ERRATUM:

P-ALCOVES AND NONEMPTINESS OF AFFINE DELIGNE-LUSZTIG VARIETIES

P-ALCÔVES ET VACUITÉ DE VARIÉTÉS DE DELIGNE-LUSZTIG AFFINES

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Abstract.

In this erratum, we would like to rectify two errors in our paper [2]. In both cases, the mistake has no consequences for other parts of the paper; only the statements of the corrected propositions below are used.

Reduction to adjoint groups. Proposition 2.2.1 does not hold as stated, and should be replaced by the following:

Proposition 0.0.1. *Assume that* $\operatorname{char}\Bbbk$ *does not divide the order of* $\pi_1(\mathbf{G}_{\mathrm{ad}})$. *Let* $\lambda \in \pi_0(\mathrm{Flag}) = \pi_1(\mathbf{G})_\Gamma$, *denote by* λ_{ad} *its image under the map* $\pi_0(\mathrm{Flag}) \to \pi_0(\mathrm{Flag}_{\mathrm{ad}})$, *and denote by* Flag_λ *and* $\mathrm{Flag}_{\mathrm{ad},\lambda_{\mathrm{ad}}}$ *the corresponding connected components. Then the projection* $\mathbf{G} \to \mathbf{G}_{\mathrm{ad}}$ *induces an isomorphism*

$$\mathrm{Flag}_\lambda \xrightarrow{\cong} \mathrm{Flag}_{\mathrm{ad},\lambda_{\mathrm{ad}}}.$$

If the map $\pi_0(\mathrm{Flag}) \to \pi_0(\mathrm{Flag}_{\mathrm{ad}})$ *is injective, then the homomorphism* $\mathbf{G} \to \mathbf{G}_{\mathrm{ad}}$ *induces an immersion*

$$\mathrm{Flag} \to \mathrm{Flag}_{\mathrm{ad}}.$$

The proof given in the paper proves the above statement. The problem with the original statement is that the map $\pi_0(\mathrm{Flag}) \to \pi_0(\mathrm{Flag}_{\mathrm{ad}})$

U. G. was partially supported by the Sonderforschungsbereich TR 45 "Periods, Moduli spaces and Arithmetic of Algebraic Varieties" of the Deutsche Forschungsgemeinschaft.

X. H. was partially supported by NSF DMS-1463852 and DMS-1128155 (from IAS).

S. N. was supported in part by QYZDB-SSW-SYS007 and NSFC grant (No. 11501547 and No. 11621061).

is *not injective in general*; this was erroneously claimed in part (2) of the original statement, and implicitly used in part (1).

Assume that $\mathbf{G}$ is semisimple. In this case the map $\pi_0(\mathrm{Flag}) \to \pi_0(\mathrm{Flag}_{\mathrm{ad}})$ is injective if and only if the coinvariants $X_*(T)_\Gamma$ are torsion-free (which is true for instance, if G is of adjoint type or simply connected, see [1] 4.4.16). In fact, using the notation of [3], we can identify

$$\pi_0(\mathrm{Flag}) = \pi_1(G)_\Gamma = X^*(\widehat{Z}(G)^\Gamma) = X_*(T)_\Gamma / X_*(T_{\mathrm{sc}})$$

(and likewise for G_{ad}), see loc. cit., page 196. In view of the commutative diagram

$$\begin{array}{ccccc} X_*(T_{\mathrm{sc}})_\Gamma & \hookrightarrow & X_*(T)_\Gamma & \longrightarrow & X_*(T)_\Gamma \otimes_{\mathbb{Z}} \mathbb{Q} \\ \downarrow{=} & & \downarrow & & \downarrow{\cong} \\ X_*(T_{\mathrm{sc}})_\Gamma & \hookrightarrow & X_*(T_{\mathrm{ad}})_\Gamma & \hookrightarrow & X_*(T_{\mathrm{ad}})_\Gamma \otimes_{\mathbb{Z}} \mathbb{Q} \end{array}$$

we see that

$$\ker(\pi_0(\mathrm{Flag}) \to \pi_0(\mathrm{Flag}_{\mathrm{ad}})) \cong X_*(T)_{\Gamma,\mathrm{tors}},$$

the torsion subgroup of $X_*(T)_\Gamma$.

Some properties on Newton points. In Proposition 3.5.1, the assumption that $[b]$ be basic is missing. The correct statement is

Proposition 0.0.2. *Let $[b]$ be a basic σ-conjugacy class in $\mathbf{G}(\mathbb{L})$ and $J \subset S$ with $\delta(J) = J$. Then $[b] \cap \mathbf{M}_J(\mathbb{L})$ contains at most one σ-conjugacy class of $\mathbf{M}_J(\mathbb{L})$.*

This statement is justified by the proof of the proposition given in [2]. The problem in the non-basic case is that in line 3 we can only really conclude that $\bar{\nu}_x = \bar{\nu}_{x'}$, i.e., that the *dominant* Newton vectors of x and x' coincide. However, in the sequel of the proof we use the stronger statement that $\nu_x = \nu_{x'}$. If x lies in a *basic* σ-conjugacy class, then $\nu_x = \bar{\nu}_x$ since $\bar{\nu}_x$ is central.

It is easy to give counterexamples to the statement for non-basic b, e.g., take $\mathbf{G} = GL_2$, $J = \emptyset$, i.e., $\mathbf{M} = \mathbf{M}_J$ is the diagonal torus. Then the diagonal matrices with entries $(\epsilon, 1)$, and $(1, \epsilon)$, resp., are σ-conjugate in $\mathbf{G}(L)$, but not in $\mathbf{M}(L)$.

Acknowledgments. We thank Haifeng Wu for discussions aound Proposition 2.2.1. We thank Eva Viehmann for pointing out the mistake in Proposition 3.5.1 and providing the counterexample given above.

References

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