

Automorphic representations - Summer term 2017

Talk 1. Topological preliminaries

- Topological groups
- Haar measures and other invariant measures
- L^2 and Hilbert spaces
- Representations of compact groups

Talk 2. Representations of \mathfrak{l} -groups

- Smooth/admissible representations
- Hecke algebras
- (compact) Induction/ Frobenius reciprocity
- Schur's Lemma

References: [Bum98] Section 4.2 and Proposition 4.5.1

Talk 3. Sheaves and distributions - Overview

- Preliminaries on distributions
- Uniqueness of Whittaker models

References: [Bum98] Section 4.3 and Theorem 4.4.1

Talk 4. Whittaker models and Jacquet functor

- (Twisted) Jacquet functor
- Existence of Whittaker models for GL_2

References: rest of [Bum98] Section 4.4

Talk 5. Principal series

- Iwasawa decomposition
- Parabolic induction
- Irreducibility of principal series (for GL_2)

References: [Bum98] Section 4.5 until Theorem 4.5.2

Talk 6. Spherical representations

- Cartan decomposition
- Spherical Hecke algebra
- Classification of spherical representations (for GL_2)

References: Section 4.6 until Theorem 4.6.4

Talk 7. New forms

- Casselman's representation theoretic version of Atkin-Lehner theory

References: [Cas73]

Talk 8. Adeles and ideles

- Ostrowski's theorem
- \mathbb{A}_K/K is compact
- \mathbb{I}_1^*/K^* is compact

References: [Neu06] Satz 3.7, Theorem 1.5 und 1.6

Talk 9. Pontryagin duality - Overview

- Dual groups
- Pontryagin duality and Fourier inversion (without proof)
- Examples: duals of local and global fields

Talk 10. Tate's thesis - Overview

- Poisson summation formula
- Local/global functional equation

- Analytic continuation

References: [Bum98] Section 3.1

Talk 11. Classical automorphic representations

- Space of cusp forms is totally reducible
- Smooth vectors are dense
- Finite multiplicity

References: [Bum98] Proposition 3.2.3, Theorem 3.2.2 and Theorem 3.2.3

Talk 12. Adelic automorphic representations

- Strong approximation
- Space of cusp forms is totally reducible

References: [Bum98] Proposition 3.3.1-3.3.3 and Theorem 3.3.1, 3.3.2 and 3.3.4

Talk 13. Tensor product theorem

- Tensor product theorem

References: [Bum98] Section 3.4

Talk 14. Whittaker models and adelization of modular forms

- Global Whittaker models
- Multiplicity one
- The representations associated to a (classical) new form

References: [Bum98] Theorem 3.5.4, Theorem 3.5.5, Theorem 3.3.6 and Section 3.6

Talk 15. L-Functions for $GL(2)$

- Functional equation
- Analytic continuation
- Local L -factors

References: [Bum98] Section 3.5

REFERENCES

- [Bum98] D. Bump. *Automorphic Forms and Representations*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 1998.
- [Cas73] W. Casselman. On some results of Atkin and Lehner. *Mathematische Annalen*, 201(4):301–314, 1973.
- [Neu06] J. Neukirch. *Algebraische Zahlentheorie*. Springer-Lehrbuch Masterclass. Springer Berlin Heidelberg, 2006.