Fakultät für Mathematik	Universität Regensburg
*	Prof. Dr. Michael Weiss	Mathematisches Institut	Universität Münster
*	Prof. Dr. Anna von Pippich	Fachbereich Mathematik	Universität Darmstadt
*	Prof. Dr. Konrad Waldorf	Institut für Mathematik und Informatik	Universität Greifswald
*	Dr. Matthias Wendt	Mathematisches Institut	Universität Freiburg
*	Prof. Dr. Annette Werner	Institut für Mathematik	Goethe Universität Frankfurt
*	Dr. Marcus Zibrowius	Fachgruppe Mathematik und Informatik	Bergische Universität Wuppertal

4 International Collaborators

• Belgium: J. Nicaise (Leuven)
• Canada: S. Gille (Edmonton), N. Karpenko (Edmonton), K. Zainoulline (Ottawa).
• France: B. Calmès (Lens), C. Cazenave (Nice), C.D. Cisinski (Toulouse), F. Déglise (Lyon), P. Gille (ENS), B. Kahn (Université Paris 7), B. Toën (Montpellier), J. Wildeshaus (Université Paris 13), O. Wittenberg (ENS).
• Spain: J. Burgos (Madrid), V. Navarro Aznar (Barcelona)
• India: A. Krishna (Tata Inst.), V. Srinivas (Tata Inst.).
• Japan: T. Geisser (Nagoya), L. Hesselholt (Nagoya), S. Saito (Tokyo), N. Yagita(Ibaraki University).
• Norway: B. Dundas (Bergen), J. Rognes (Oslo), C. Schlichtkrull (Bergen), P.A. Østvær (Oslo).
• Denmark: I. Madsen (Copenhagen), M. Szymik(Copenhagen)
• Russia: V. Kiritchenko (Moscow), I. Panin (St. Petersburg), S. Yagunov (St. Petersburg).
• Switzerland: J. Ayoub (Univ. Zürich), R. Pandharipande (ETH).
• UK: A. Baker (Glasgow), D. Benson (Aberdeen), V. Guletski (Liverpool), T. Pirashvili (Leicester), A. Robinson (Warwick), M. Schlichting (Warwick), N. Strickland (Sheffield), A. Vishik (Nottingham), Ranicki (Edinburgh), Tillmann (Oxford)
• USA: A. Asok(USC), C. Barwick (MIT), M. Basterra (New Hampshire), M. Behrens (MIT), E. Friedlander (USC), S. Garibaldi (Emory Univ.), P. Goerss (Northwestern), C. Haesemeyer (UCLA), M. Hill (Univ. Virginia), M. Hopkins (Harvard), D. Isaksen (Wayne State Univ.), T. Lawson (Univ. Minnesota), Y.P. Lee (Univ. Utah), J. Lurie (Harvard), M. A. Mandell (Univ. Indiana), J.P. May (Univ. Chicago), A. Merkurjev (UCLA), H. Miller (MIT), C. Rezk (Univ. Ill. Urbana-Champaign), B. Shipley (Univ. Ill. Chicago), Y.J. Tzeng (Harvard), C. Weibel (Rutgers).

5 Literature not in print

All works referred to in this application, which have not appeared as a published journal article, book chapter or book, are available for download, either from the Math ArXiv `http://arxiv.org/archive/math` using the arXiv identifier listed in the article reference or at the URL listed in the article reference. For the reader's convenience, we have also set up a web-site for the down-loading of all such material, at

`http://www.esaga.uni-due.de/marc.levine/SPP/`