

**Algebraic Geometry 2, Exercises**

*Sheet 7, due May 31*

**Exercise 19**

Let  $k$  be an algebraically closed field, let  $f(X) \in k[X]$  be a polynomial, and let  $V = V(Y^2 - f(X)) \subset \mathbb{A}_k^2$ . Prove that  $V$  is smooth over  $k$  if and only if  $f$  has no multiple zero.

**Exercise 20**

Let  $k$  be an algebraically closed field of characteristic  $\neq 2$ . Let  $Q = V_+(X_0^2 + \cdots + X_i^2) \subset \mathbb{P}_k^n$  be a quadric of dimension  $n - 1$ . Prove that  $Q$  is smooth if and only if  $i = n$ .

**Exercise 21**

Let  $p$  be a prime number. Prove that the morphism  $\text{Spec } \mathbb{F}_p(T) \rightarrow \text{Spec } \mathbb{F}_p(T^p)$  is not smooth.