Using Optically Induced Forces in Numerical Structural Optimization

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A numerical structural optimization procedure is presented that utilizes optically induced force fields as a "natural" strategy for shape deformations in photonic components. The proposed methodology has supposedly proven successful to reveal largest possible quality factors $Q$ for e.g. two-dimensional (2D) simply-connected dielectric microcavity volumes.

Summary

The force field is calculated from the Maxwell’s stress tensor $\vec{T}$ at the boundary of a dielectric particle that is illuminated by a TM-polarized plane wave at the particle’s resonance. The time-averaged optical force $\langle \vec{F} \rangle_t$ exerted on such object enclosed by the surface $S$ is calculated as [1, 2]:

$$\langle \vec{F} \rangle_t = \int_S \langle \vec{T} \cdot \vec{n} \rangle_t \, ds = \int_S \left\{ \frac{1}{2} \Re[(D \cdot \vec{n})E^*] - \frac{1}{4}(D \cdot E^*)n + \frac{1}{2} \Re[(B \cdot \vec{n})H^*] - \frac{1}{4}(B \cdot H^*)n \right\} ds \tag{1}$$

Shape relaxations of objects according to optically induced force fields are governed by energy minimization. Inverting the forces (i.e. the area-density of the force $\langle \vec{T} \cdot \vec{n} \rangle_t$ on the surface $S$) will therefore provide a ”natural” strategy for energy maximization. In the context of a resonant microcavity this is tantamount to tracking down the largest possible quality factor $Q$. Figure 1 shows the evolution of such an optimization scenario, where the boundary of a circular dielectric micropillar that supports a resonant whispering-gallery mode is deformed using a maximal stepsize of 0.1nm per iteration step. With this optimization strategy we were able to enhance the quality factor $Q$ by 40%. Further investigations include waveguides as well as grating structures.

Fig. 1. Computed distribution of the magnetic field $|H_z|$ within an evolving 2D dielectric micropillar ($r = 1\mu m$, $\epsilon_r = 10$, embedded in air) at three different iteration stages $N$. The structure is excited by a plane wave from the left at a resonance wavelength of 1649 nm.

References

Using Optically Induced Forces in Numerical Structural Optimization  
– A Quest for the Ultimate Optical Resonator –

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1. Introduction

A numerical structural optimization procedure is presented that utilizes optically induced force fields as a “natural” strategy for shape deformations in photonic components. The proposed methodology has supposedly proven successful to maximize quality factors \( Q \) for e.g. two-dimensional (2D) simply-connected dielectric micropillar volumes.

The force field is calculated from the Maxwell’s stress tensor \( \mathbf{T} \) at the boundary of a dielectric particle that is illuminated by a TM-polarized plane wave at the particle’s resonance. The time-averaged optical force \( \langle F \rangle \), exerted on such an object enclosed by the surface \( \mathcal{S} \) is calculated as [1, 2]

\[
\langle F \rangle = \oint_{\mathcal{S}} \mathbf{n} \cdot \mathbf{E} \times \mathbf{n} \cdot \mathbf{H} \, dS.
\]

Shape relaxations of objects according to optically induced force fields are governed by energy minimization. Inverting the forces (i.e. the area-density of the force \( F \cdot \mathcal{S} \), on the surface \( \mathcal{S} \)) will therefore provide a “natural” strategy for energy maximization. In the context of a resonant micropillar this is tantamount to tracking down largest possible quality factors \( Q \).

2. Applying Optically Induced Forces

A dielectric micropillar supporting different resonant whispering-gallery modes (WGMs) is being deformed according to the inverted optical force while following the iterative scheme shown in Fig. 1.

- start
- realize to resonance
- solve optical stress tensor
- apply optical forces to resonator boundaries
- reached some end criteria?

Figure 1: Numerical iteration scheme for resonator optimization

In Fig. 2 a) and c) the evolution of micropillars with WGM order of 10 and 16 can be observed. Figures 2 b) and d) compare different stepsizes per iteration.

Figure 2: Computed distribution of the magnetic field \( |\mathbf{H}| \) within an evolving 2D dielectric micropillar with initial \( r = 1000 \text{nm} \) (a-b) and \( r = 657 \text{nm} \) (c-d), embedded in air at three different iteration stages (a-d). The structure is excited by a plane wave from the left at a resonance wavelength of 1649nm (a-b) and 1550nm (c-d).

- Force induced modifications yield to a virtually identical relative increase of the quality factor irrespective the mode order (see Fig. 2 b) and d)).
- Conversely reasoning: the prior statement on force magnitude explains the high sensitivity of WGM resonators against small deformation and roughness.

Q: Is this the ultimate optical resonator?

3. Micрогear Resonator as a Reference System

In [3, 4] it was shown that microgear-shaped dielectric resonators exhibit comparable enhancements regarding the quality factor. They will therefore be investigated as a reference system. The iteration scheme in this case increases the grating depth gradually followed by a subsequent alteration of the aspect ratio (gap width to the grating period). For keeping the resonance wavelength constant the microgear is afterwards resized accordingly. Figure 3 show the resulting 2D-plot and some points of interest.

Figure 3: (a-b) Magnetic field \( |\mathbf{H}| \) distribution of some points of interest with increasing aspect ratios, 2D topology of the quality factor \( Q \), and \( Q_{\text{m}} \), with respect to the microgear resonator’s relevant shape parameters

- For certain aspect-ratios and grating depths Figs. 3, 4 and 5 show that the quality factor can be easily enhanced to even higher values (up to a value of 410% at a WGM order of 16) compared to the ‘force-shaped’ micropillars (max. 148%).
- The application of optically induced forces have, again, revealed a resonator with further increased quality factor \( Q \) before relaxing into the optimal “force-shaped” resonator topology.

Q: Is there such a thing like an ultimate optical resonator?

4. Evolving the Micрогear Resonator by Optical Forces

In the following investigation the optically induced forces are applied to the best performing microgear resonator (b) from Fig. 3 yielding an evolution as depicted in Fig. 6 and 7.

- From Section 2: Applying the inverted optical induced forces on a dielectric micropillar provide a “natural” strategy for optimizing the quality factor \( Q \).
- From Section 3: The microgear reference system provides even higher \( Q \) compared to the optimized micropillars.
- From Section 4: Applying optically induced forces to deform the best found microgear resonator yields even further increased \( Q \) factors.

Q: Where does this leave us?
Q: Is the “force strategy” just an optimality test?
Q: Are we tied up to full-blown numerical structural optimization?

5. Literature