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Problem sheet 2

Due date: Oct. 27, 2017.

Problem 5

Let K be a non-archimedean field.

(1) Show that K° is a ring and $K^{\circ\circ} \subset K^\circ$ is the only maximal ideal.

(2) For any $x, y \in K^\circ$, we have $(x) \subset (y)$ or $(x) \supset (y)$.

Problem 6

Let K be an algebraically closed field which is complete with respect to a non-archimedean absolute value. Let $f\colon K \to K$ be a bounded function which is represented by a formal power series converging on all of K. Prove that f is constant ("Liouville's theorem").

Problem 7

Let K be an algebraically closed field which is complete with respect to a non-archimedean absolute value. Denote by $\mathcal{O}_K$ its valuation ring, and write $\overline{z}$ for the image of $z \in \mathcal{O}_K$ in the residue class field κ. Identify $\operatorname{Spm} \mathbb{T}_1 = \mathbb{B}^1(K)$.

(1) Which of the following subsets are rational subdomains?

   (a) $\{0\}$,
   (b) $\{z \in \mathbb{B}^1(K);\ |z| < 1\}$,
   (c) $\{z \in \mathbb{B}^1(K);\ |z| = 1\}$,
   (d) $\{z \in \mathcal{O}_K;\ \overline{z} = 1\}$,
   (e) $\{z \in \mathbb{B}^1(K);\ z \neq 0\}$.

(2) Use Tate's acyclicity theorem to show that $\mathbb{B}^n$ is not a disjoint union of finitely many affinoid subdomains ($\neq \mathbb{B}^n$).

Problem 8

What is the completion of the non-archimedean field $F(t)$ from [Perfectoid Spaces], Example (1.1.1)? Describe the valuation ring and the residue field.