The Armstrong-Frederick Plasticity model with Cosserat effects

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We propose an extension of equations formulated by Armstrong and Frederick which includes micropolar effects. We study existence of solutions to the quasistatic Armstrong-Frederick model with Cosserat effects which is still of non-monotone type. It was shown that the limit in the Yosida approximation process satisfies the energy inequality. The limit functions have a better regularity than it could be found in the literature, where the original Armstrong-Frederick model was studied.

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1 Formulation of the problem

The original Armstrong-Frederick model describes the inelastic deformation process in metals. It is a modification of Melan-Prager model, which is well know in the literature and it can be treated as an approximation of the Prandtl-Reuss model. The modification is to add a nonlinear correction term to the equations for the backstress. New term entails the Prager model, which is well known in the literature and it can be treated as an approximation of the Prandtl-Reuss model. The original Armstrong-Frederick model describes the inelastic deformation process in metals. It is a modification of Melan-

The system (1.1) is considered with Dirichlet boundary conditions for the displacement and the microrotation:

\[ \text{axl : tensor.} \]

\[ \text{due to the Cosserat effects.} \]

\[ \text{constants,} \]

\[ \text{dev (} \]

\[ \text{set of admissible stresses} \]

\[ \text{with the following initial-boundary value problem: we are looking for the displacement field} \]

\[ \text{used in practice. Unfortunately there is a drawback: This model is of non-monotone type and not of gradient type and the} \]

\[ \text{of the backstress. This new property is required in many applications, hence the Armstrong-Frederick model is very often} \]

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\[ \text{functions have a better regularity than it could be found in the literature, where the original Armstrong-Frederick model was studied.} \]

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Main result

Let us consider convex set (the construction of this set appears in [2] and it will be used as set of test functions further on)

\[ \mathcal{K}^* = \{ (\text{dev} (T_E), -\frac{1}{c}b) \in S_{\text{dev}}^3 \times S_{\text{dev}}^3 : |\text{dev} (T_E) - b| + \frac{d}{2c} |b|^2 \leq \sigma_y \}. \]

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\textbf{Definition 2.1} (solution concept – energy inequality)

Fix \( T > 0 \). Suppose that the given data satisfy some natural regularity, which are specified in [5]. We say that a vector \((u, T, A, \varepsilon, b) \in L^\infty (0, T; H^1 (\Omega; \mathbb{R}^3) \times L^2 (\Omega; S^3) \times H^1 (\Omega; \mathfrak{so}(3)) \times L^2 (\Omega; S_{\text{dev}}^3) \times L^\infty (\Omega; S_{\text{dev}}^3) \) solves the problem (1.1)-(1.3) if

\[ (u_t, T_t, A_t, \varepsilon_t, b_t) \in L^2 (0, T; H^1 (\Omega; \mathbb{R}^3) \times L^2 (\Omega; S^3) \times H^1 (\Omega; \mathfrak{so}(3)) \times (L^2 (\Omega; S_{\text{dev}}^3))^2), \]

the equations (1.1) and (1.1)_3 are satisfied pointwise almost everywhere on \( \Omega \times (0, T) \) and for all test functions \((\hat{T}_E, \hat{b}) \in L^2 (0, T; L^2 (\Omega; S^3) \times L^2 (\Omega; S_{\text{dev}}^3))\) such that

\[ (\text{dev} (\hat{T}_E), \hat{b}) \in \mathcal{K}^* , \quad \text{div} \hat{T}_E \in L^2 (0, T; L^2 (\Omega; \mathbb{R}^3)) \]

the inequality

\[ \begin{align*}
\frac{1}{2} \int_\Omega \mathbb{C}^{-1} T_E (x, t) T_E (x, t) dx + \mu_c \int_\Omega |\text{skew} (\nabla x u (x, t)) - A (x, t)|^2 dx \\
+ 2 \mu_c \int_\Omega |\nabla \text{axl} (A (x, t))|^2 dx + \frac{1}{2c} \int_\Omega |b (x, t)|^2 dx - \frac{1}{2} \int_\Omega \mathbb{C}^{-1} T_E (x) T_E (x) dx \\
+ \mu_c \int_\Omega |\text{skew} (\nabla x u (x, 0)) - A (x, 0)|^2 dx + \frac{1}{2c} \int_\Omega |b (x, 0)|^2 dx + 2 \mu_c \int_\Omega |\nabla \text{axl} (A (x, 0))|^2 dx \\
+ \int_0^t \int_\Omega u_t (x, \tau) f (x, \tau) dx d\tau + \int_0^t \int_\Omega u_t (x, \tau) \text{div} \hat{T}_E (x, \tau) dx d\tau \\
+ \int_0^t \int_{\partial \Omega} g_{D,E} (x, \tau) (T (x, \tau) - \hat{T}_E (x, \tau)) \cdot n (x) dS d\tau + \int_0^t \int_\Omega \mathbb{C}^{-1} T_E, t (x, \tau) \hat{T}_E (x, \tau) dx d\tau \\
+ \frac{1}{c} \int_0^t \int_{\partial \Omega} b_t (x, \tau) \hat{b} (x, \tau) dx d\tau + \frac{4 \mu_c}{c} \int_0^t \int_{\partial \Omega} \nabla \text{axl} (A (x, \tau)) \cdot n \text{axl} (A_{D,E} (x, \tau)) dS d\tau
\end{align*} \]

holds for all \( t \in (0, T) \), where \( T_E^0 \in L^2 (\Omega; S^3) \) and \((u(0), A(0)) \in H^1(\Omega; \mathbb{R}^3) \times H^1(\Omega; \mathfrak{so}(3)) \) are unique solution of the problem (1.1)_1 – (1.1)_3 at the time equals zero and \( \mathbb{C}^{-1} : S^3 \rightarrow S^3 \) is a positive definite operator such that \( \mathbb{C}^{-1} T_E = \varepsilon - \varepsilon^p \).

\textbf{Theorem 2.2} (Main result)

Let us assume that the given data and initial data satisfy some natural regularity, which are specified in [5]. Then there exists a global in time solution (in the sense of Definition 2.1) of the system (1.1) with boundary condition (1.2) and initial condition (1.3).

The proof of Theorem 2.2 is divided into two parts. First, we use the Yosida Approximation to the maximal monotone part of the inelastic constitutive equation. Next, we pass to the limit to obtain a solution in the sense of Definition 2.1. The details can be found in the article [5].

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\textbf{References}


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