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Problem sheet 1

Due date: Oct. 20, 2017.

Let K be a field which is complete with respect to a non-trivial non-archimedean absolute value.

Problem 1

Prove that K with the topology induced by the absolute value is totally disconnected, i.e., whenever $X \subseteq K$ is a subset containing at least 2 elements, then X is disconnected.

Problem 2

(1) Assume for simplicity that K is algebraically closed, so that we can identify $\mathbb{B}^n = \operatorname{Spm} \mathbb{T}_n$. Prove that rational subdomains are open in $\mathbb{B}^n$ (with respect to the topology induced by the absolute value on K) and form a basis of this topology.

(2) Give an example of functions $g, f_1, \dots, f_r \in \mathbb{T}_1$ such that

$$\{x \in \operatorname{Spm} \mathbb{T}_1;\ \forall i : |f_i(x)| \leqslant |g(x)|\}$$

is not open (and in particular, is not a rational subdomain).

Problem 3

Let $n \geqslant 1$. Show that the polynomial ring $K[T_1, \dots, T_n]$ is not a Tate algebra over K.

Hint: One way to argue is to use that $K[T_1, \dots, T_n]^\times = K^\times$.

Problem 4

Let A be a Tate algebra over K, $X = \operatorname{Spm} A$, and let $f \in A$. Then

$$X = X\left(\frac{f}{1}\right) \cup X\left(\frac{1}{f}\right)$$

is a covering of X by rational subdomains. Check that the statement of Tate's acyclicity theorem holds for this covering.