Perfectoid spaces PD Dr. A. Chatzistamatiou
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Problem sheet 11

Due date: January 12, 2018

Problem 41

Let K be a non-archimedean field. Show that $\mathrm{Spa}(K, K^o)$ consists of one point.

Problem 42

Let K be a perfectoid field. Show that there is exactly one preimage for the Gaußpoint under the map

$$g : \mathrm{Spa}(K\langle t^{\frac{1}{p^\infty}}\rangle, K^o\langle t^{\frac{1}{p^\infty}}\rangle) \to \mathrm{Spa}(K\langle t\rangle, K^o\langle t\rangle).$$

Problem 43

Let (A, A^+) be a Huber pair. Denote by $\mathrm{Spa}(A, A^+)_a \subseteq \mathrm{Spa}(A, A^+)$ the subset of those points whose support is *not open* in A (these points are called the *analytic points* of $\mathrm{Spa}(A, A^+)$).

Now assume that A is a Tate ring. Show that there are natural homeomorphisms

$$\mathrm{Spa}(A, A^+) = \mathrm{Spa}(A, A^+)_a \cong \mathrm{Spa}(A^+, A^+)_a.$$

Problem 44

Let (A, A^+) be a Huber pair, and assume that A is a Tate ring. Prove that an element $f \in A$ is topologically nilpotent, if and only if $|f(x)|^n \to 0$ as $n \to \infty$ for all $x \in \mathrm{Spa}(A, A^+)$.

Hint. Let $g \in A$ denote a topologically nilpotent unit. Use that $X = \mathrm{Spa}(A, A^+)$ is quasi-compact together with a suitable open covering of X in order to show that for n sufficiently large, $|f(x)|^n < |g(x)|$ holds for all x.