Perfectoid spaces PD Dr. A. Chatzistamatiou
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Problem sheet 3

Due date: Nov. 3, 2017.

Problem 9

Suppose $f : S \to R$ is a surjective morphism of rings with nilpotent kernel. Then f_{perf} and f^{perf} are isomorphisms.

Problem 10

Show that the morphism $f : \mathbb{F}_p[t^{\frac{1}{p^\infty}}] \to \Big(\mathbb{F}_p[t^{\frac{1}{p^\infty}}]/(t)\Big)^{\mathrm{perf}}$, induced by $t \mapsto (t^{\frac{1}{p^n}})_{n\geqslant 0}$, yields an isomorphism between the t-adic completion of $\mathbb{F}_p[t^{\frac{1}{p^\infty}}]$ and $\Big(\mathbb{F}_p[t^{\frac{1}{p^\infty}}]/(t)\Big)^{\mathrm{perf}}$.

Problem 11

Let K be an algebraically closed field with a complete non-archimedean absolute value. Let $\mathbb{A}^1_K$ denote the affine line considered as a rigid analytic space over K.

(1) Prove that $\Gamma(\mathbb{A}^1_K, \mathcal{O}_{\mathbb{A}^1_K})$ is the ring of power series with coefficients in K which converge on all of K.

(2) Prove that $\mathbb{A}^1_K$ is not isomorphic, as a rigid space, to the open unit disk. (*Hint:* You can use Liouville's Theorem, Pbm. 6.)

Problem 12

Consider the group $\Gamma = \mathbb{R}_{>0} \times \mathbb{R}_{>0}$ with the lexicographic ordering, i.e.,

$$(a, b) < (a', b') \quad :\Longleftrightarrow \quad a < a' \text{ or } (a = a' \text{ and } b < b').$$

Let $\gamma = (1, \frac{1}{2}) \in \mathbb{R}_{>0} \times \mathbb{R}_{>0}$. For $r \in \mathbb{R}_{>0}$, we write r for $(r, 1)$. Check that $r < \gamma < 1$ for all real numbers $0 < r < 1$.

We extend the multiplication ($0x = 0$ for all x) and the ordering ($0 < x$ for all $x \in \Gamma$) to the disjoint union $\Gamma \sqcup \{0\}$ in the obvious way.

Now let K be a complete non-archimedean field. Consider the map

$$v\colon K\langle T\rangle \to \Gamma \sqcup \{0\}, \quad \sum a_n T^n \mapsto \max_{n \geqslant 0} |a_n| \gamma^n.$$

Show that v is multiplicative and satisfies the strong triangle inequality.