Harish-Chandra modules and applications

Winter term 2015/16

Talk 1. Overview

Talk 2. Lie-groups, Lie-algebras and the exponential map

- Lie groups and their Lie algebras
- The exponential map
- Closed subgroups as Lie groups

Talk 3. Semi-simple Lie-algebras

- Solvable and semi-simple Lie algebras
- Killing form and Casimir operator
- Representations of semi-simple Lie algebras are totally reducible
- Cartan subalgebras

References: [Kna] Section 1

Talk 4. Representation theory of semi-simple Lie-algebras

- Roots and the Weyl group
- Highest weight classification
- Full proof in the case $\mathfrak{sl}_2$

References: [Kna] Section 2, [FH91] 11.1

Talk 5. Lie algebra cohomology

- Lie algebra cohomology as a derived functor
- Poincare-Berkhoff-Witt Theorem
- Chevalley-Eilenberg complex
- Comparison with deRham cohomology of connected compact Lie groups
- The relative case

References: [Wei95] Chapter 7.7, [Sol] Sections 2.1-2.3

Talk 6. Compact Lie groups

- Compact forms of semi-simple Lie algebras
- Maximal tori
- Weyl group of compact Lie group equals Weyl group of its Lie algebra
- Peter-Weyl Theorem

References: [Kna] Section 3, [Bum98] Chapter 2.4 for Peter-Weyl

Talk 7. Structure theory of noncompact semisimple groups

- Cartan deocmposition
- Iwasawa decomposition
- Bruhat decomposition
- Decomposition of Haar measures

References: [Kna] Section 4

Talk 8. From G-representations to $(\mathfrak{g}, K)$-modules

- Definition of (admissible) $(\mathfrak{g}, K)$-modules
- $(\mathfrak{g}, K)$-modules from Hilbert space representations

References: [Bum98] Chapter 2.4

Talk 9. Irreducible $(\mathfrak{g}, K)$-modules for $\mathrm{GL}_2(\mathbb{R})$

- Classification
- The $(\mathfrak{g}.K)$-module generated by a cusp form
- Lie algebra action and Shimura-Maass operators
- If time permits: globalization and unitarizability

References: [Bum98] Chapter 2, mainly Chapter 2.5

Talk 10. Relative Lie algebra cohomology

- Vanishing theorems
- Zuckerman functors
- Poincare duality

References: [BW00] Chapter 1 and 2

Talk 11. Scalar product, Laplacian and Casimir Element

- Hodge theory of $(\mathfrak{g}, K)$-cohomology
- Matsushima's vanishing theorem

References: [BW00] Chapter 2

Talk 12. Discreteness of the spectrum

- Full reducibility in the cocompact case

References: [Bum98] Chapter 2.4

Talk 13. Cohomology of discrete subgroups

- Comparison of Betti/deRham cohomology with group cohomology
- Full discussion of the cocompact case

References: [BW00] Chapter 7

Talk 14. Cuspidal cohomology

- Cusp forms
- Differential forms with growth conditions
- Cohomological cuspidal representations show up in cohomology

References: [Bor81]

References

[Bor81] A. Borel. Stable real cohomology of arithmetic groups II. In Jun-ichi Hano, A. Morimoto, S. Murakami, K. Okamoto, and H. Ozeki, editors, *Manifolds and Lie Groups*, volume 14 of *Progress in Mathematics*, pages 21–55. Birkhäuser Boston, 1981.

[Bum98] D. Bump. *Automorphic Forms and Representations*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 1998.

[BW00] A. Borel and N.R. Wallach. *Continuous Cohomology, Discrete Subgroups, and Representations of Reductive Groups*. Mathematical surveys and monographs. American Mathematical Society, 2000.

[FH91] W. Fulton and J. Harris. *Representation Theory: A First Course*. Graduate Texts in Mathematics / Readings in Mathematics. Springer New York, 1991.

[Kna] Anthony W. Knapp. Structure theory of semisimple lie groups. notes.

[Sol] Maarten Solleveld. Lie algebra cohomology and Mcdonalds conjecture. Master thesis.

[Wei95] C.A. Weibel. *An Introduction to Homological Algebra*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 1995.