Formal groups and Tate cohomology in local fields

In 2004 Köck has shown that in a Galois extension L/K of local fields with group G the ideals $\mathfrak{P}_L^n$ of L, where $\mathfrak{P}_L$ is the maximal ideal of L, are cohomologically trivial if and only if the extension is weakly ramified and $n \equiv 1 \; mod \; |G_1|$ where G_1 is the first ramification group.
In the case of local number fields the ideals are (at least for large n) isomorphic to the higher unit groups $U_L^n = 1 + \mathfrak{P}_L^n$. So this characterization of cohomological triviality is also true for those higher unit groups.
We show that it remains valid for small n (other than $n = 1$) and also for local function fields. We further give in the local number field case a generalization to any formal group over the integers of the lower field. Finally we give application to elliptic curves and ray class groups.