Exercise 4
Let $K$ be a local field, let $L/K$ be a finite unramified extension, and let $M/K$ be an arbitrary finite extension. Show that $ML/M$ is unramified.

Exercise 5
Let $K$ be a local field. A monic polynomial $f = X^n + a_{n-1}X^{n-1} + \cdots + a_0 \in O_K[X]$ is called an Eisenstein polynomial, if $a_0, \ldots, a_{n-1} \in m$, $a_0 \notin m^2$, where $O_K$ denotes the ring of integers of $K$, and $m$ is its maximal ideal. Prove that

1. Every Eisenstein polynomial $f$ is irreducible over $K$. If $\alpha$ is a root of $f$, then the extension $K(\alpha)/K$ is totally ramified, and $\alpha$ is a uniformizer in $K(\alpha)$.

2. If $L/K$ is a totally ramified separable extension of degree $n$, and $\pi \in L$ is a uniformizer, then the minimal polynomial of $\pi$ over $K$ is an Eisenstein polynomial of degree $n$.

Exercise 6
Let $K$ be a local field with ring of integers $O$ and residue field $k$, and let $\pi \in O$ be a uniformizer. Set $U = O^\times$, and for each $i \geq 1$, set $U_i = \{ u \in U; \ u \equiv 1 \mod \pi^i\}$.

We obtain a filtration $F^\times \supset U_2 \supset U_1 \supset U_0 \supset \cdots$. Prove that

1. $F^\times/U \cong \mathbb{Z}$,
2. $U/U_1 \cong k^\times$,
3. for all $i \geq 1$, $U_i/U_{i+1} \cong k$.

Show that for every $i \geq 1$ and every prime number $\ell$ different from the characteristic of $k$, the map $U_i \to U_i$, $u \mapsto u^\ell$ is an isomorphism.

Exercise 7
Let $K$ be a local field. If $\text{char} \ K = 0$, then $(K^\times)^n$ is an open subgroup of $K^\times$ for every $n \geq 1$. If $\text{char} \ K = p > 0$, then $(K^\times)^n$ is an open subgroup of $K^\times$ if and only if $p \mid n$.