

Tame derivatives and the Eisenstein ideal

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Abstract

As was made famous by Mazur, the mod-5 Galois representation associated to the elliptic curve $X_0(11)$ is reducible. Less famously, but also noted by Mazur, the mod-25 Galois representation is reducible. We'll talk about this kind of extra reducibility phenomenon more generally, for cuspforms of even weight k and prime level. We'll observe that the characters appearing in the reducible representation are related, on one hand, to an algebraic invariant (the 'tame derivative' of an L -function), and, on the other hand, to an algebraic invariant (the 'tame \mathcal{L} -invariant'). This type of 'algebraic=analytic' relation is predicted by a version of the Bloch–Kato conjecture for families of motives formulated by Kato.