

Algebraic Geometry 2, Exercises

Sheet 8, due June 7

Exercise 22

Show that the following morphisms are non-smooth morphisms between regular schemes.

1. $\text{Spec } \mathbb{Z}[T]/(T^2 + 1) \rightarrow \text{Spec } \mathbb{Z}$,
2. $\text{Spec } R[T_1, \dots, T_n]/(T_1 \cdots T_n - \pi) \rightarrow \text{Spec } R$, where R is a discrete valuation ring with uniformizer π , $n \geq 2$,
3. $\mathbb{A}_k^m \rightarrow \mathbb{A}_k^n$, $(x_1, \dots, x_m) \mapsto (x_1, \dots, x_m, 0, \dots, 0)$, where k is a field and $0 \leq m < n$.

Exercise 23

Give an example of a field extension L/K and a normal K -scheme X , such that $X \times_{\text{Spec } K} \text{Spec } L$ is not normal.

Exercise 24

Let A be a domain, such that all localizations of A with respect to prime ideals are unique factorization domains (e.g. if A is a unique factorization domain, or if A is regular). Show that

$$A = \bigcap_{\mathfrak{p}} A_{\mathfrak{p}}$$

where the intersection is taken inside the field of fractions of A , and the index set is the set of prime ideals $\mathfrak{p} \subset A$ with $\dim A_{\mathfrak{p}} = 1$.

Hint: First reduce to the case that A is a unique factorization domain.

Remark: The result holds more generally if A is only assumed to be an integrally closed domain.